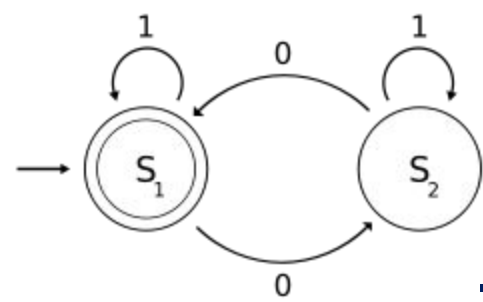
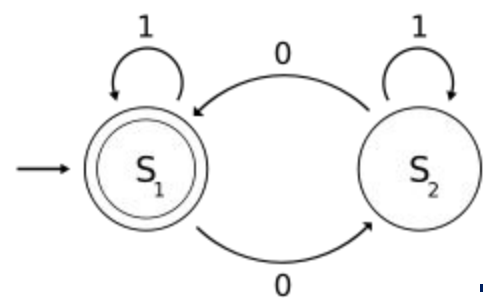




Turing Machines

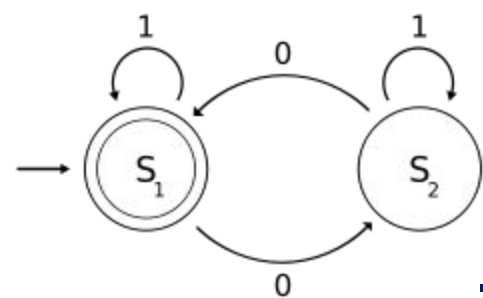


Today's adventures



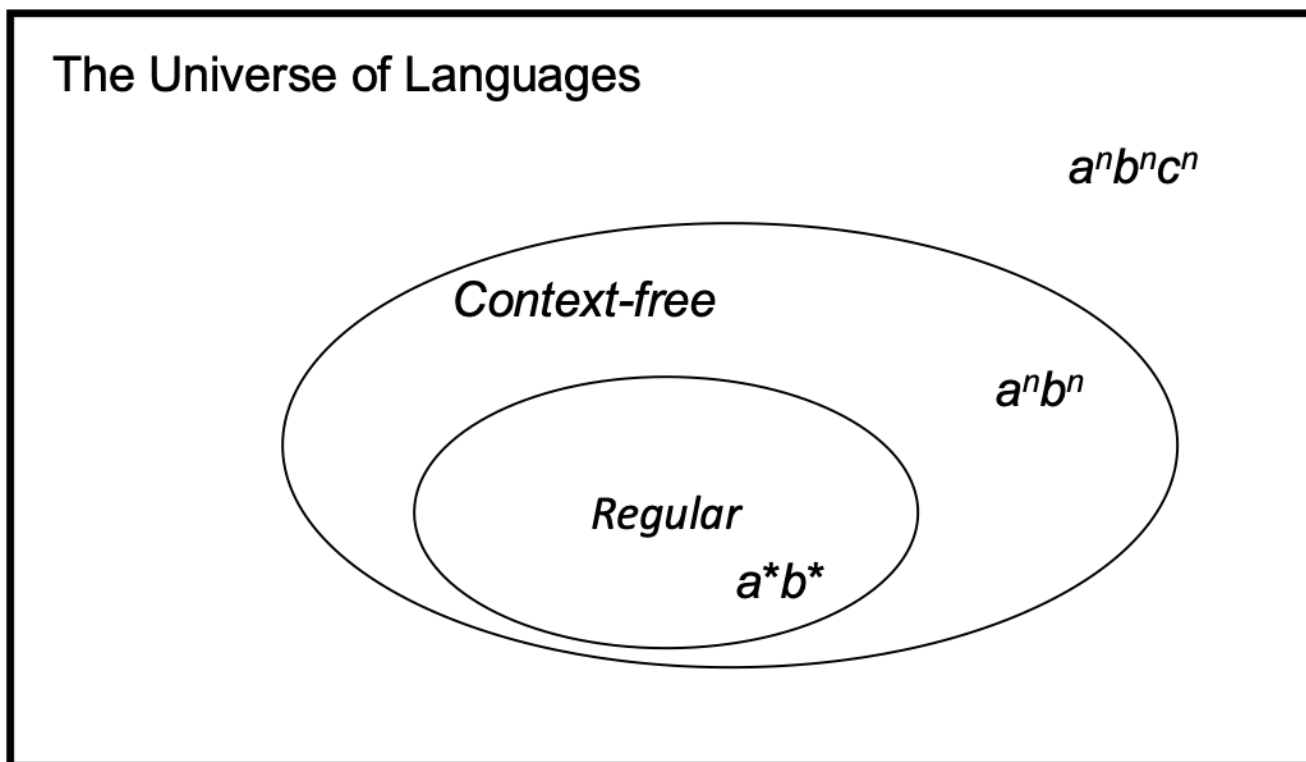
- A language that context-free grammars cannot generate
- Defining Turing Machines
- Configurations and yields
- Recursively enumerable languages
- Recognizing using TMs

