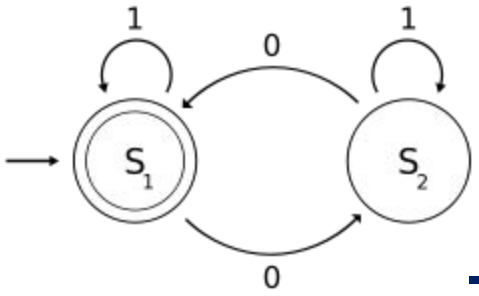
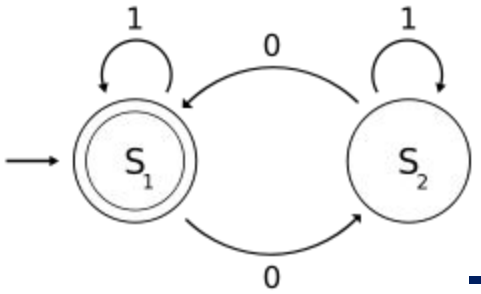


The Classes P and NP

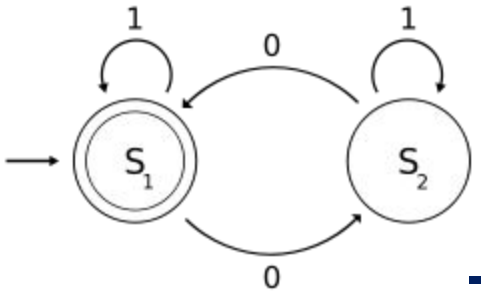
Sipser: Section 7.2 pages 284 – 292; Section 7.3 pages 292 – 298



