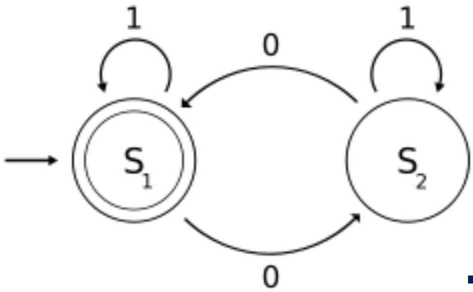
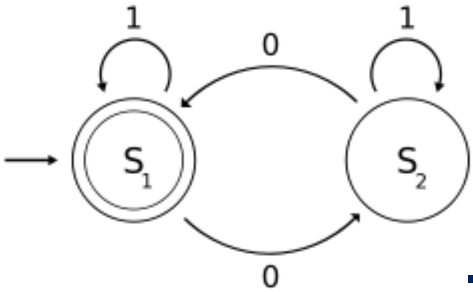


What Machines Can Do

Sipser: Section 4.1 pages 193 - 201



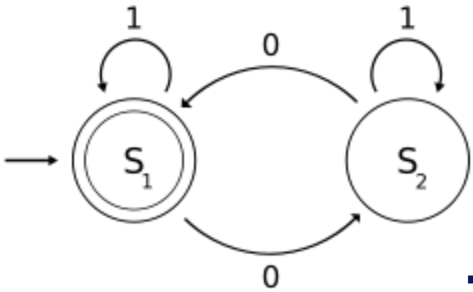
Problems Concerning Regular Languages: Acceptance

Definition.

A_{DFA} = {<*B*, *w*> | *B* is a DFA that accepts input string *w*}

A_{NFA} = {<*B*, *w*> | *B* is an NFA that accepts input string *w*}

A_{REX} = {<*R*, *w*> | *R* is a regular expression that generates string *w*}



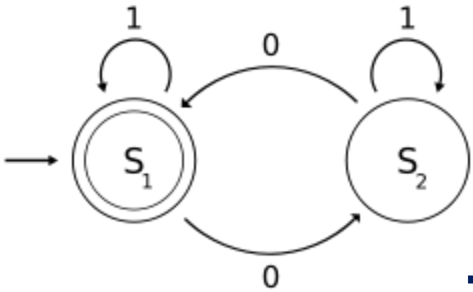
Deciding Regular Languages

Theorem. A_{DFA} is decidable.

Proof. M = "On input <B, w>,

1. Simulate B on input w.

2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*."

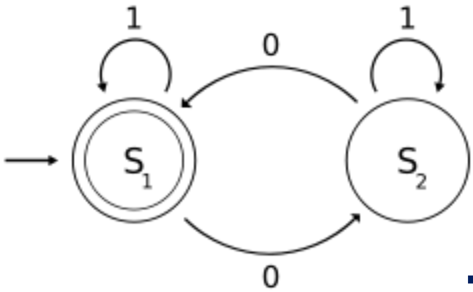


Guessing is No Problem

Theorem. A_{NFA} is decidable.

Proof. N = "On input <*B*, *w*>,

1. Convert NFA *B* to an equivalent DFA *C*.
2. Simulate TM *M* on input <*C*, *w*>.
3. If *M* accepts, *accept*. Otherwise, *reject*."

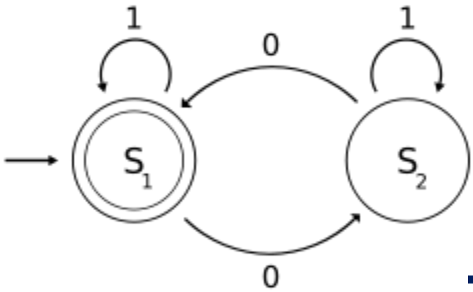


Deciding Regular Expressions

Theorem. A_{REX} is decidable.

Proof. P = "On input <R, w>,

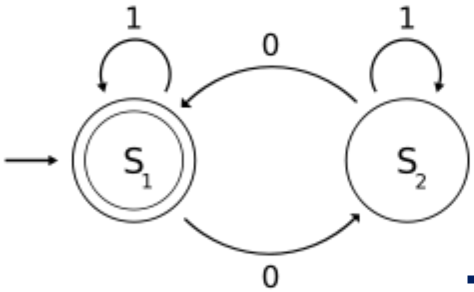
1. Convert RE R to an equivalent DFA C.
2. Simulate TM M on input <C, w>.
3. If M accepts, *accept*. Otherwise, *reject*."



Emptiness Testing

Definition. E_{DFA} = {‹A› | A is a DFA and $L(A) = \emptyset$ }

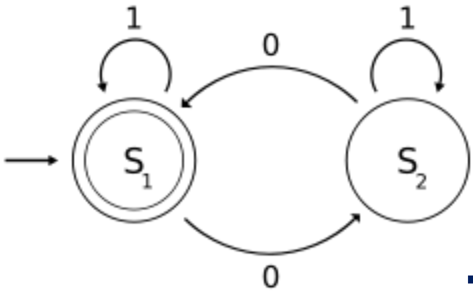
Is E_{DFA} decidable?



