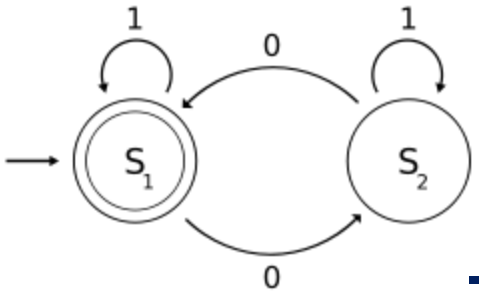
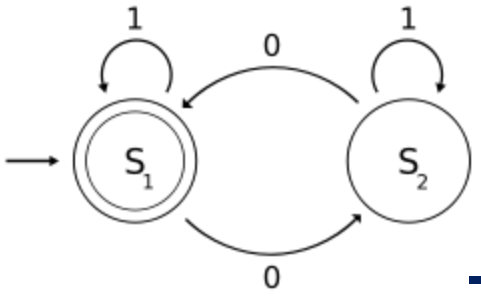
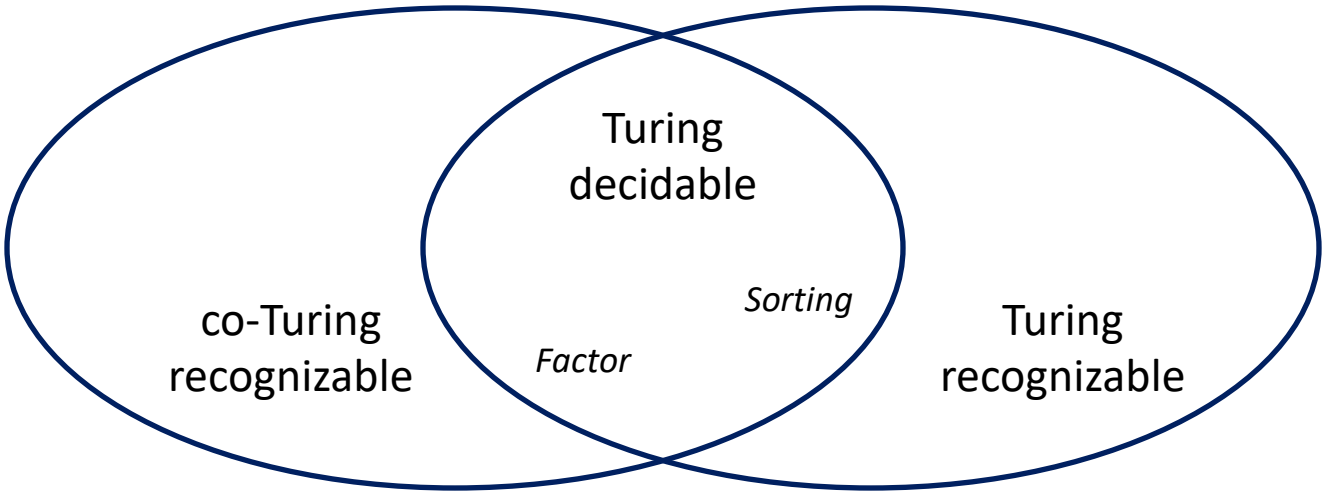


