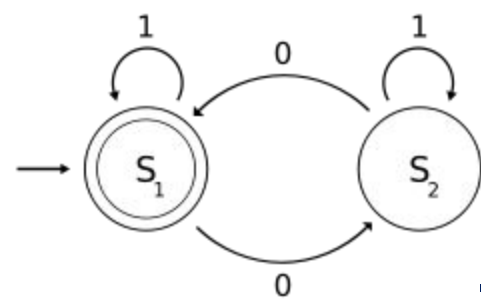
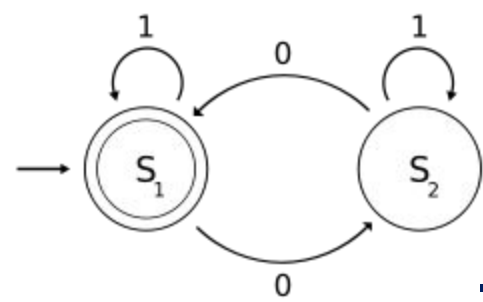




What Machines Can Do

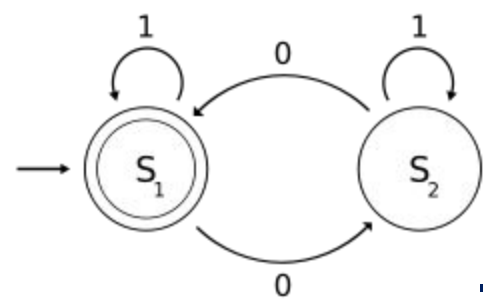


Today's adventures



- Decidability of acceptance problem for DFAs, NFAs, regular expression (i.e. A_{DFA} , A_{NFA} , A_{REX})
- Emptiness and equivalence testing for DFAs
- Acceptance problem for CFGs
- Emptiness and equivalence testing for CFGs

Weekly tip: Everyone and everything can teach us something



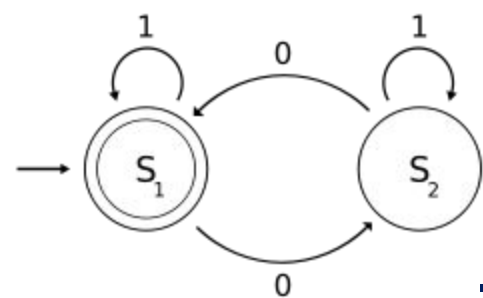
Problems Concerning Regular Languages: Acceptance

Definition.

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

$$A_{\text{NFA}} = \{ \langle B, w \rangle \mid B \text{ is an NFA that accepts input string } w \}$$

$$A_{\text{REX}} = \{ \langle R, w \rangle \mid R \text{ is a regular expression that generates string } w \}$$

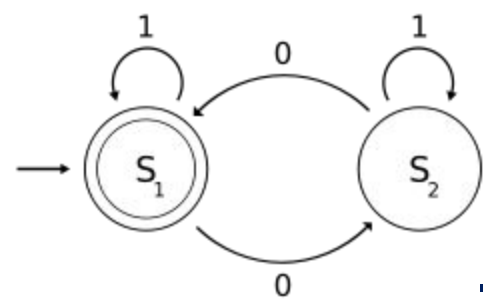


Deciding Regular Languages

Theorem. A_{DFA} is decidable.

Proof. $M =$ “On input $\langle B, w \rangle$,

1. Simulate B on input w .
2. If the simulation ends in an accept state, *accept*.
If it ends in a nonaccepting state, *reject*.”

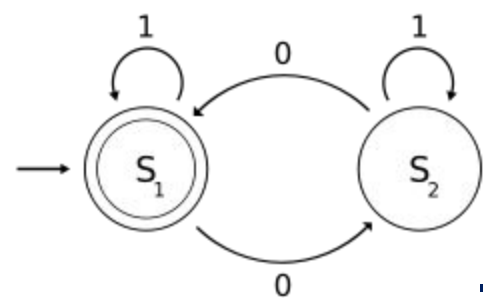


Guessing is No Problem

Theorem. A_{NFA} is decidable.

