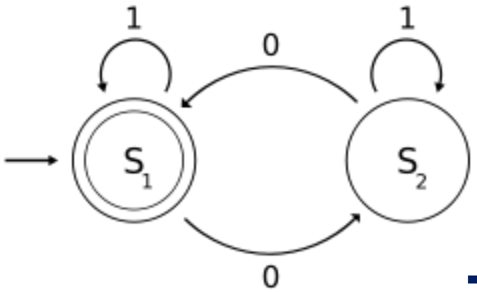
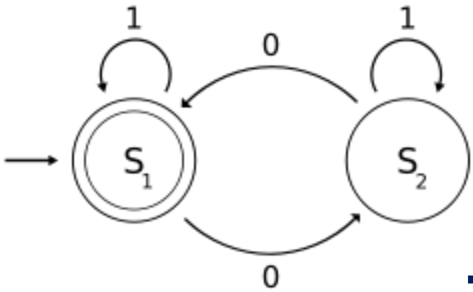


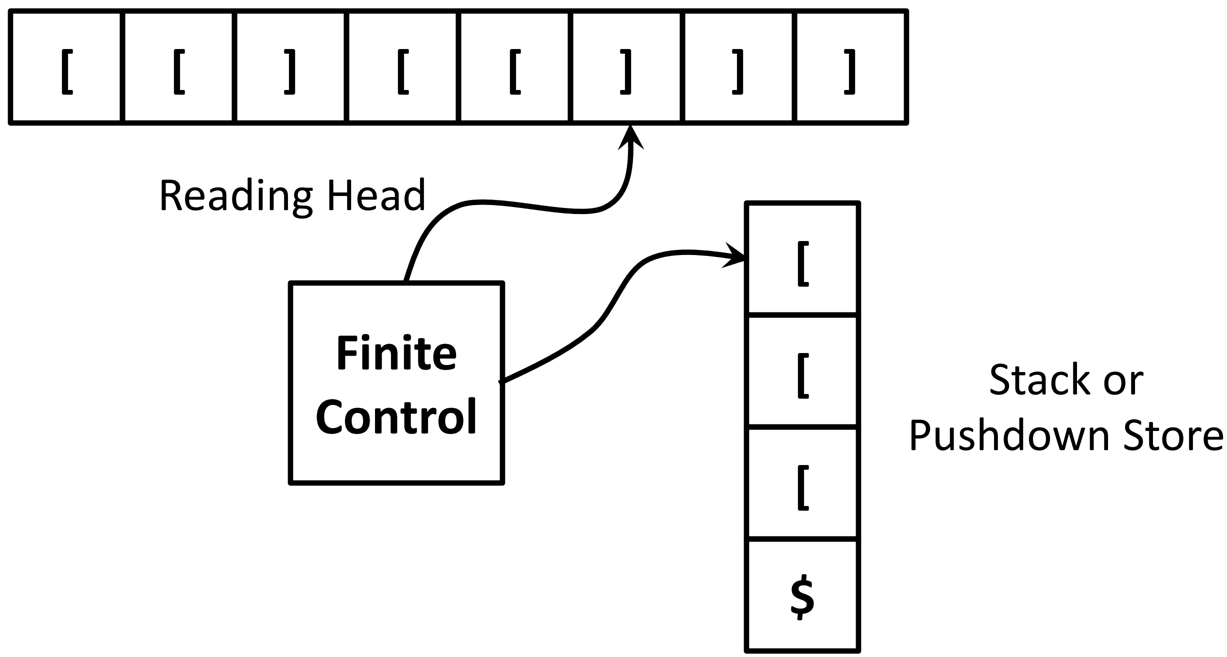
Pushdown Automata

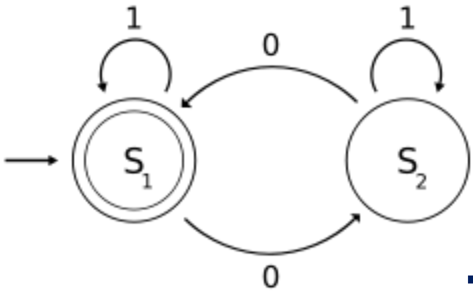
Sipser: Section 2.2 pages 111 - 116



Pushdown Automaton

The last left bracket seen matches the first right bracket.
This suggests a stack storage mechanism.