Weekly tip: Curiosity is a muscle

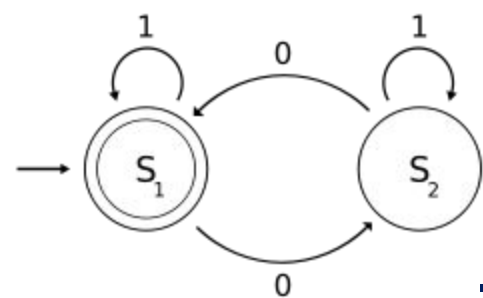


A Context-Free Grammar for $\{a^n b^n c^n : n \geq 0\}$?

As it turns out, we can't generate one!*

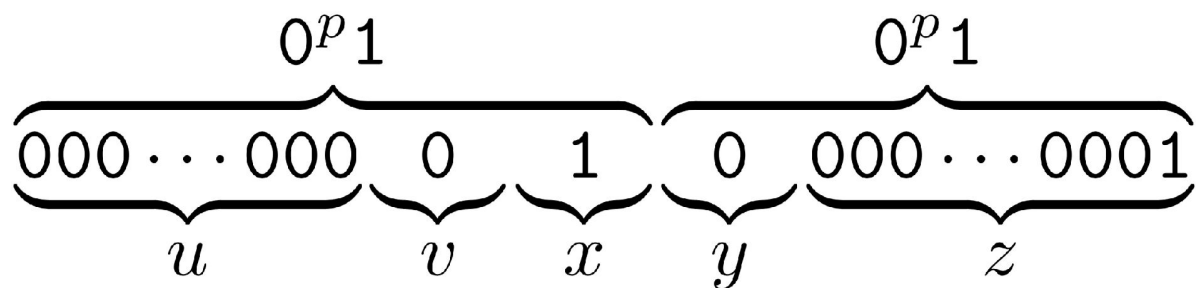


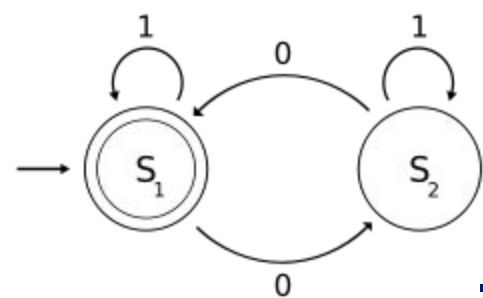
*We can prove this using an analog of the pumping lemma for context-free grammars.



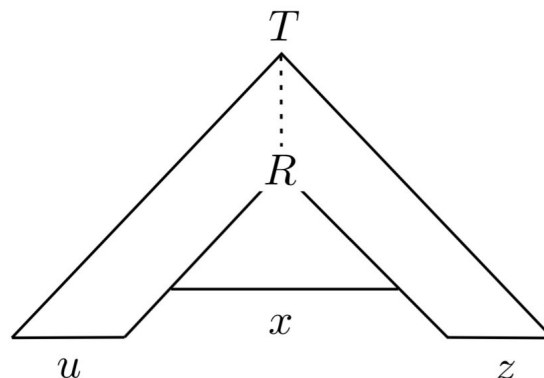
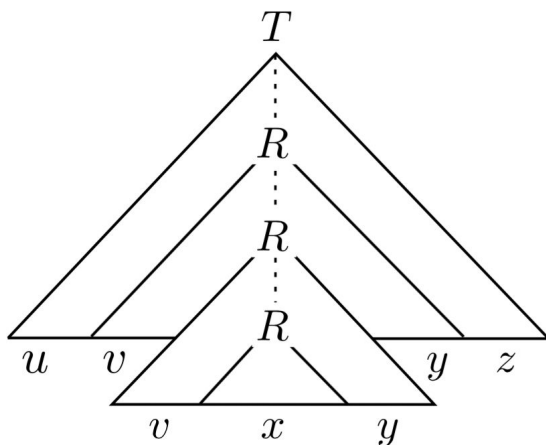
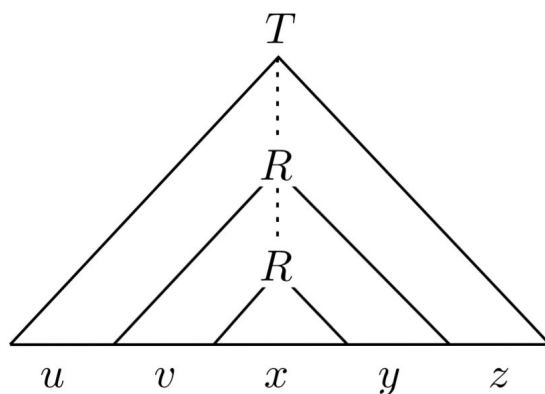
A Context-Free Grammar for $D = \{ww : w \in \{0,1\}^*\}$?