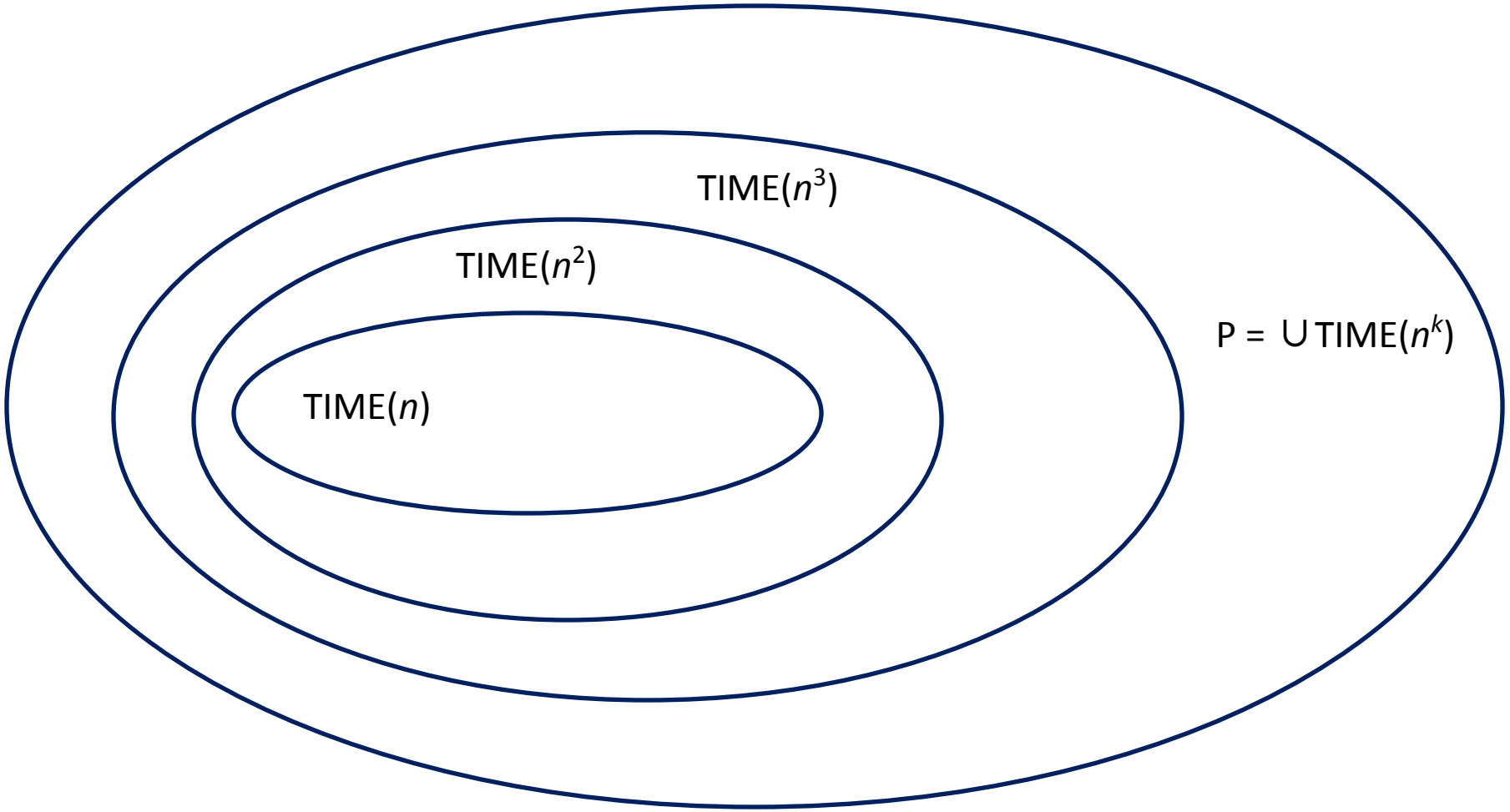
Today's adventures

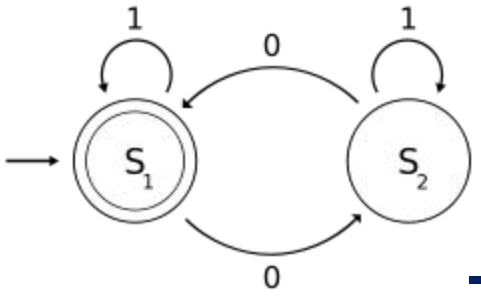
- Recap what we know about P
- Define NP (two definitions! one new kind of TM!)
- Examples in P and NP