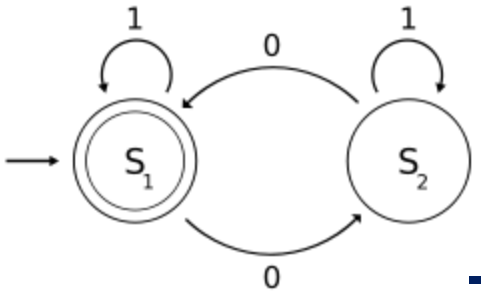
Measuring Time Complexity

Sipser: Section 7.1 pages 275 - 284



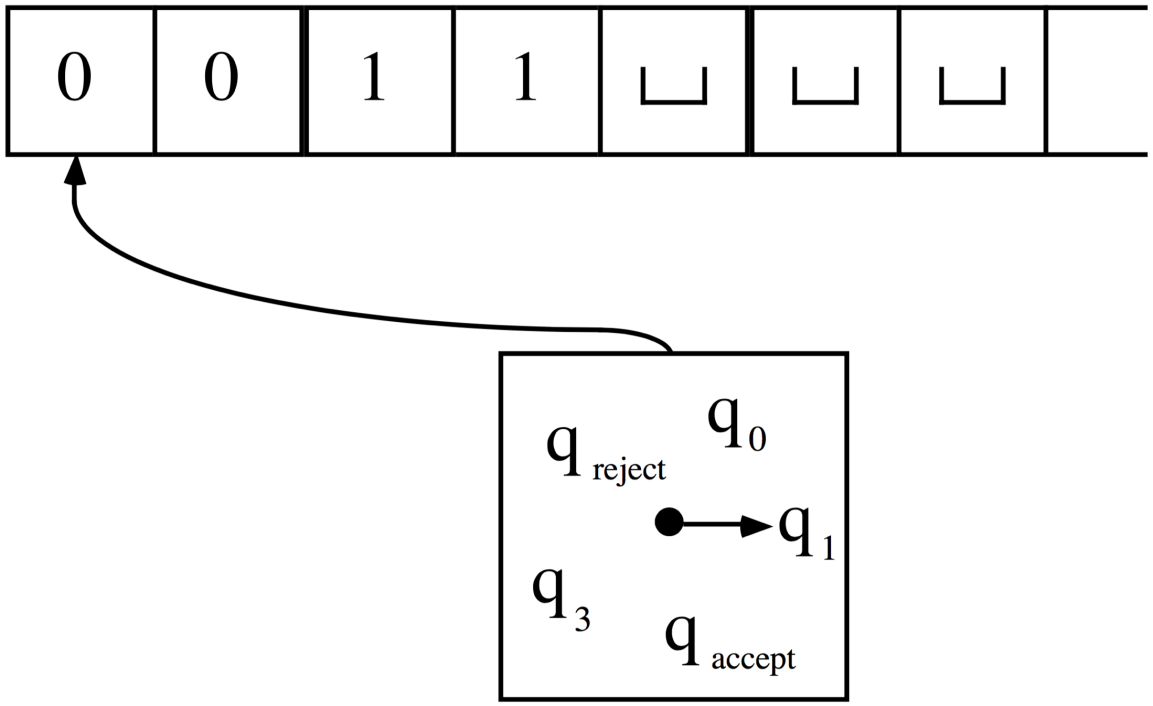
Solvable in Theory …

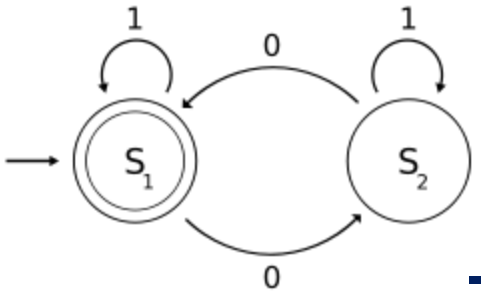




Measuring Difficultly

How hard is it to recognize $\{ 0^k1^k \mid k \geq 0 \}$?