Pushdown Automata

A ***pushdown automaton*** is a sextuple $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$, where

Q is a finite set of *states*,

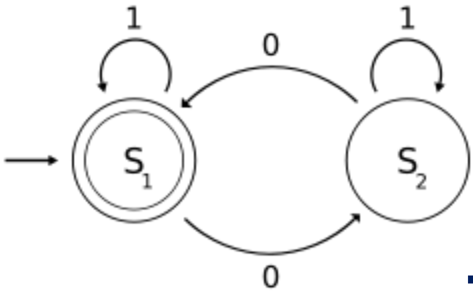
Σ is a finite alphabet (the *input symbols*),

Γ is a finite alphabet (the *stack symbols*),

δ: $(Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon) \rightarrow P(Q \times \Gamma_\varepsilon)$ is the *transition function*,

$q_0 \in Q$ is the *initial state*, and

$F \subseteq Q$ is the set of *accept states*.



Balanced Brackets

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$, where

$Q = \{q_1, q_2, q_3\}$,

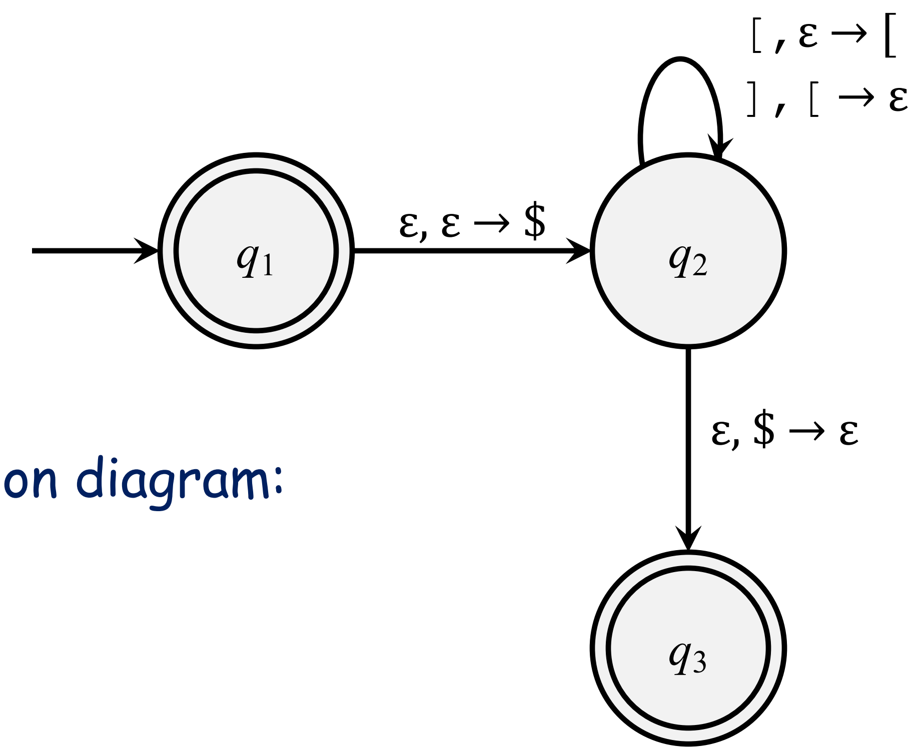
$\Sigma = \{[,]\}$,

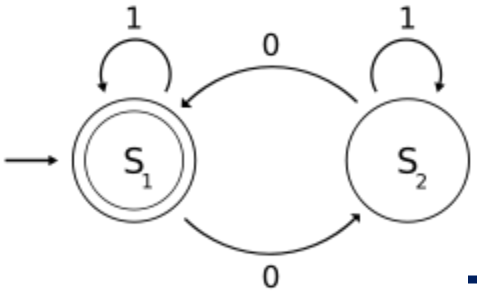
$\Gamma = \{ [, \$ \}$,

$q_0 = q_1$,

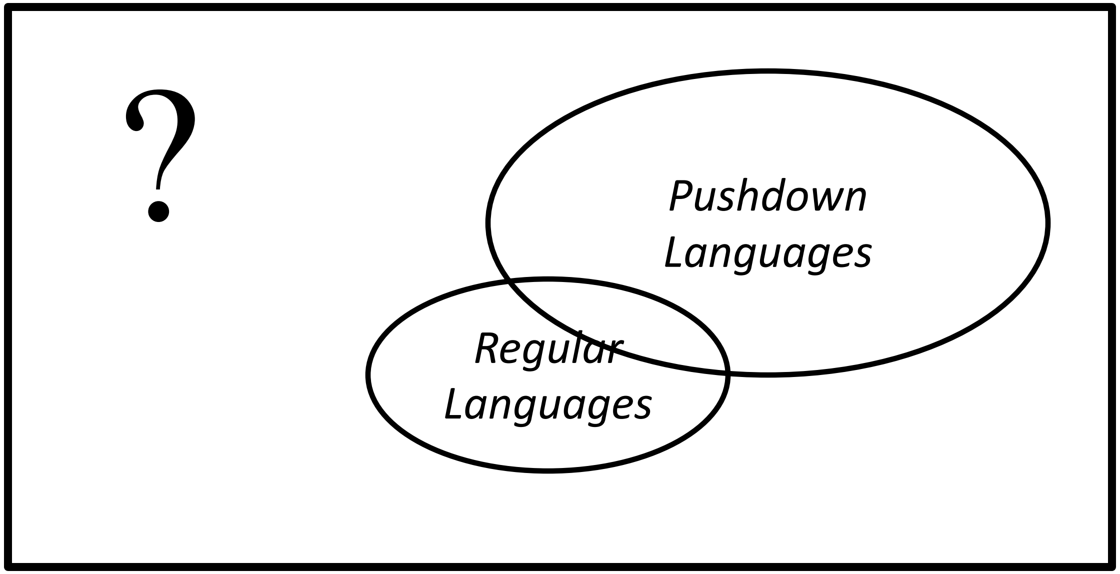
$F = \{q_1, q_3\}$, and

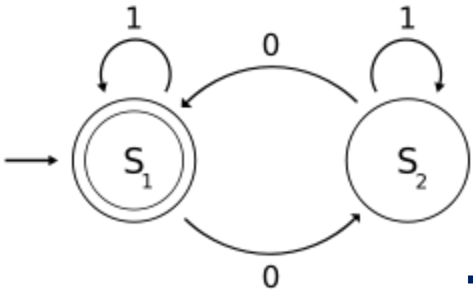
δ is given by the transition diagram:





Finite Automata and Pushdown Automata

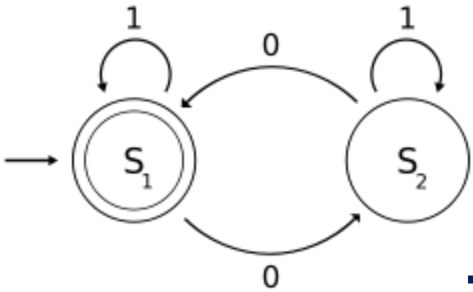




Regular Languages ⇒ Pushdown Accept

Proposition. Every finite automaton can be viewed as a pushdown automaton that never operates on its stack.