Assume D is a context-free language and reach a contradiction.
 Let p be the pumping length given by the pumping lemma.
 Choose $s = 0^p 1 0^p 1$

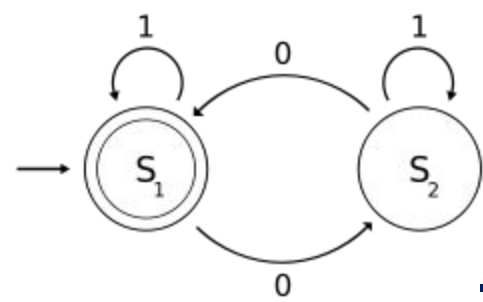




Surgery on Parse Trees

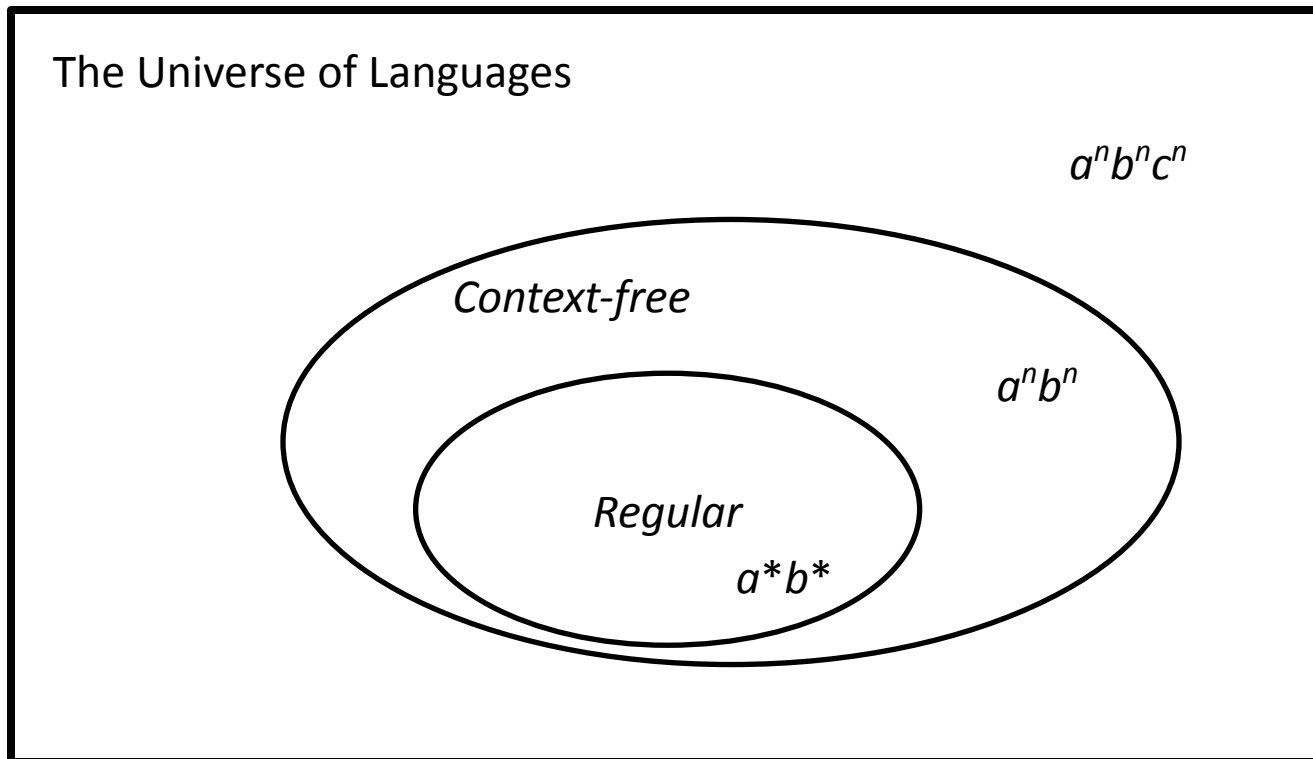


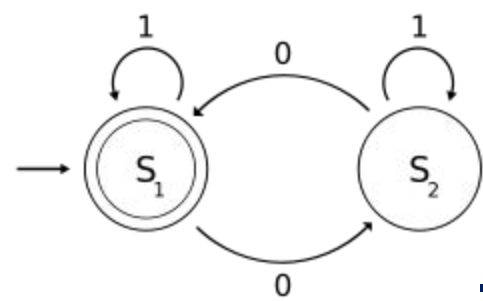
Observation. A k -ary tree of height n has at most k^n leaves.