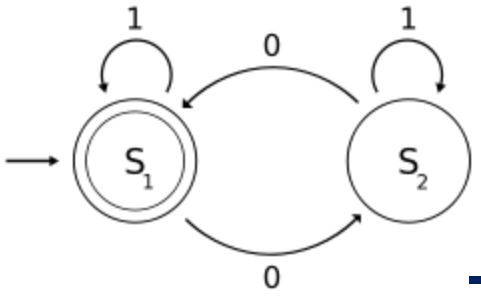




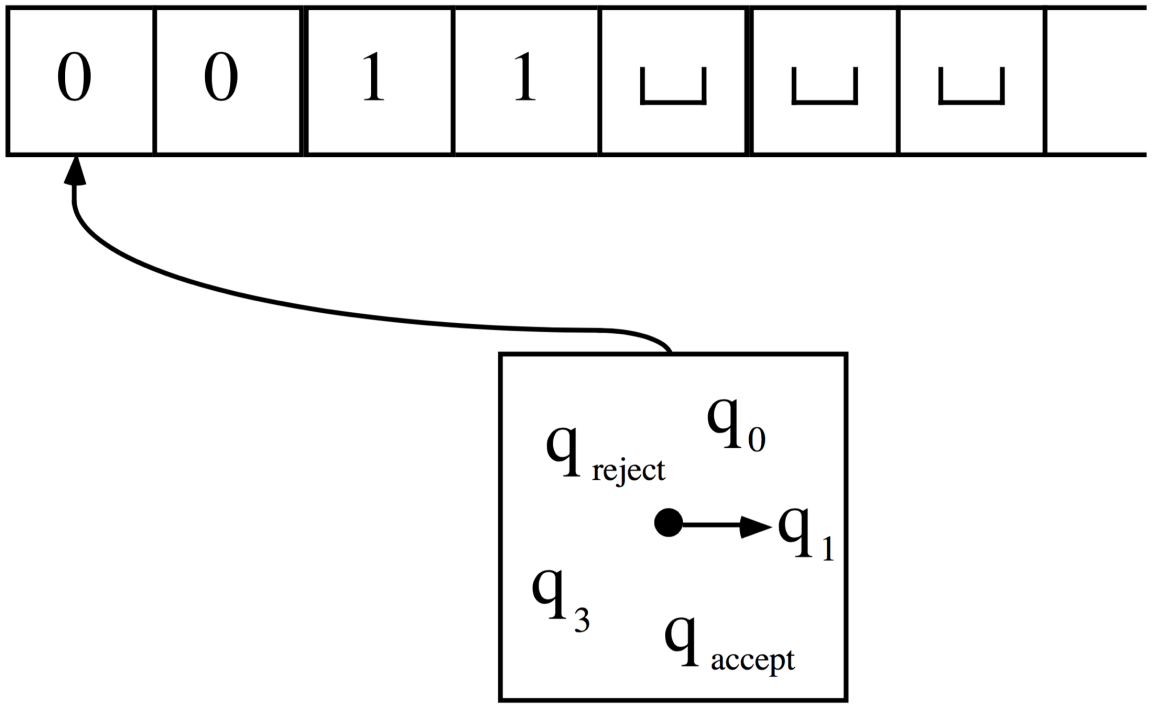
An Algorithm

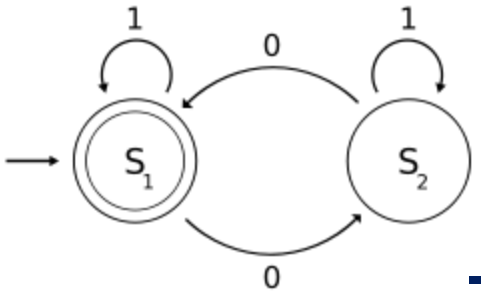
M_1 = "On input string w:

1. Scan across the tape and *reject* if a 0 is found to the right of a 1.
2. Repeat the following if both 0s and 1s remain.
3. Scan across tape, crossing off a single 0 and a single 1.
4. If either 0 or 1 remains, *reject*. Otherwise, *accept*."