Weekly tip: You are the architect and designer of your own success



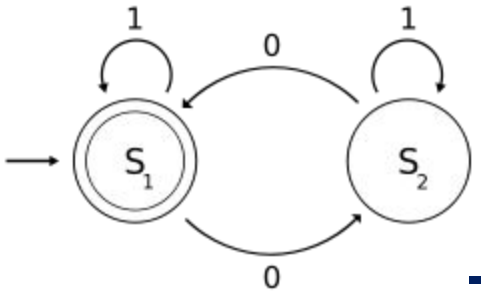
Polynomial Time



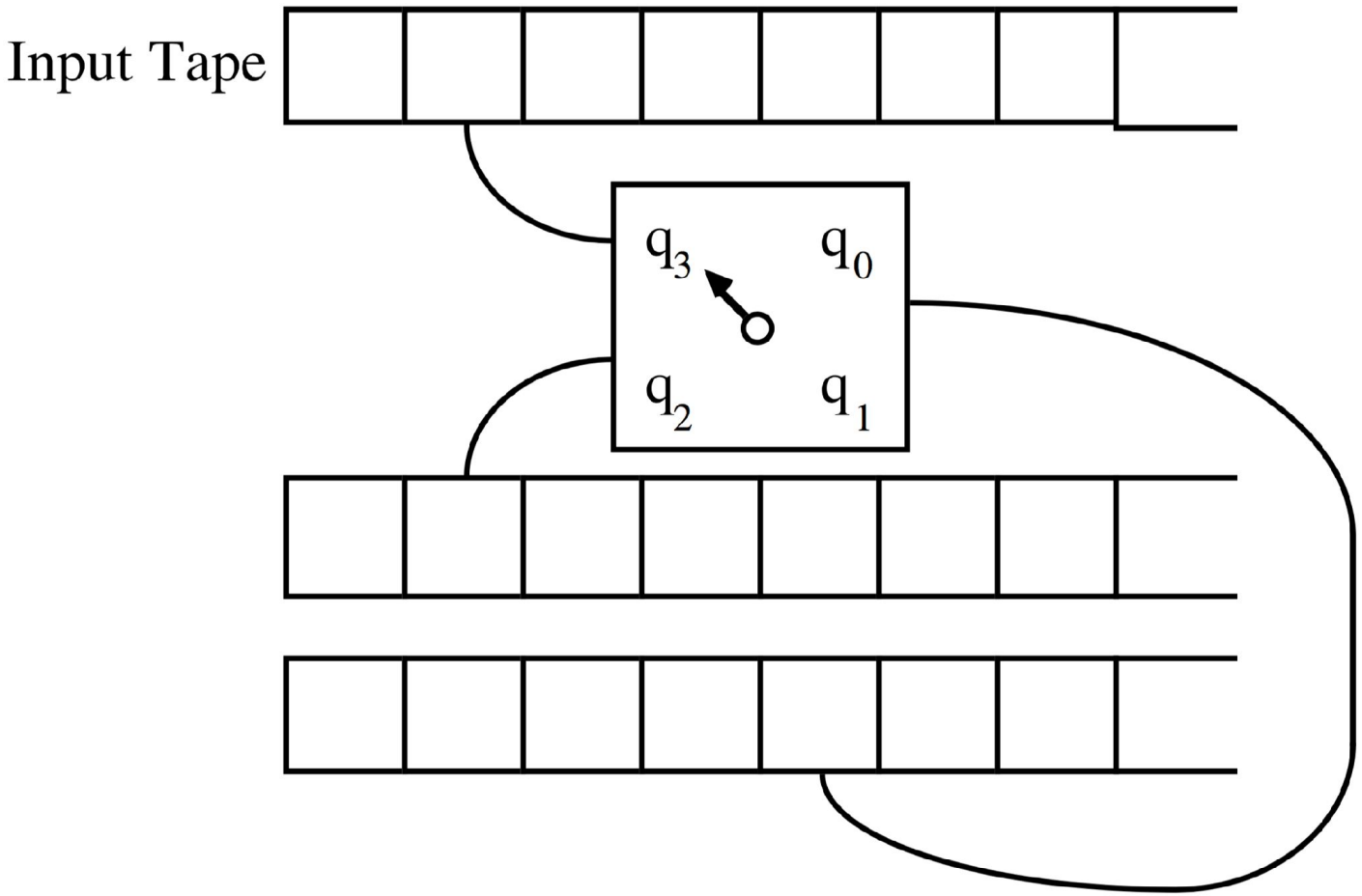


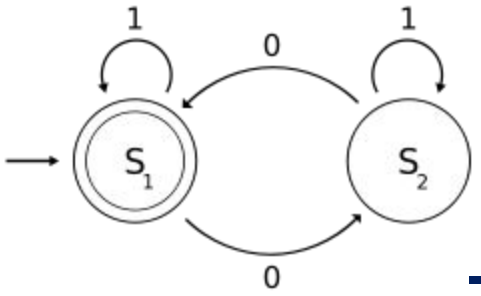
Tractable Problems

Time complexity function	Size n: 10	20	30	40	50	60
n	.00001 second	.00002 second	.00003 second	.00004 second	.00005 second	.00006 second
n^2	.0001 second	.0004 second	.0009 second	.0016 second	.0025 second	.0036 second
n^3	.001 second	.008 second	.027 second	.064 second	.125 second	.216 second
n^5	.1 second	3.2 seconds	24.3 seconds	1.7 minutes	5.2 minutes	13.0 minutes
2^n	.001 second	1.0 second	17.9 minutes	12.7 days	35.7 years	366 centuries
3^n	0.59 second	58 minutes	6.5 years	3855 centuries	2×10^8 centuries	1.3×10^{13} centuries