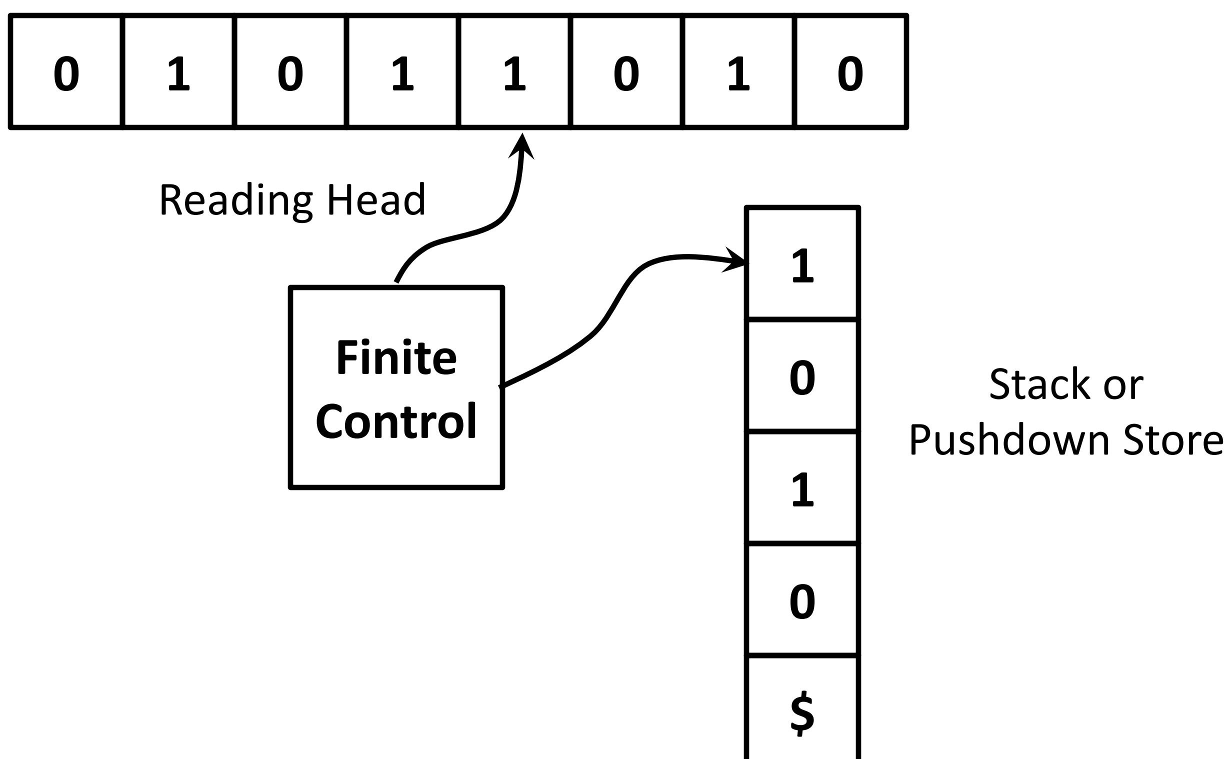
Proof. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton. Define $M' = (Q, \Sigma, \Gamma, \delta', q_0, F)$, where …

