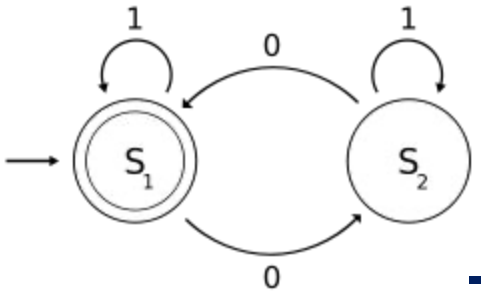


NP-Complete Problems

Sipser: Section 7.5 pages 311 - 322

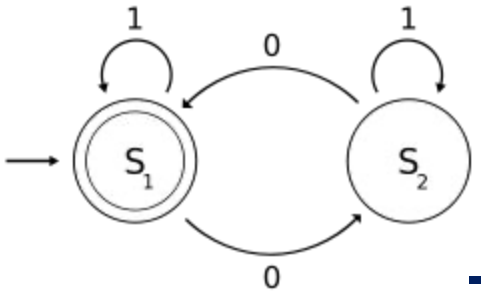


Today's adventures



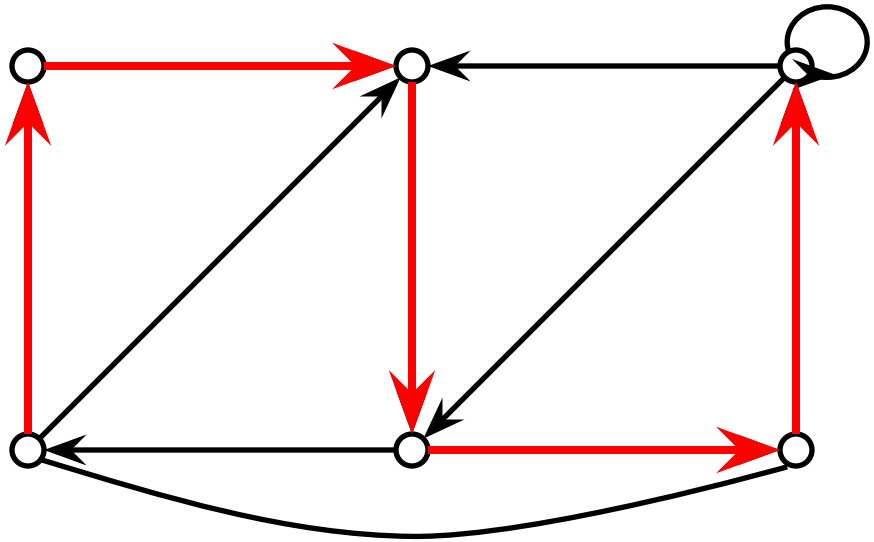
- Reduce 3SAT to Hampath
- Reduce 3SAT to Subset-Sum
- More NP-Complete examples

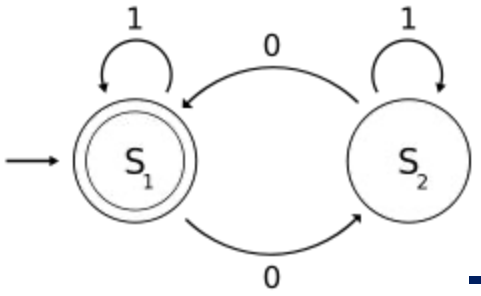
Weekly tip: Strive for excellence, not perfection