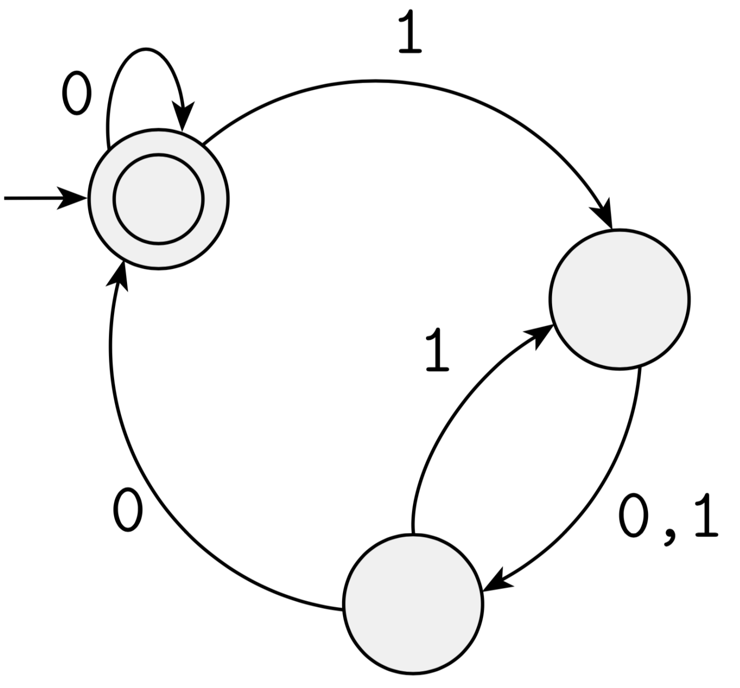
Equivalence Testing

Definition. $EQ_{\mathrm{DFA}} = \{\langle A, B \rangle \mid A$ and B are DFAs and $L(A) = L(B)\}$

Is EQ_{DFA} decidable?

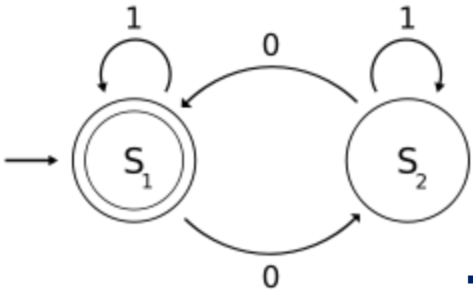


Exercises



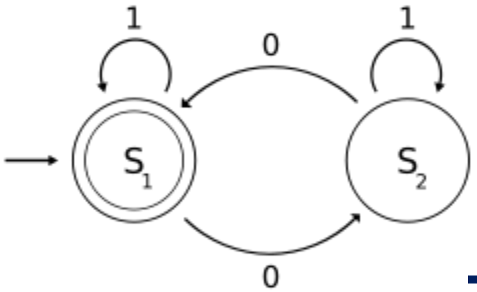
1. For the DFA *M* on the right
   a. Is <*M*, 0100> ∈ A_{DFA}?
   b. Is <*M*, 010> ∈ A_{DFA}?
   c. Is <*M*> ∈ A_{DFA}?
   d. Is <*M*, 0100> ∈ A_{REX}?
   e. Is <*M*> ∈ E_{DFA}?
   f. Is <*M*, *M*> ∈ EQ_{DFA}?

2. Let ALL_{DFA} = {<*A*> | *A* is a DFA and $L(A) = \Sigma^*$ }.
Show that ALL_{DFA} is decidable.



Deciding Context-Free Languages?

Definition. A_{CFG} = {<*G*, *w*> | *G* is a CFG that generates *w* }



Yes! (What a Relief)

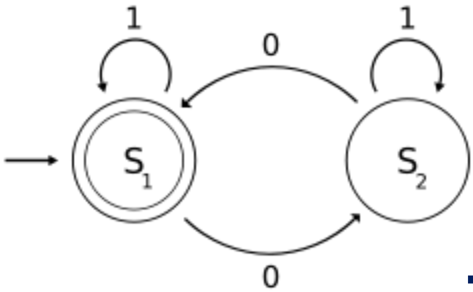
Theorem. A_{CFG} is decidable.

Proof. S = "On input <G, w>,

1. Convert G to an equivalent grammar in Chomsky normal form.

2. List all derivations with 2n-1 steps, where n is the length of w.

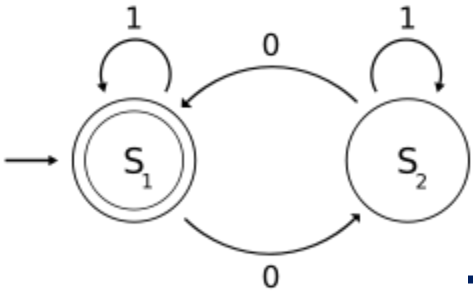
3. If any of these derivations generate w, *accept*. Otherwise, *reject*."



Emptiness Testing

Definition. E_{CFG} = {<G> | G is a CFG and L(G) = ∅ }

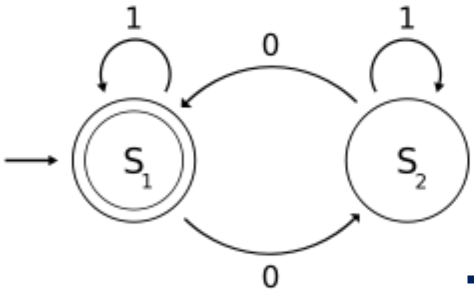
Is E_{CFG} decidable?



Equivalence Testing

Definition. EQ_{CFG} = {<*G*, *H*> | *G* and *H* are CFGs and *L*(*G*) = *L*(*H*) }

Is EQ_{CFG} decidable?



The Missing Piece

Theorem. Every context-free language is decidable.

Proof. Let *G* be a CFG for *A*. We design a TM M_G that decides *A* as follows.

M_G = "On input *w*.

1. Run TM *S* on input <*G*, *w*>.

2. If this machine accepts, *accept*. Otherwise, *reject*."