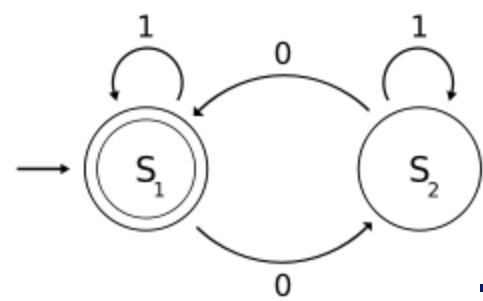


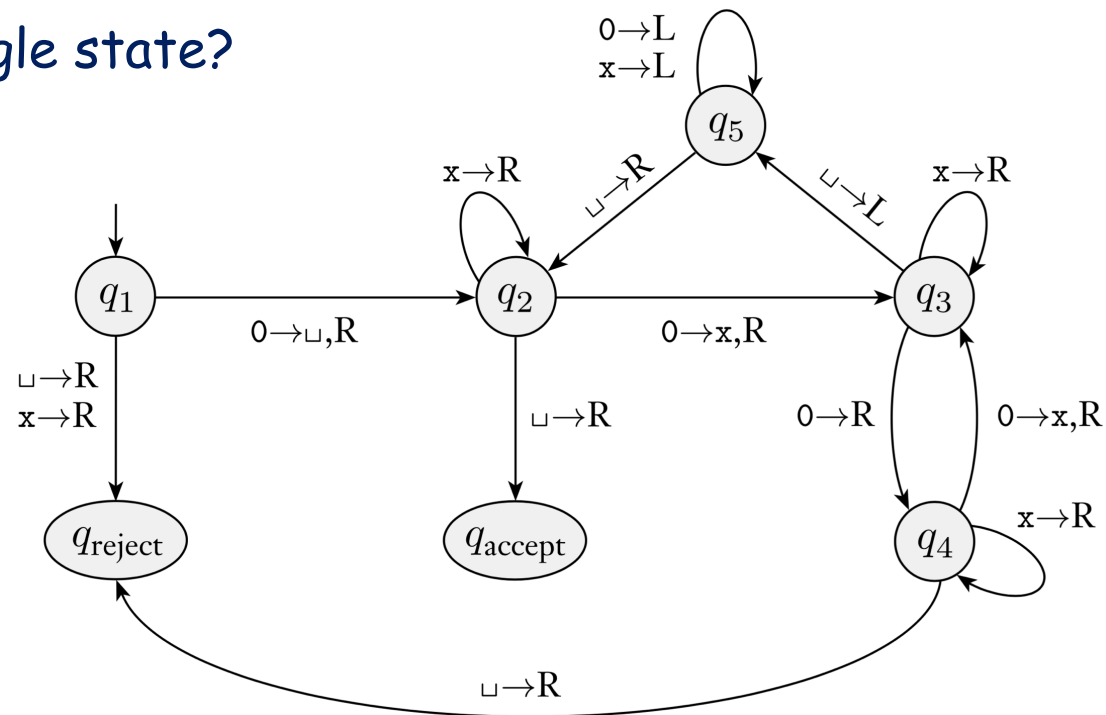
What is an Algorithm?

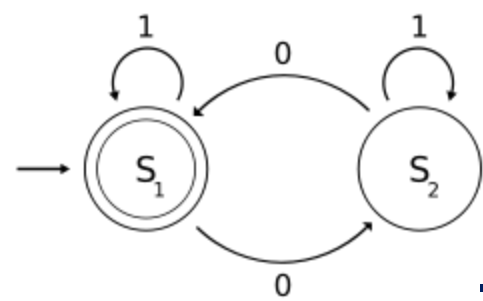


Quiz

- Can a TM ever write the blank symbol \sqcup on its tape?
- Can the tape alphabet Γ of a TM be the same as the input alphabet Σ ?
- Can a TM's head ever be in the same location in two successive steps?
- Can a TM contain just a single state?

- Give the sequence of configurations that the TM on the right enters when started on the input string 00.





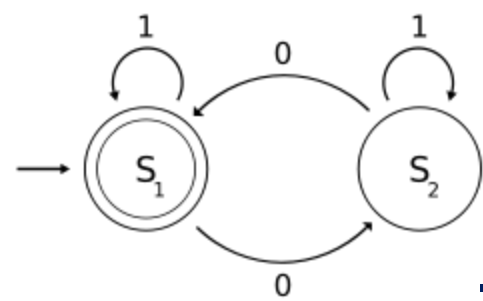
Computability

Hilbert's Tenth Problem.