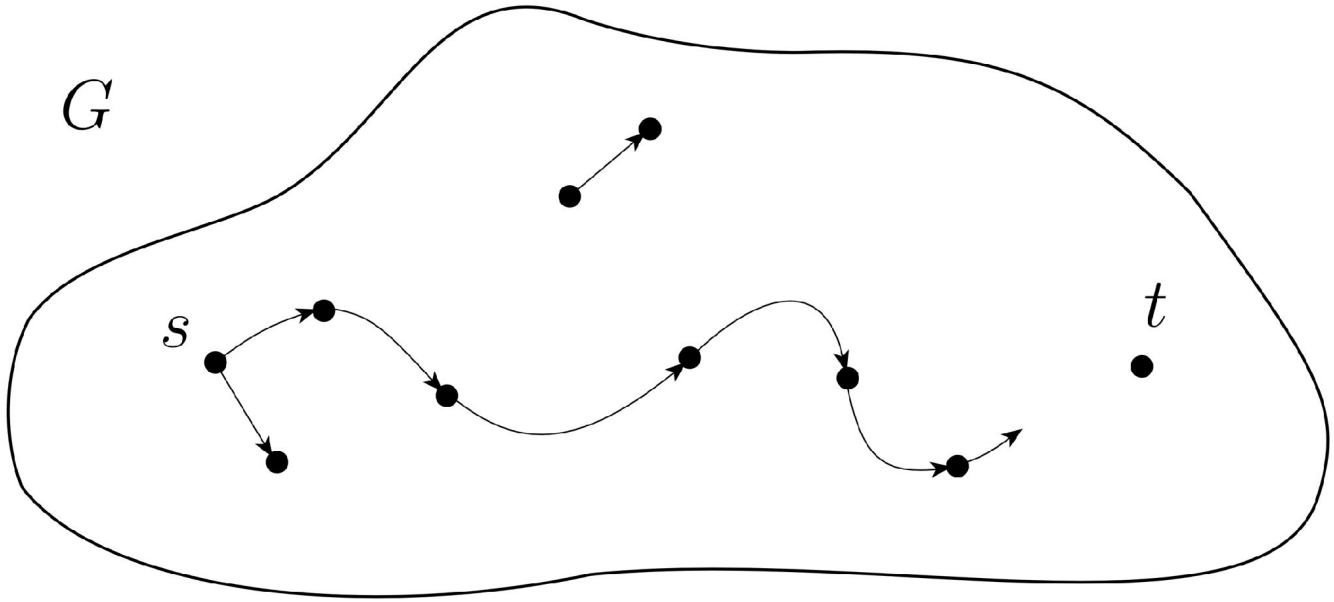
Polynomially Equivalent Models

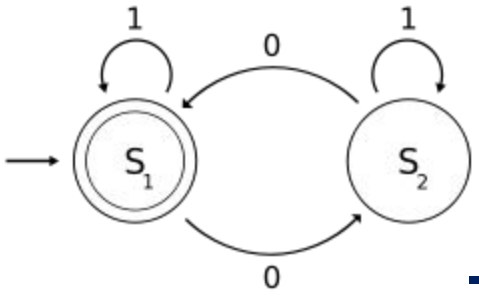




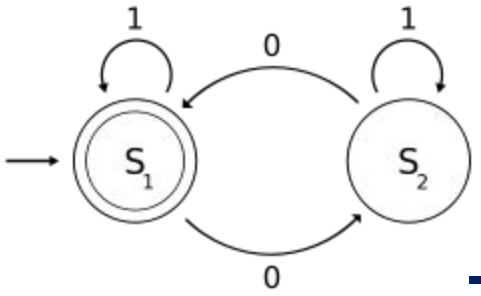
Graph Theory

Definition. $PATH = \{\langle G, s, t\rangle \mid \exists$ a directed path from s to t in $G\}$