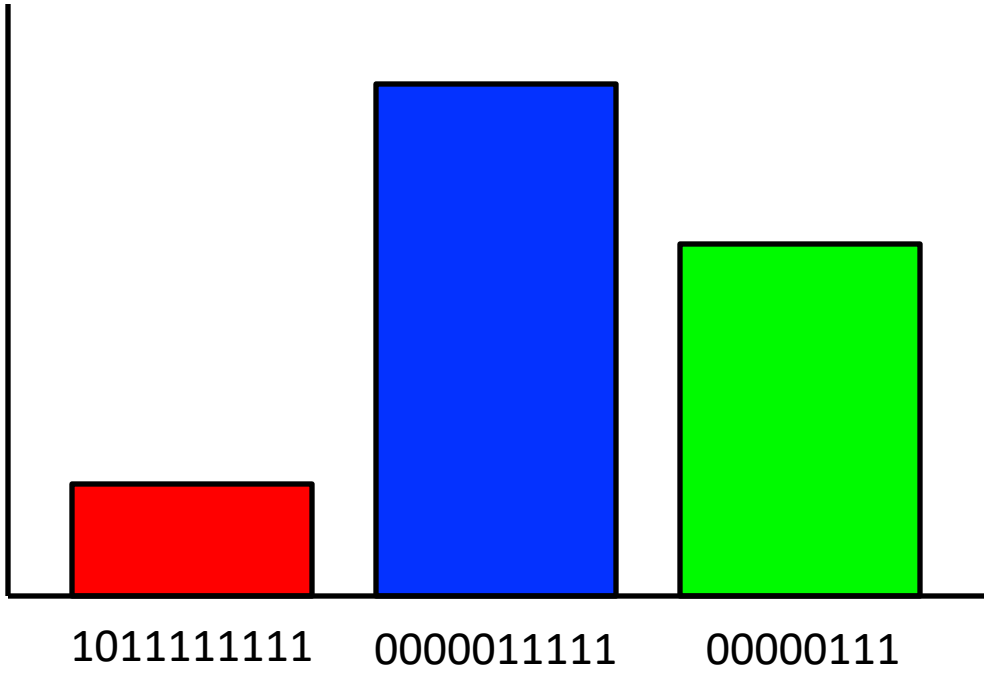


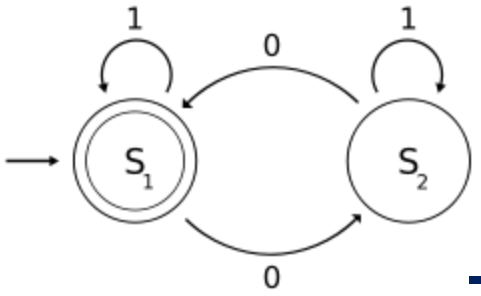
Number of Steps Depends on Size of Input





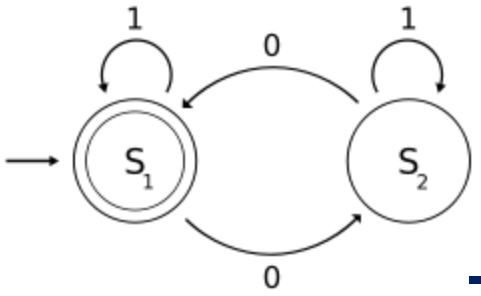
The Good, the Bad, and the Average



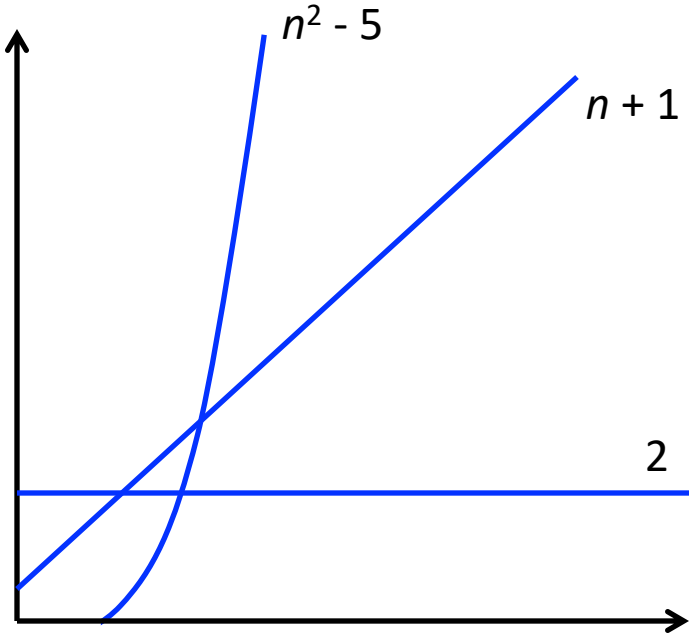


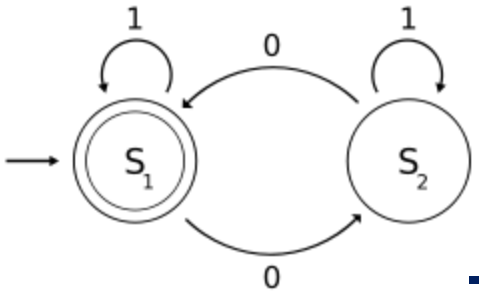
Time Complexity

Definition. The ***time complexity*** of TM M is the function $f: \mathbb{N} \longrightarrow \mathbb{N}$, where $f(n)$ is the maximum number of steps that M uses on any input of length n.