Find a process according to which it can be determined by a finite number of operations whether a given polynomial

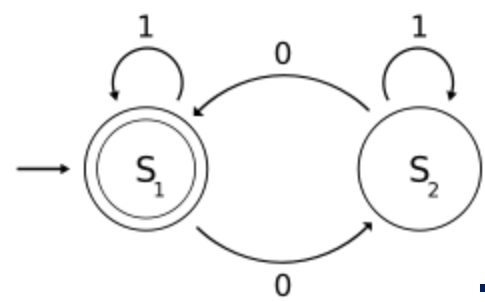
$$p(x_1, x_2, \dots, x_n)$$

has an integral root.



Recipes and Processes

Definition. An *algorithm* is a finite sequence of operations, each chosen from a finite set of well-defined operations, that halts in a finite time.

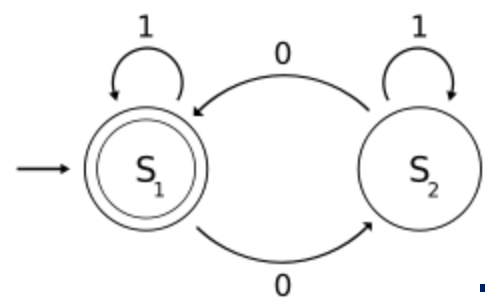


Church-Turing Thesis

Algorithm

=

Turing Machine

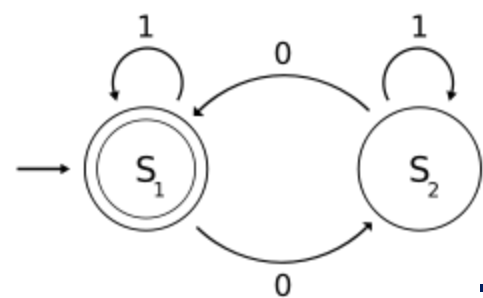


Languages and Problems

Definition. Let $D = \{p \mid p \text{ is a polynomial with an integral root}\}$.

Hilbert's Tenth Problem.

Determine whether D is Turing-decidable.



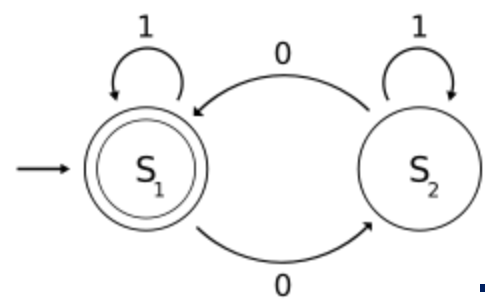
$D = \{p \mid p \text{ is a polynomial with integral roots}\}$

$M =$ "The input is a polynomial $p(x_1, x_2, \dots, x_n)$."

1. Lexicographically generate integer values for

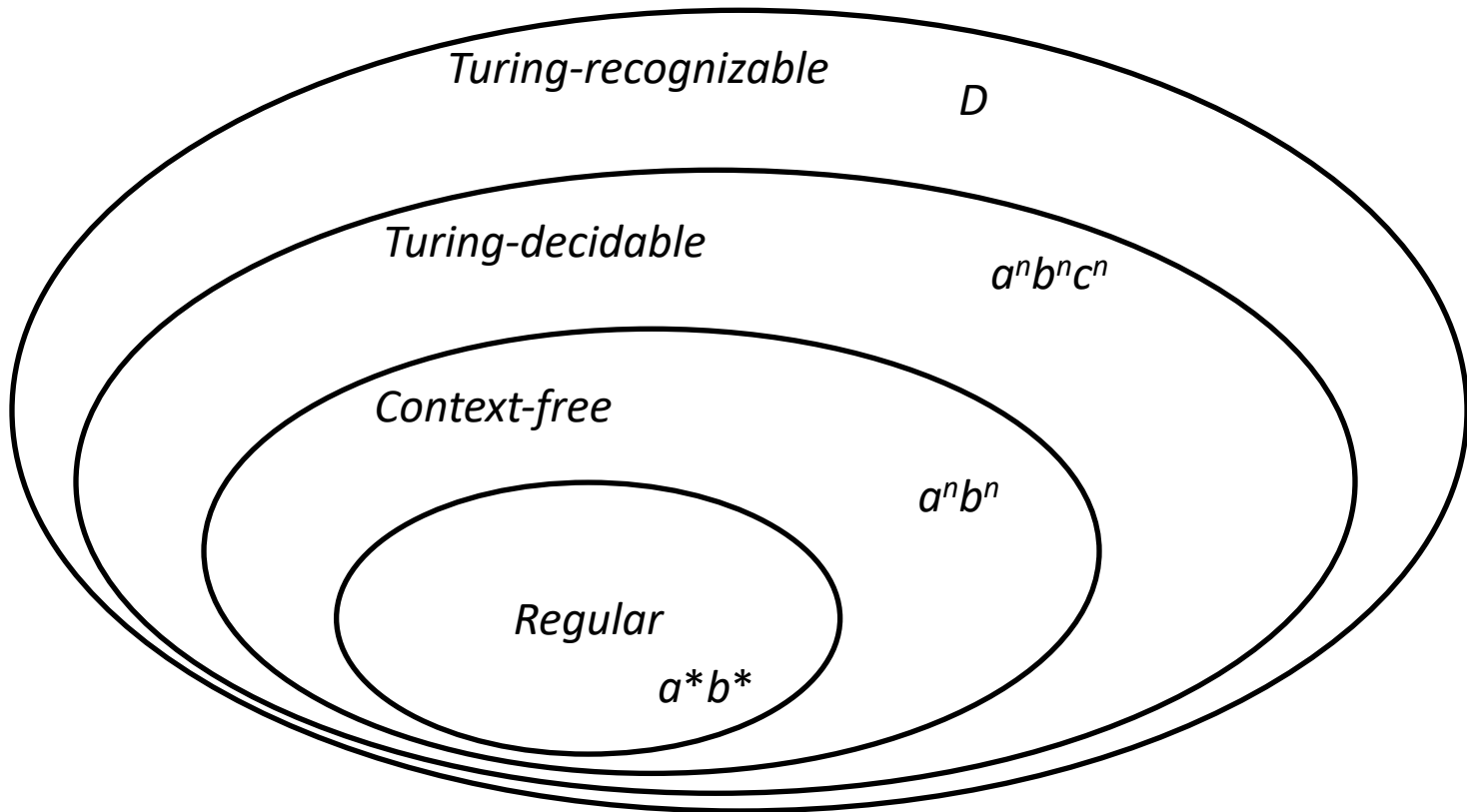
(x_1, x_2, \dots, x_n) .