Proof. $N =$ “On input $\langle B, w \rangle$,

1. Convert NFA B to an equivalent DFA C .
2. Simulate TM M on input $\langle C, w \rangle$.
3. If M accepts, *accept*. Otherwise, *reject*.”

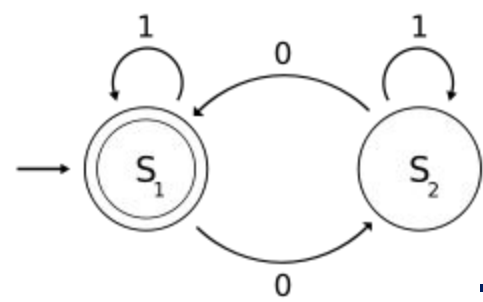


Deciding Regular Expressions

Theorem. A_{REX} is decidable.

Proof. $P =$ “On input $\langle R, w \rangle$,

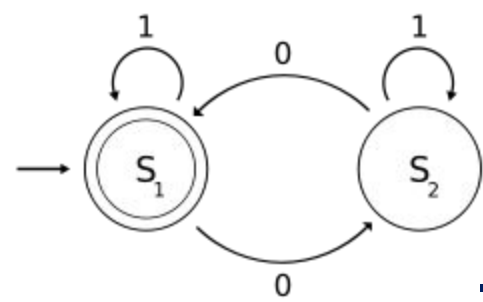
1. Convert RE R to an equivalent DFA C .
2. Simulate TM M on input $\langle C, w \rangle$.
3. If M accepts, *accept*. Otherwise, *reject*.”



Emptiness Testing

Definition. $E_{\text{DFA}} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset \}$

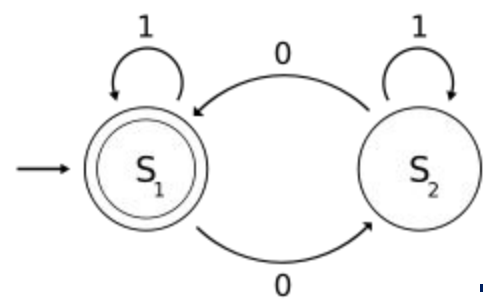
Is E_{DFA} decidable?



Equivalence Testing

Definition. $EQ_{DFA} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B) \}$

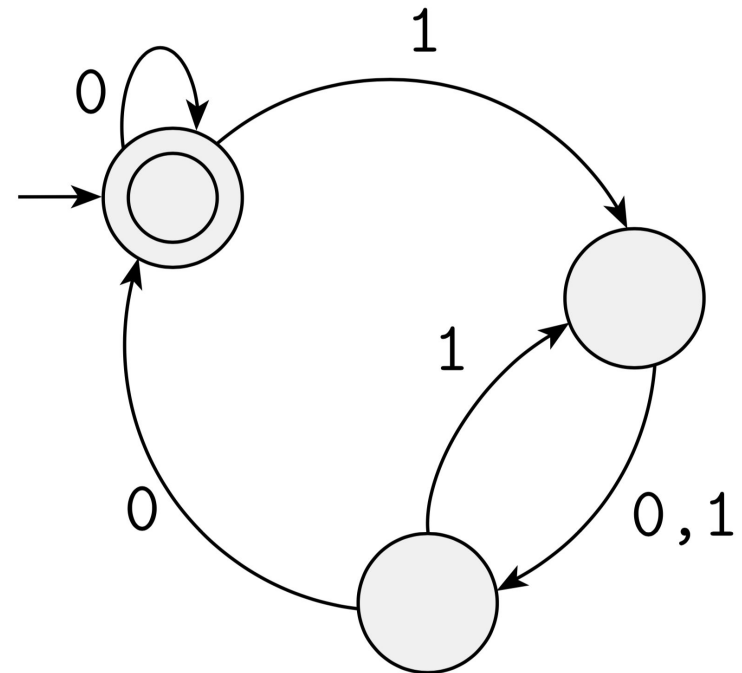
Is EQ_{DFA} decidable?