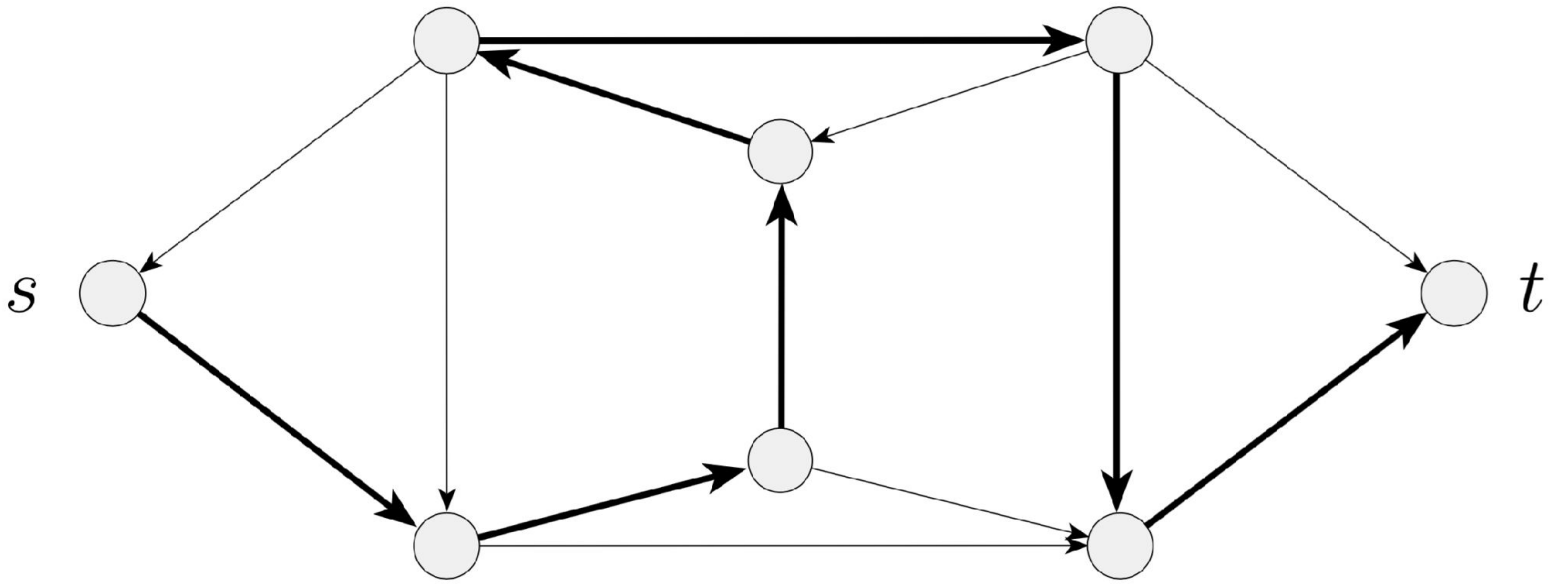


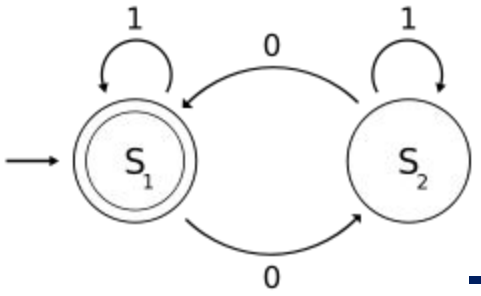
$PATH \in \mathrm{P}$



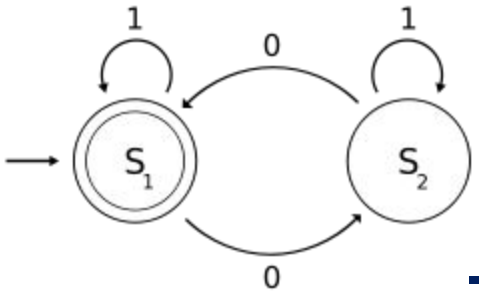
Hamiltonian Paths

Definition. $HAMPATH = \{\langle G, s, t\rangle \mid \exists$ a Hamiltonian path from s to $t\}$