Chomsky Strikes Again

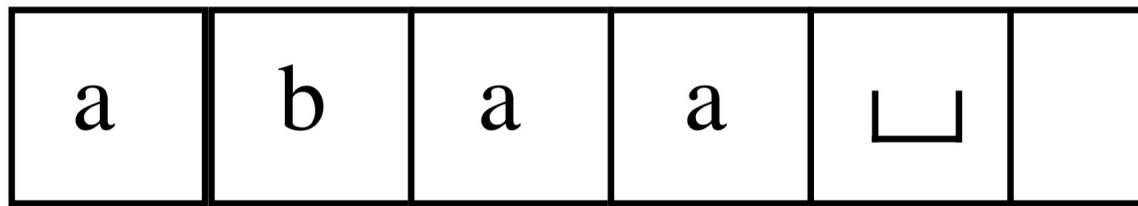
Corollary. Every regular language is context-free.





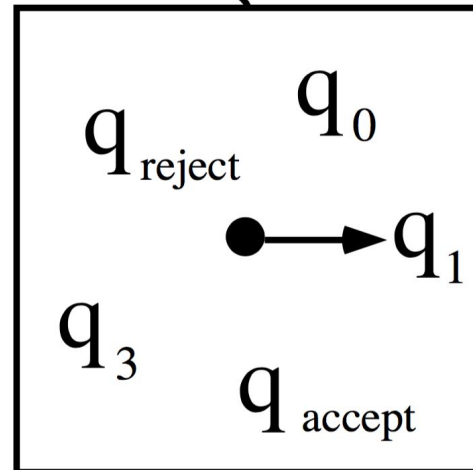
Turing Machines

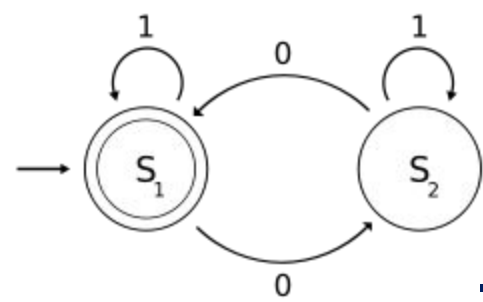
Right Infinite Tape



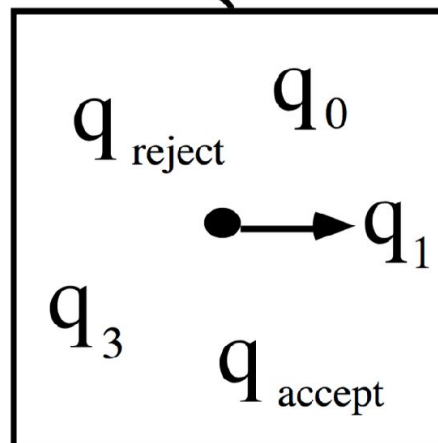
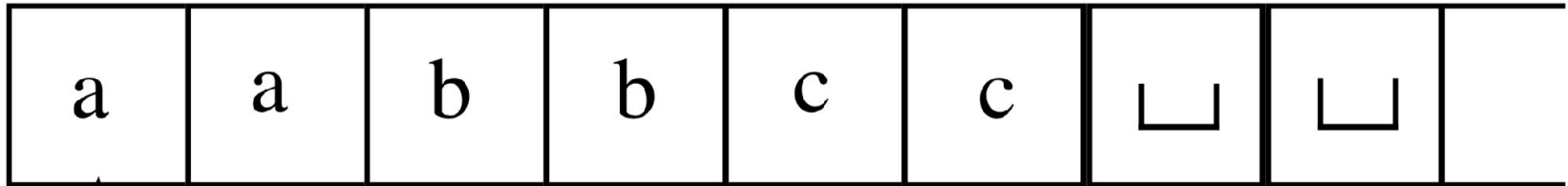
Bidirectional
Read/Write
Head