Asymptotic Analysis





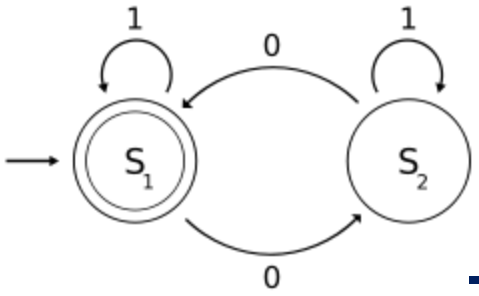
Big O, Little o

Definition. Consider $f, g : \mathbb{N} \longrightarrow \mathbb{R}^+$. Say that $f(n) = O(g(n))$ if positive integers c and n_0 exist such that for every $n \geq n_0$

$$f(n) \leq c\, g(n)$$

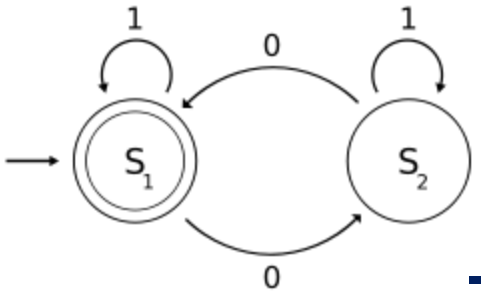
Definition. Consider $f, g : \mathbb{N} \longrightarrow \mathbb{R}^+$. Say that $f(n) = o(g(n))$ if

$$\lim_{n \longrightarrow \infty} f(n)/g(n) = 0$$

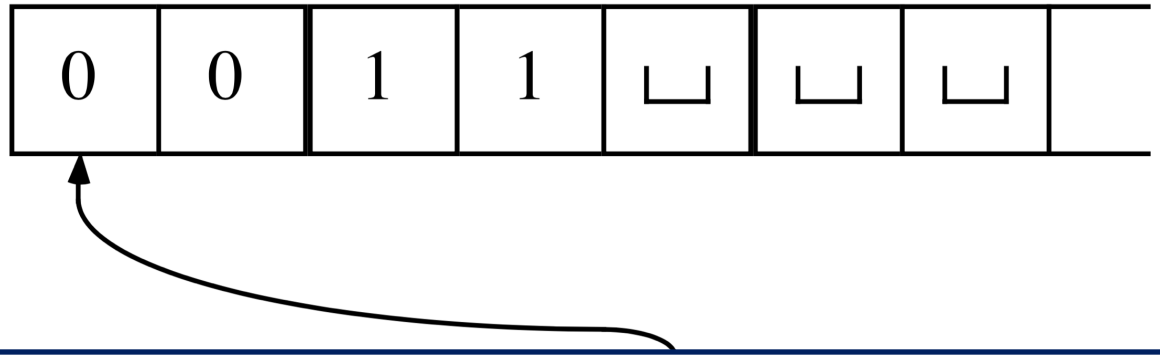


True or False?

1. $8n + 5 = O(n)$
2. $8n + 5 = o(n)$
3. $\sqrt{n} = o(n)$
4. $\log_2 n = o(\ln n)$
5. $n \log \log n = o(n \log n)$
6. $n^2 = o(n \log n)$

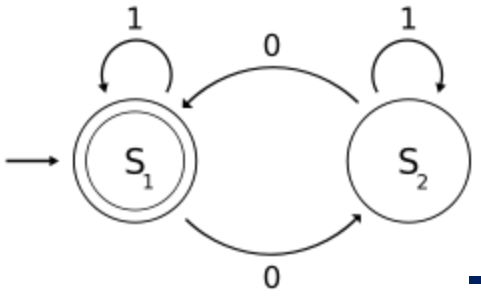


The Verdict Please …



M_1 = "On input string w:

1. Scan across the tape and *reject* if a 0 is found to the right of a 1.
2. Repeat the following if both 0s and 1s remain.
3. Scan across tape, crossing off a single 0 and a single 1.
4. If either 0 or 1 remains, *reject*. Otherwise, *accept*."