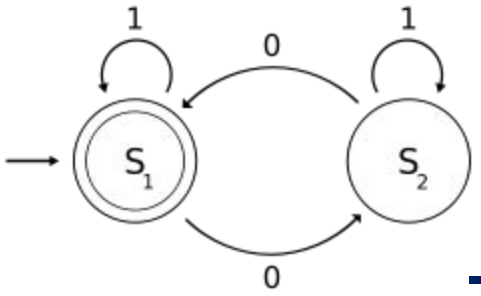




Checking for Hamiltonian Paths

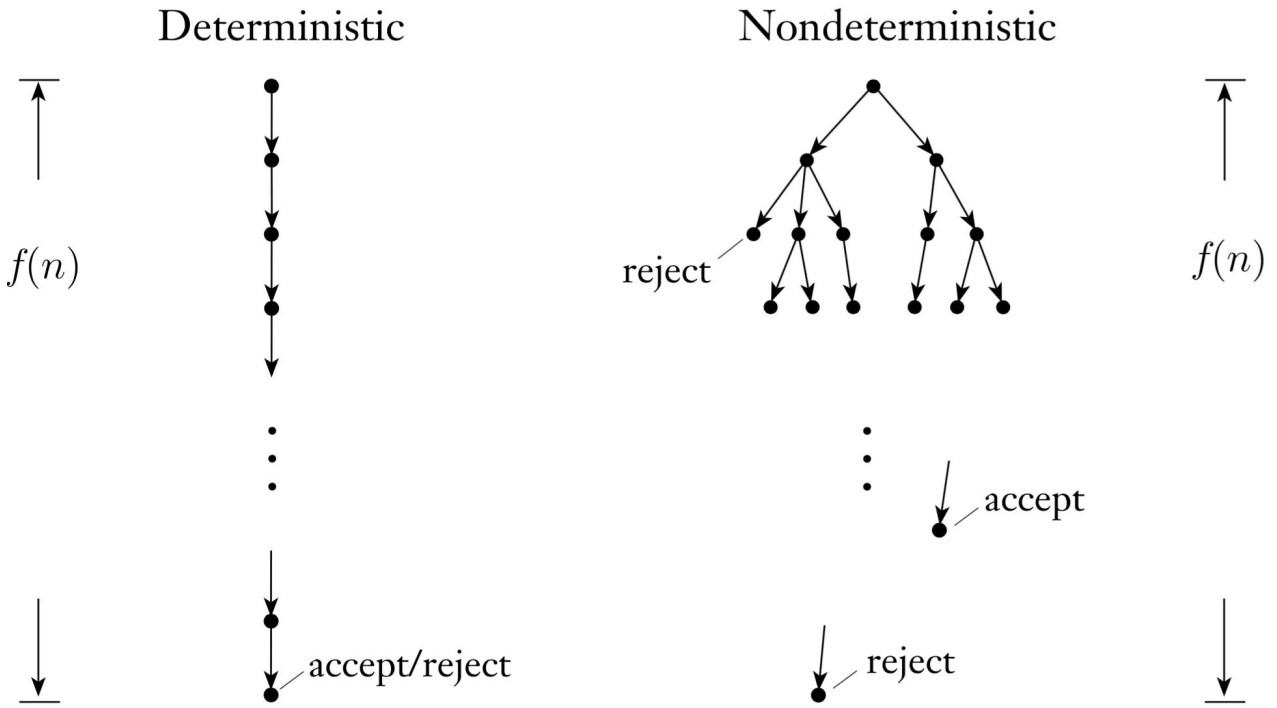


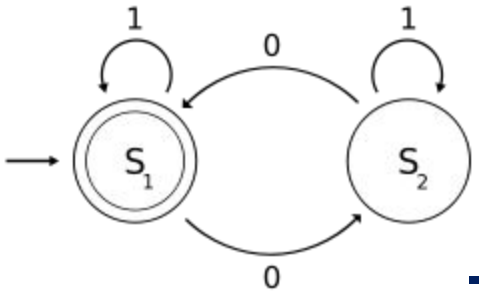
Guessing a Solution



Nondeterministic Time Complexity

Definition. Let N be a NTM. The ***running time*** of N is a function $f: \mathbb{N} \longrightarrow \mathbb{N}$, where $f(n)$ is the maximum number of steps that N uses on any branch of its computation on any input of length n.