2. Evaluate p as each set of values is generated. If at any point the polynomial evaluates to 0, *accept*."

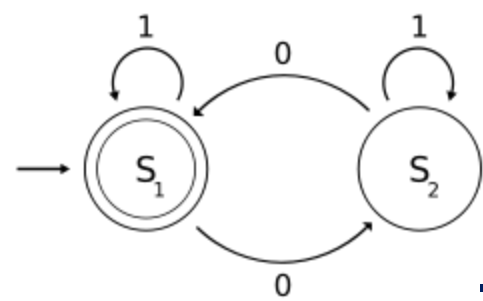


Here We Go Again

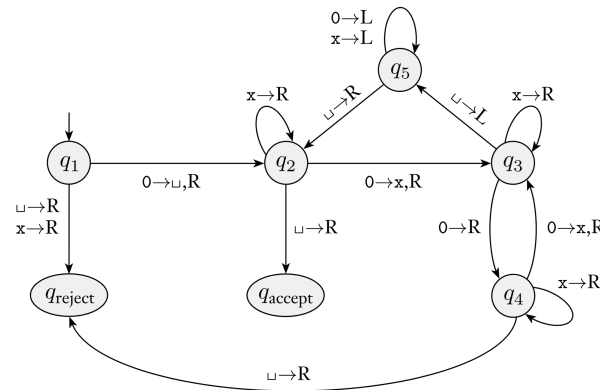
The Universe of Languages



Describing Turing Machines



- *Formal*



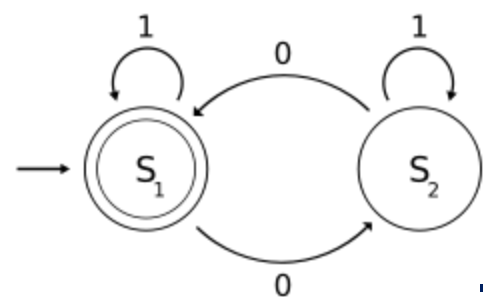
- *Implementation*

$M =$ "On input string w :

1. Sweep across tape, crossing off every other 0.
2. If tape contained one 0, *accept*.
3. Else, if number of 0's is odd, *reject*.
4. Return head to left-hand end of tape.
5. Go to step 1.

- *High-level*

repeat until $n = 1$
 exit if $n \bmod 2 \neq 0$
 set $n = n \text{ div } 2$



Exercises

- Is a TM with a doubly infinite tape (infinite to the left as well as the right) more powerful than an ordinary TM?
- Let a k -PDA be a pushdown automaton that has k stacks. Is a 1-PDA more powerful than a 0-PDA? Is a 2-PDA more powerful than a 1-PDA? Is a 3-PDA more powerful than a 2-PDA?