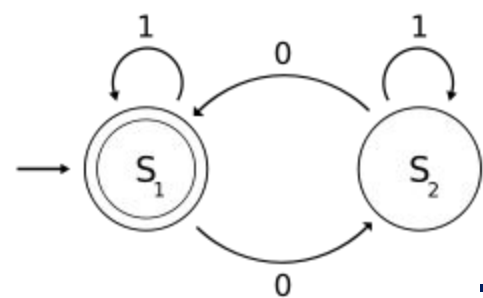
Finite-State
Control





Recognizing $\{a^n b^n c^n : n \geq 0\}$

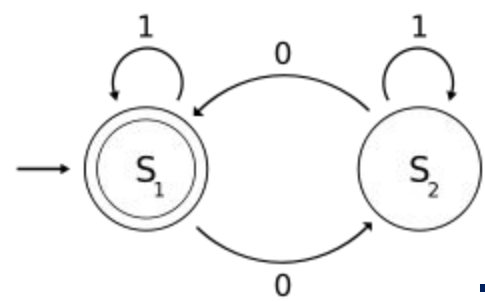




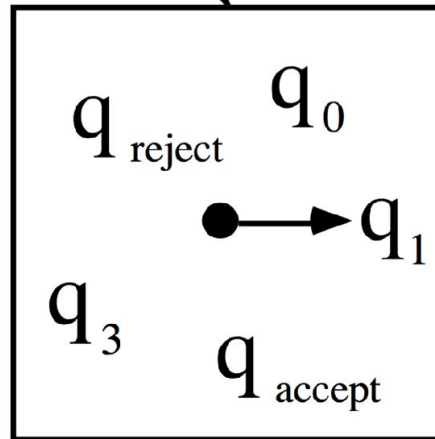
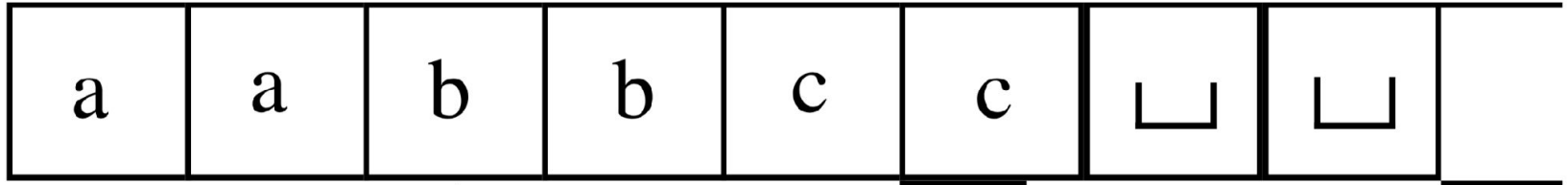
Boring ...

Definition. A **Turing Machine** is a 7-tuple,
 $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, Q, Σ, Γ are finite sets,

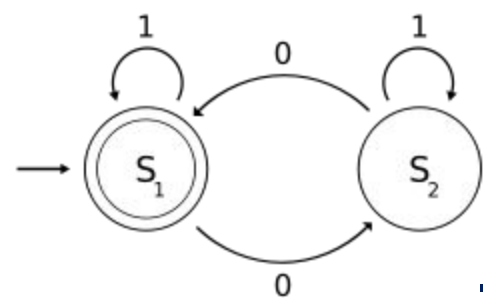
1. Q is the set of states,
2. Σ is the input alphabet not containing the special **blank** symbol $\sqcup$,
3. Γ is the tape alphabet, where $\{\sqcup\} \in \Gamma$ and $\Sigma \subseteq \Gamma$,
4. $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the transition function,
5. $q_0 \in Q$ is the start state,
6. $q_{accept} \in Q$ is the accept state, and
7. $q_{reject} \in Q$ is the reject state, where $q_{reject} \neq q_{accept}$