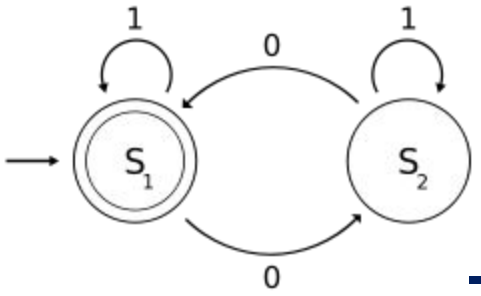




The Class NP

Definition. $\mathrm{NTIME}(t(n)) = \{ L \mid L \text{ is decided in } O(t(n)) \text{ time by an NTM.} \}$

Corollary. $\mathrm{NP} = \cup_k \mathrm{NTIME}(n^k)$



Certificates

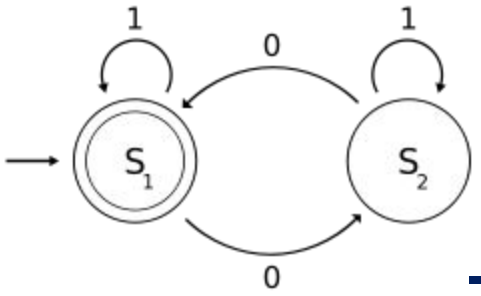
NP is the class of languages that have polynomial time verifiers.

What is a certificate for *PATH*?

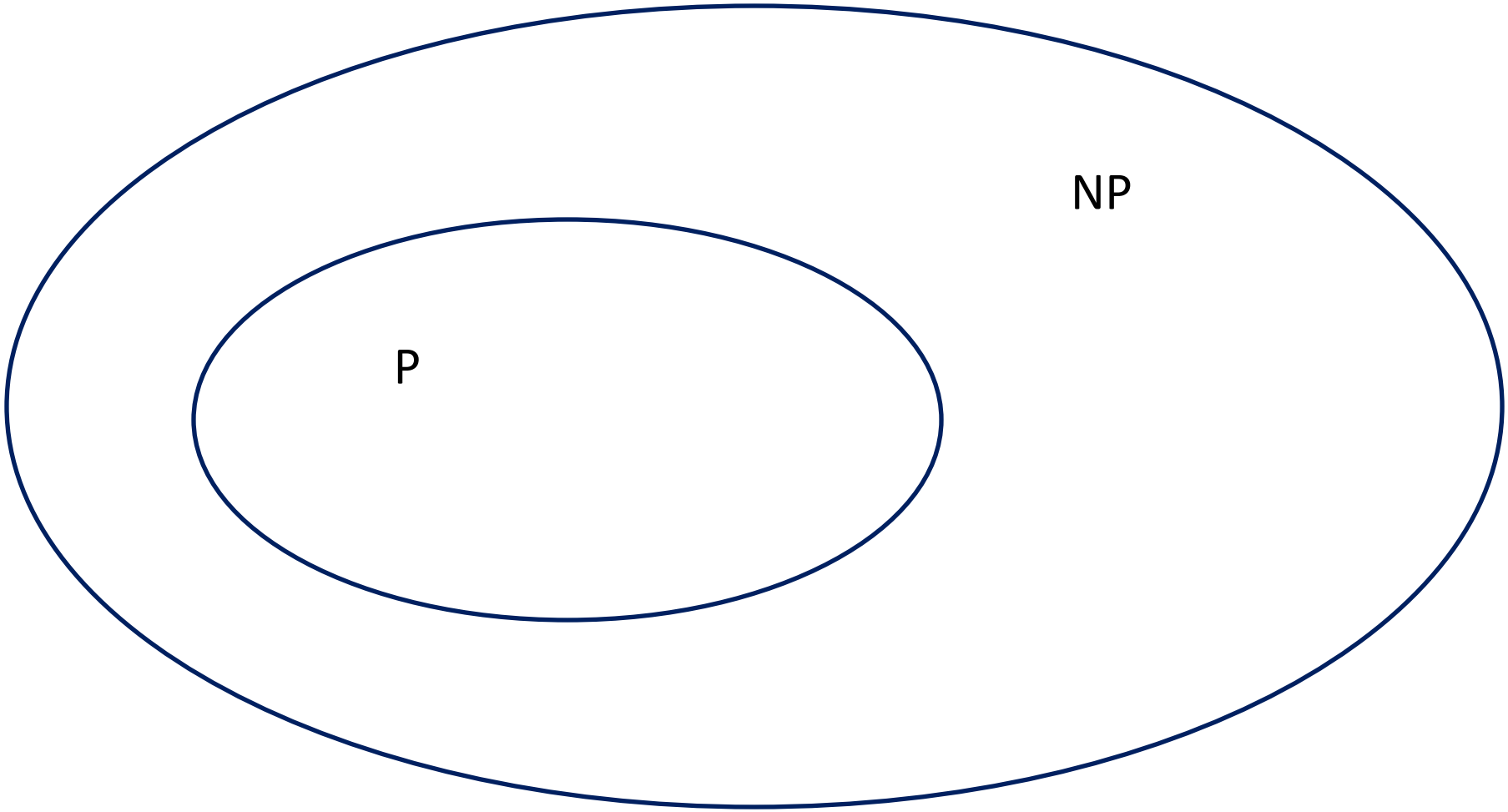
What is a certificate for *HAMPATH*?