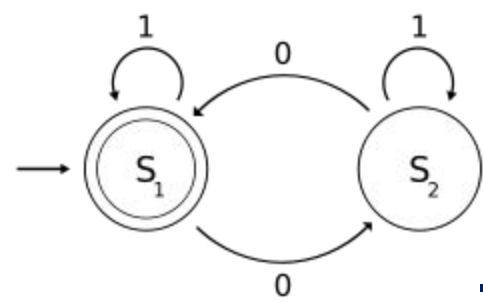
Exercises

1. For the DFA M on the right

- Is $\langle M, 0100 \rangle \in A_{\text{DFA}}$?
- Is $\langle M, 010 \rangle \in A_{\text{DFA}}$?
- Is $\langle M \rangle \in A_{\text{DFA}}$?
- Is $\langle M, 0100 \rangle \in A_{\text{REX}}$?
- Is $\langle M \rangle \in E_{\text{DFA}}$?
- Is $\langle M, M \rangle \in EQ_{\text{DFA}}$?

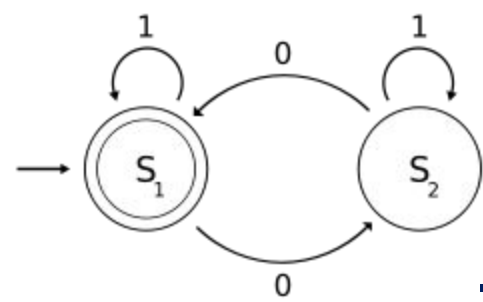


2. Let $ALL_{\text{DFA}} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^* \}$.
Show that ALL_{DFA} is decidable.



Deciding Context-Free Languages?

Definition. $A_{\text{CFG}} = \{ \langle G, w \rangle \mid G \text{ is a CFG that generates } w \}$

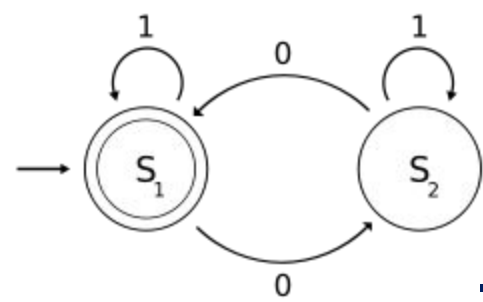


Yes! (What a Relief)

Theorem. A_{CFG} is decidable.

Proof.

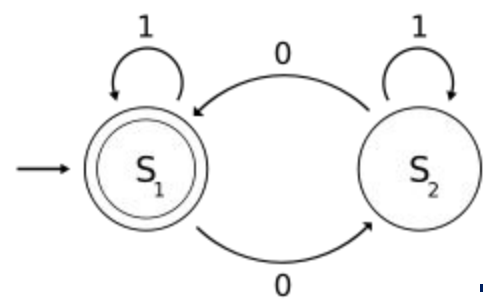




Emptiness Testing

Definition. $E_{\text{CFG}} = \{ \langle G \rangle \mid G \text{ is a CFG and } L(G) = \emptyset \}$

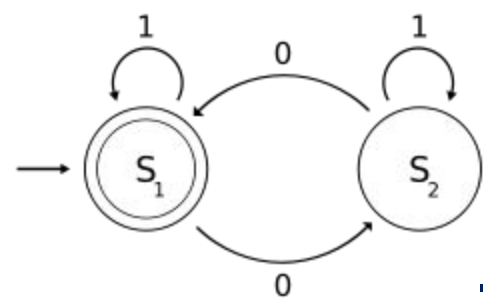
Is E_{CFG} decidable?



Equivalence Testing

Definition. $EQ_{CFG} = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are CFGs and } L(G) = L(H) \}$

Is EQ_{CFG} decidable?



The Missing Piece

Theorem. Every context-free language is decidable.

Proof.