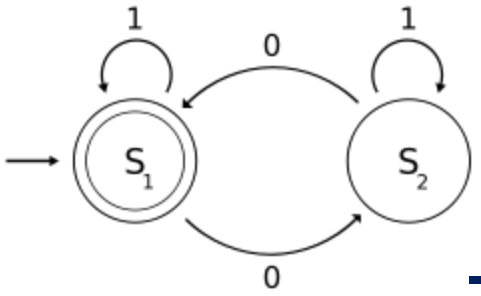
Time Complexity Classes

Definition. Let $t : \mathbb{N} \longrightarrow \mathbb{N}$ be a function. Define the time complexity class, TIME($t(n)$), to be

$$\text{TIME}(t(n)) = \{ L \mid L \text{ is decided by } O(t(n)) \text{ time TM} \}$$

Example. The language $\{ 0^k1^k \mid k \geq 0 \} \in \text{TIME}(n^2)$.*

* And TIME(n^3) and TIME(n^4) and …



Losing TIME Complexity

M_2 = "On input string w:

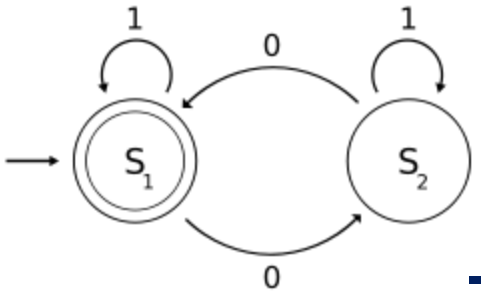
1. Scan across the tape and *reject* if a 0 is found to the right of a 1.

2. Repeat the following if both 0s and 1s remain.

3. Scan across tape, checking whether the total number of 0s and 1s remaining on the tape is even or odd. If odd, *reject*.

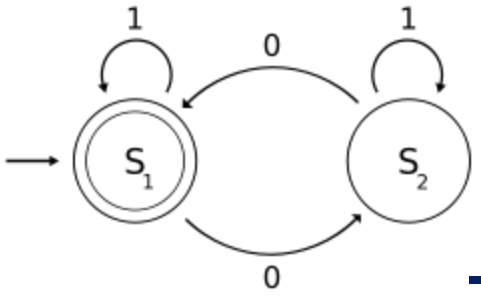
4. Scan again across tape, crossing off every other 0 and every other 1.