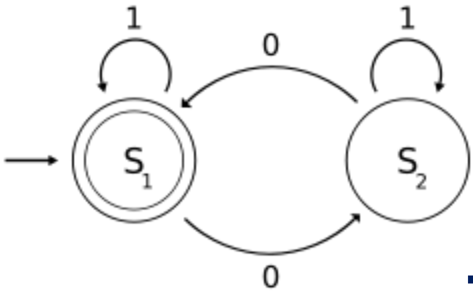


Pushdown Automata are Nondeterministic

Build a machine to recognize

$L(G) = \{\ ww^R \mid w \in \{0,1\}^*\ \}$

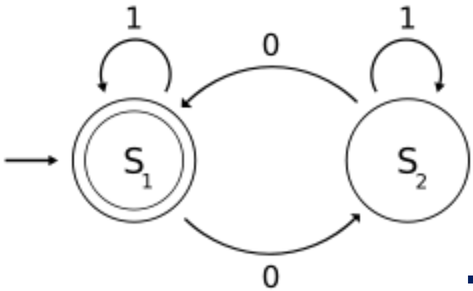




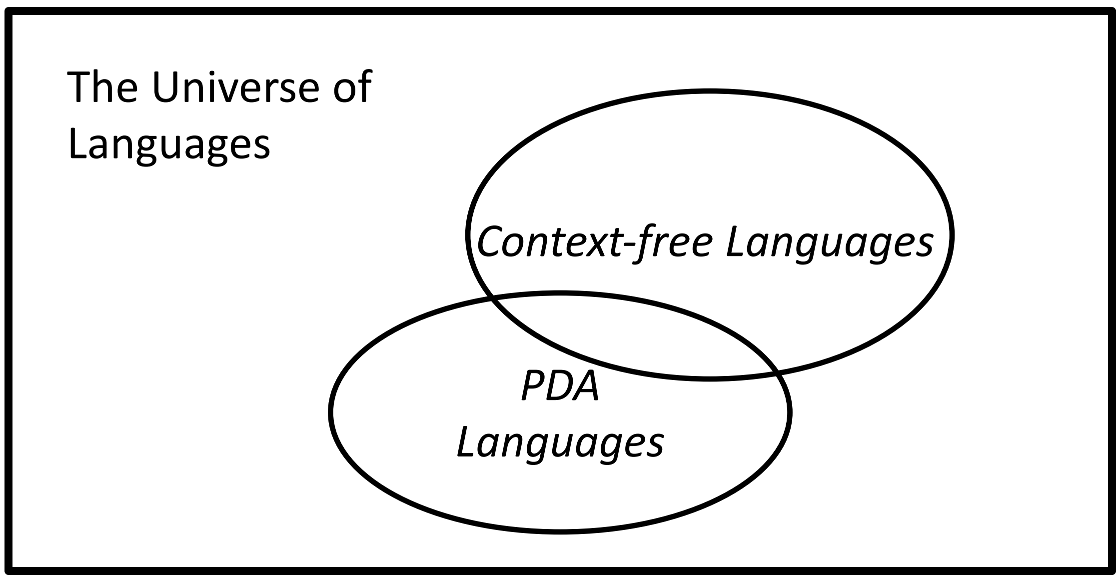
Pushdown Automata are Nondeterministic

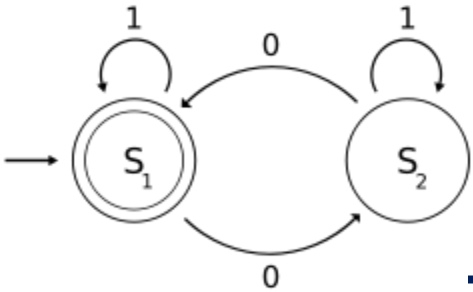
Build a machine to recognize

$$L(G) = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k \}$$



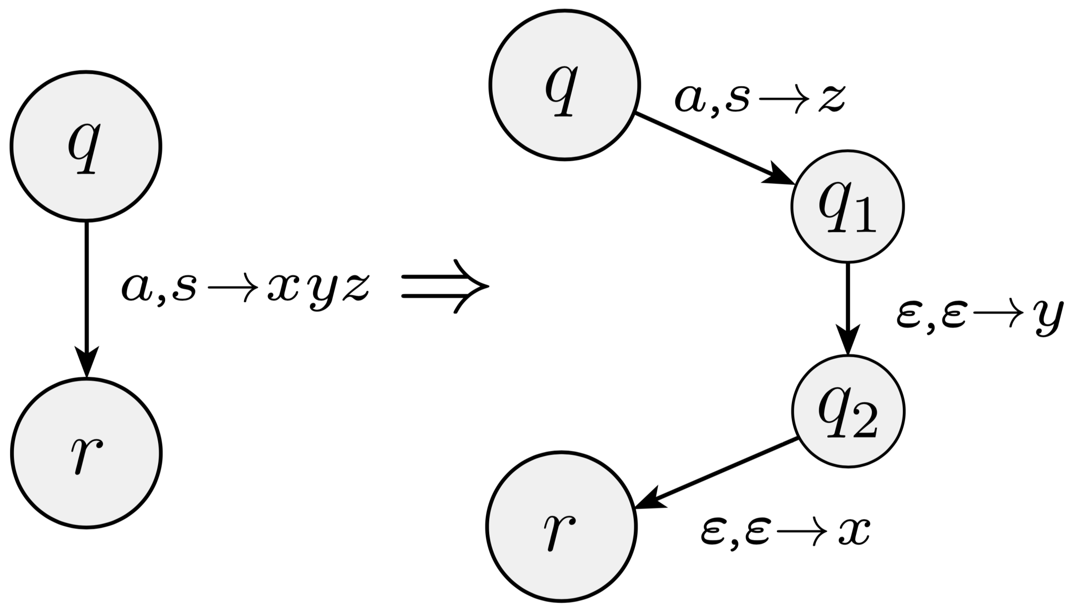
Context-free generation and pushdown recognition