Configurations and Yields



aaq_1bbcc *yields* aaq_3bcc

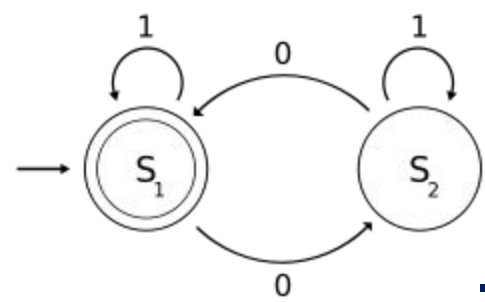


Recursively Enumerable Languages

Definition. A Turing machine M accepts input w if a sequence of configurations $C_1, C_2, \dots, C_k$ exists where

1. C_1 is the start configuration of M on input w ,
2. each C_i yields C_{i+1} , and
3. C_k is an accepting configuration.

Definition. Call a language ***Turing-recognizable*** if it is the language accepted by some Turing machine.



Recognizing $A = \{0^{2^n} \mid n \geq 0\}$

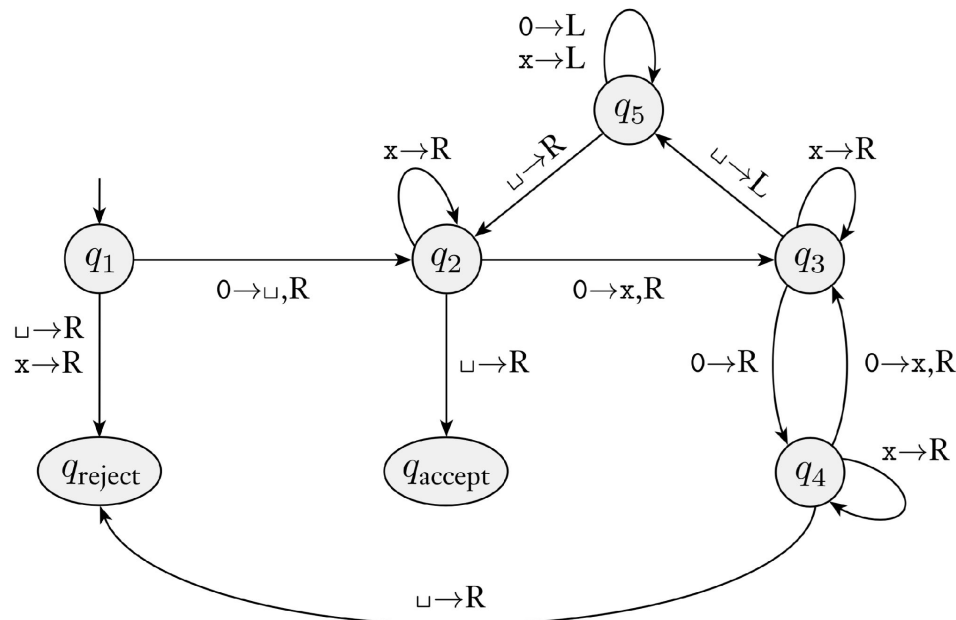
Define $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where

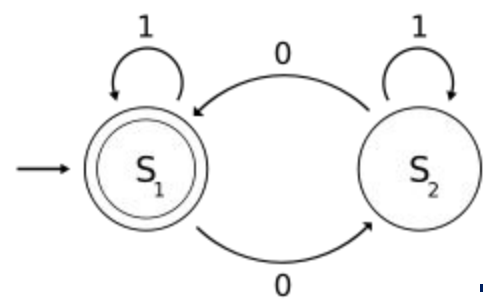
$$Q = \{q_1, q_2, q_3, q_4, q_5, q_{accept}, q_{reject}\},$$

$$\Sigma = \{0\},$$

$$\Gamma = \{0, x, \sqcup\}, \text{ and}$$

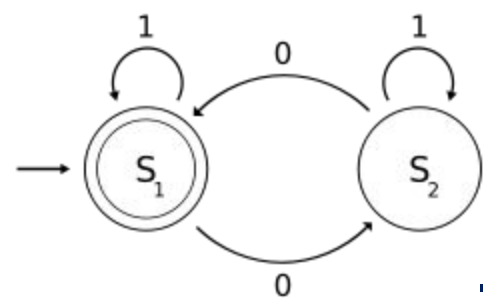
$\delta = Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is given by