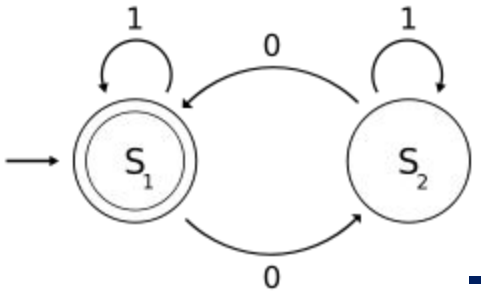
What is a certificate for *COMPOSITE*?

What is a certificate for $\overline{HAMPATH}$?



The Classes P and NP

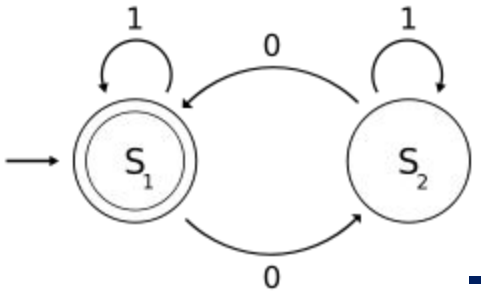




P = NP?

$$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME$$

Proper containment



Exercises

Let *CONNECTED* = { <*G*> | *G* is a connected undirected graph}.
Is *CONNECTED* in NP? (can we verify a candidate solution in polynomial time on a nondeterministic machine?)
Is *CONNECTED* in P? (can we find an algorithm to decide if a graph is connected in polynomial time on a deterministic TM?)

A ***triangle*** in an undirected graph is a 3-clique.
Let *TRIANGLE* = { <*G*> | *G* contains a triangle}.
Is *TRIANGLE* in NP? Is *TRIANGLE* in P?

Call the graphs *G* and *H* ***isomorphic*** if the nodes of *G* may be reordered so that it is identical to *H*.
Let *ISO* = { <*G*, *H*> | *G* and *H* are isomorphic graphs}.
Is *ISO* in NP? Is *ISO* in P?