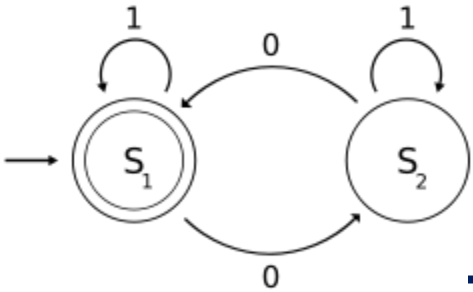




Writing Strings to Stack

Suppose we want to write strings (multiple characters) to a PDA stack...

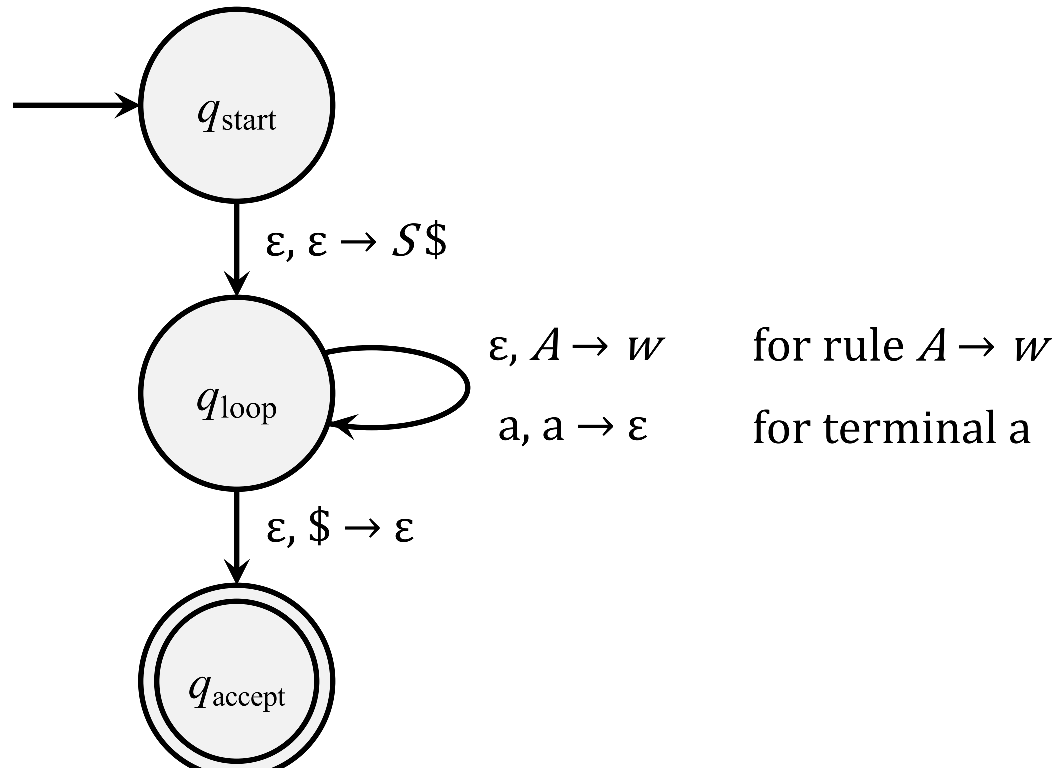


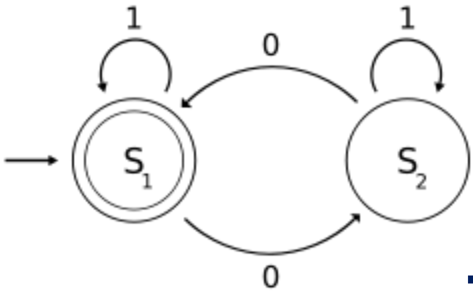


Recognizing Context-Free Languages

Lemma. If a language is context-free, then some pushdown automaton recognizes it.

Proof.