Recognizing $B = \{w\#w \mid w \in \{0,1\}^*\}$

1. Come up with a strategy (in English!)
2. Write the state diagram for the TM capturing your algorithm



Recognizing $B = \{w\#w \mid w \in \{0,1\}^*\}$

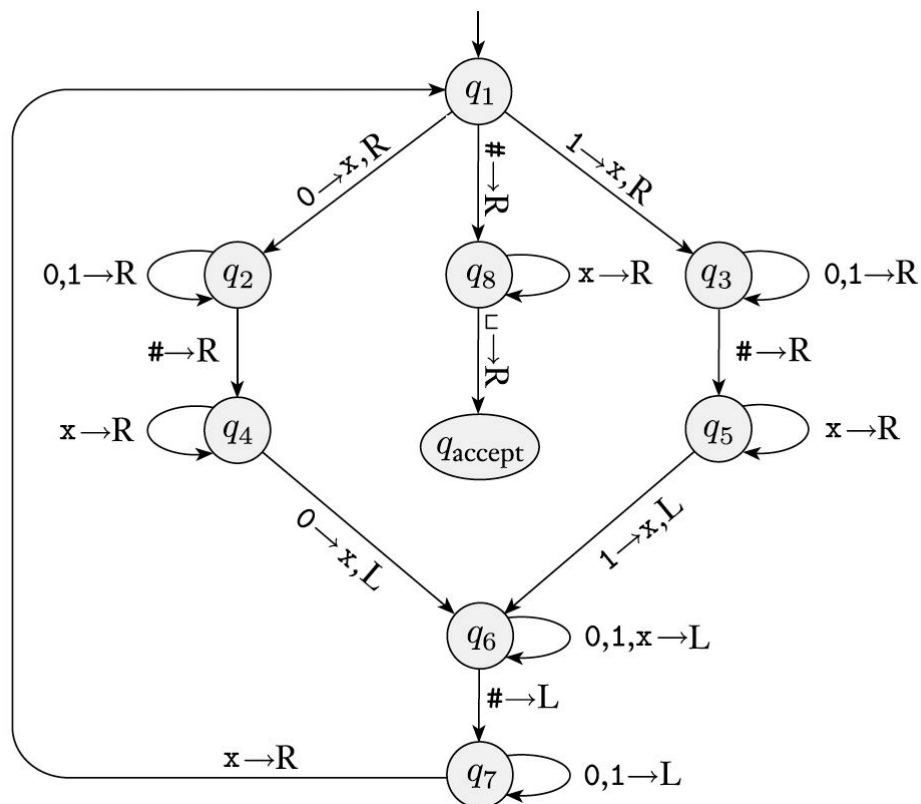
Define $M_I = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where

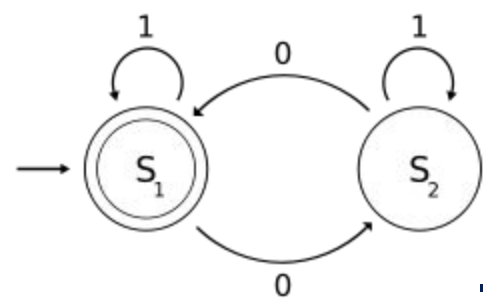
$Q = \{q_1, \dots, q_8, q_{accept}, q_{reject}\}$, q_{reject} not in the picture for simplicity

$\Sigma = \{0\}$,

$\Gamma = \{0, x, \sqcup\}$, and

$\delta = Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$





Recognizing $B = \{w\#w \mid w \in \{0,1\}^*\}$

At a higher level, we can also *describe* M , slightly differently, by *specifying its implementation*.

M_1 = “On input string w :

1. Check if the string is in the correct format (i.e. contains $\#$). If not, *reject*.
2. Start on the left side of the $\#$, at the first non-crossed symbol. Cross off a symbol.
3. Scan right, past the $\#$, to the first non-crossed symbol. If it matches the one crossed off in step 2, cross this one off. Otherwise, *reject*.
4. Go back to the left-hand side of the tape. Go to step 2.
5. When all symbols to the left of the $\#$ have been crossed off, check for remaining symbols on the right of the $\#$. If any remain, *reject*. Otherwise, *accept*.”