5. If no 0s or 1s remain, *accept*. Otherwise, *reject*."

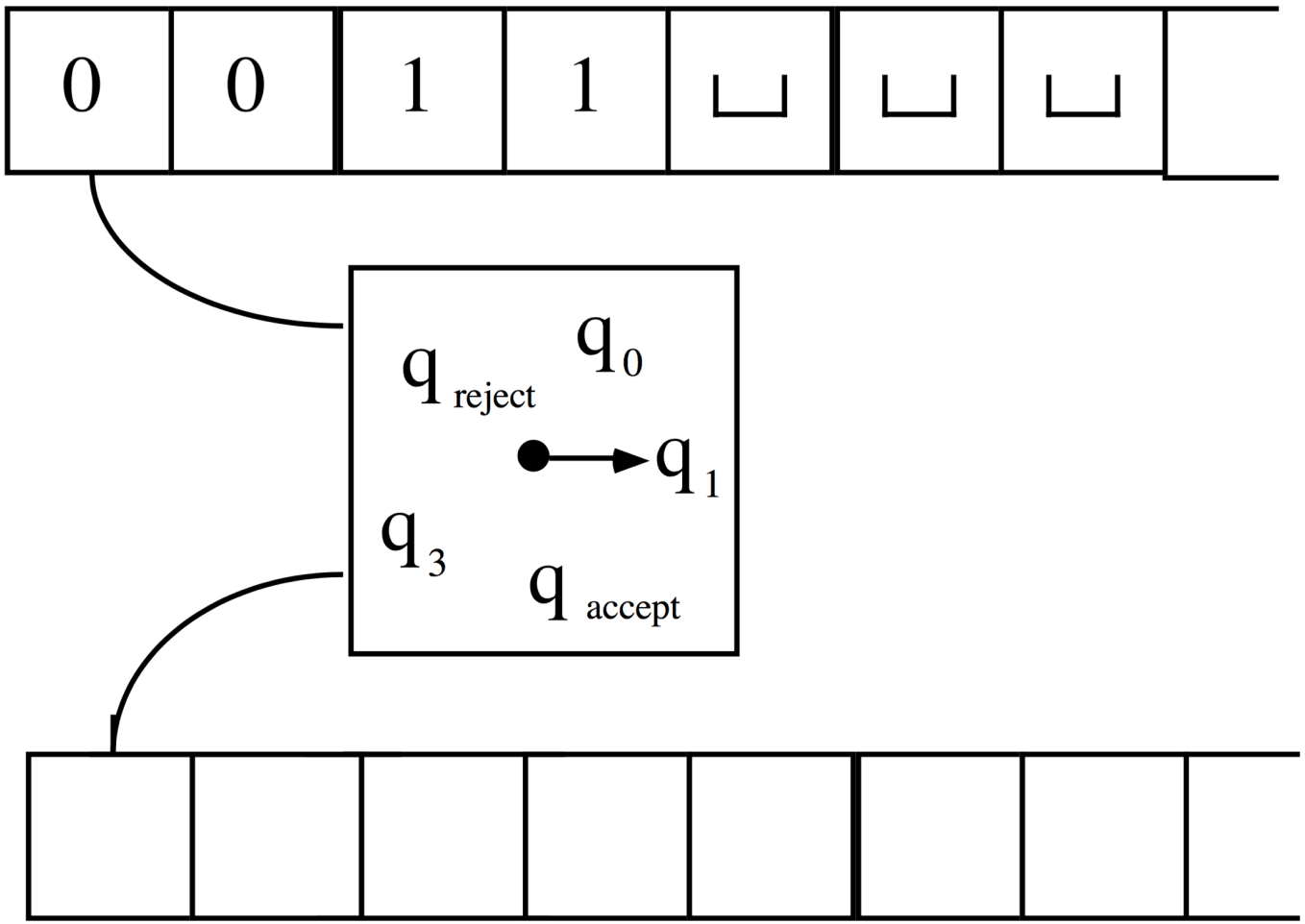


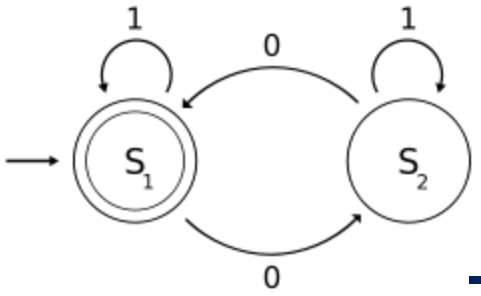
Better Still???

Theorem. Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ belong to $o(n \log n)$. If language $A \in \mathrm{TIME}(f)$, then A is regular.



Ah, But What If We Had Two Tapes …



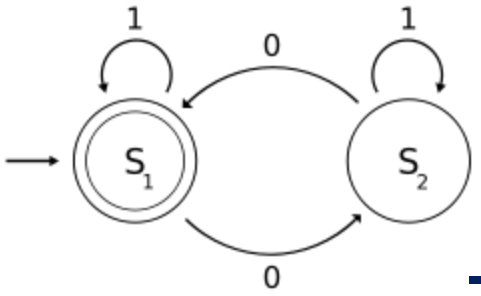


Well, Then We Could Do Better Still!*

Theorem. Let $t(n)$ be a function, where $t(n) \geq n$. Then every $t(n)$ time multitape Turing machine has an equivalent $O(t^2(n))$ time single-tape Turing machine.

Proof. Compare the time complexity of the given multitape machine with the equivalent single tape machine that simulates it.

* But only slightly.



How Costly Is Nondeterminism?

Theorem. Let $t(n)$ be a function, where $t(n) \geq n$. Then every $t(n)$ time nondeterministic single-tape Turing machine has an equivalent $2^{O(t(n))}$ time deterministic single-tape Turing machine.

Proof.

Compare the time complexity of the given nondeterministic machine with the equivalent single tape deterministic machine that simulates it.