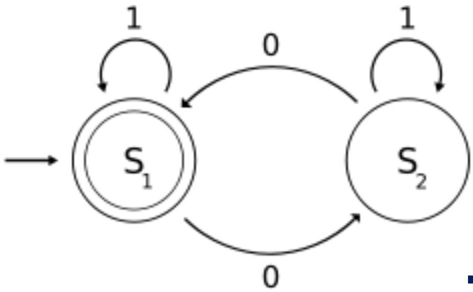
For Example

We apply this construction to $G = (V, \Sigma, R, S)$, where

$V = \{S\}$,

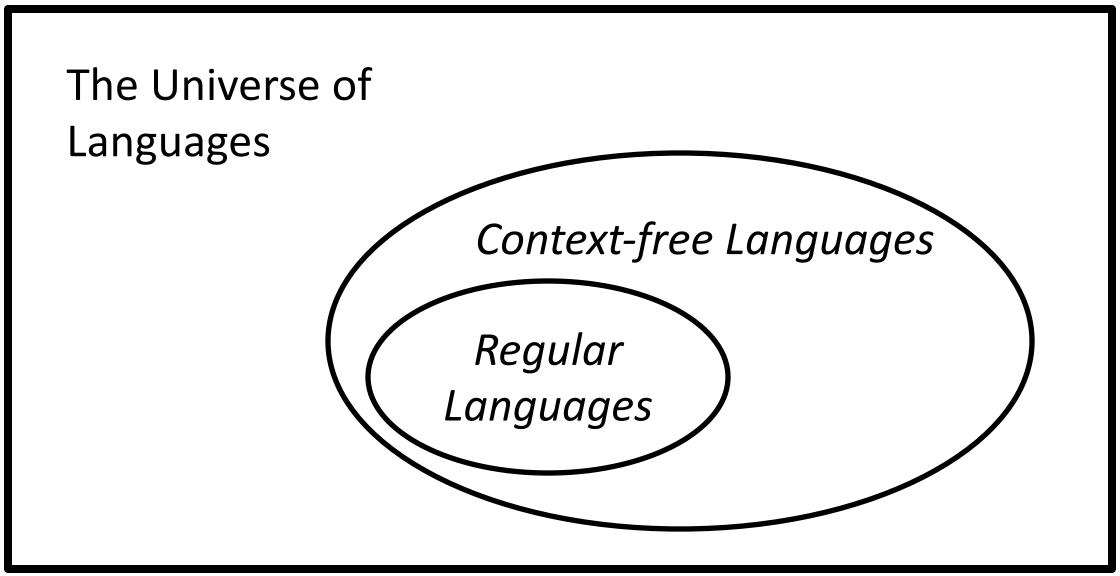
$\Sigma = \{[,]\}$,

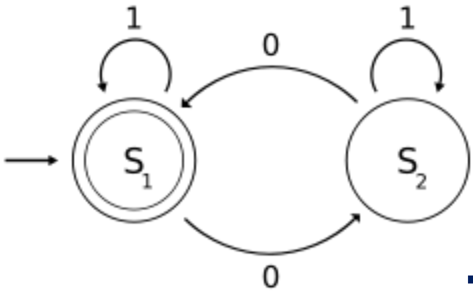
$R = \{ S \rightarrow \varepsilon \mid SS \mid [S] \}$.



Chomsky Hierarchy of Languages (Partial)

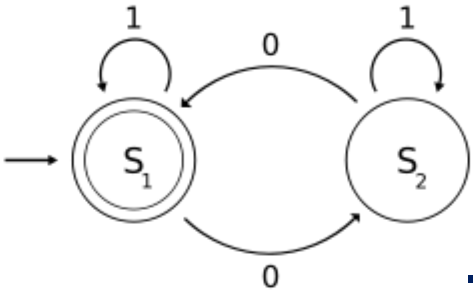
Corollary. Every regular language is context-free.





PDA Exercise

Show the state diagram for a PDA that recognizes $\{ 0^n1^n \mid n \geq 0 \}$.



Exercise

Convert each of the two grammars below to an equivalent PDA using the procedure just described.

$$S \to \mathtt{a}T\mathtt{b} \mid \mathtt{b}$$
$$T \to T\mathtt{a} \mid \varepsilon$$

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid \mathtt{a}$$