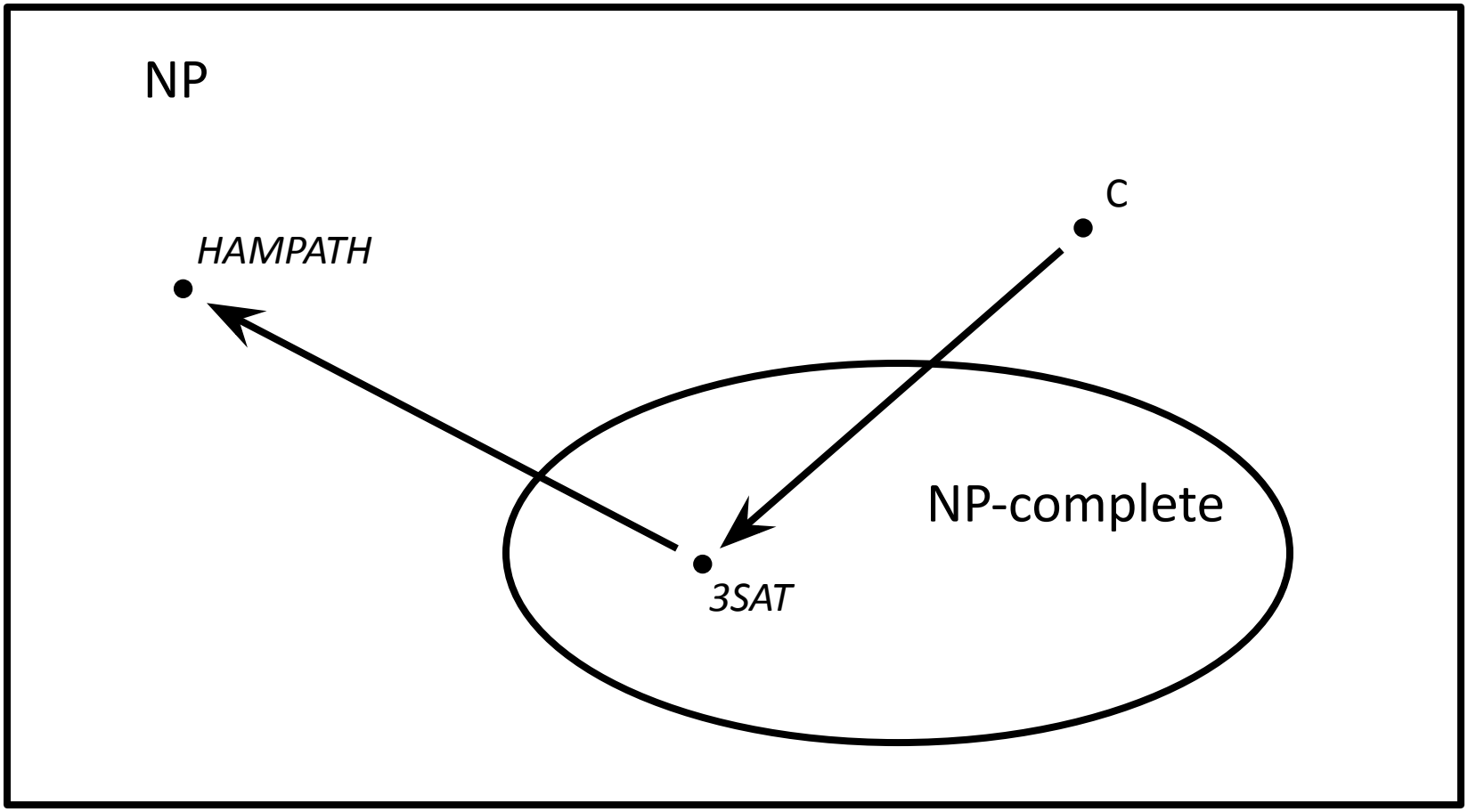
More NP-Complete Problems

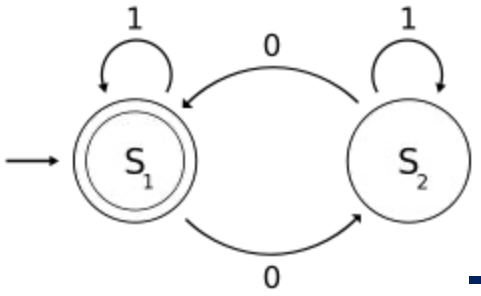
Theorem. HAMILTONIAN PATH is NP-complete.





We Reduce *3SAT* to *HAMPATH*





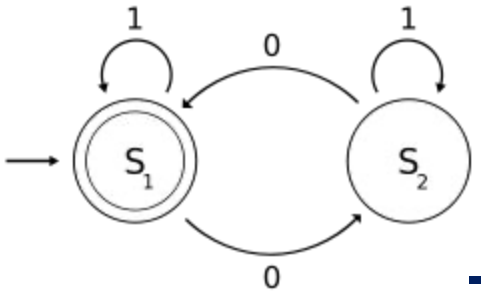
More Precisely,

Proof Outline.

Given an instance of *3SAT*,

$$\varphi = (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3 \vee \neg x_4) \wedge (x_2 \vee \neg x_4 \vee x_5)$$

we construct a directed graph G so that φ is satisfiable iff G has a Hamiltonian path.



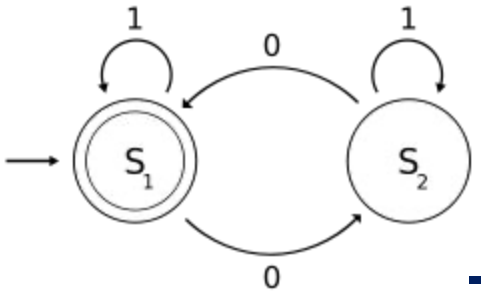
3SAT's Salient Features*

Choice. Each variable has a choice between two truth values.

Consistency. Different occurrences of the same variable have the same value.

Constraints. Variable occurrences are organized into *clauses* that provide constraints that must be satisfied.

* We model each of these three features by a different "gadget" in the graph *G*.



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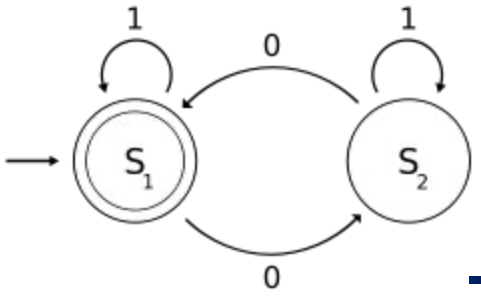
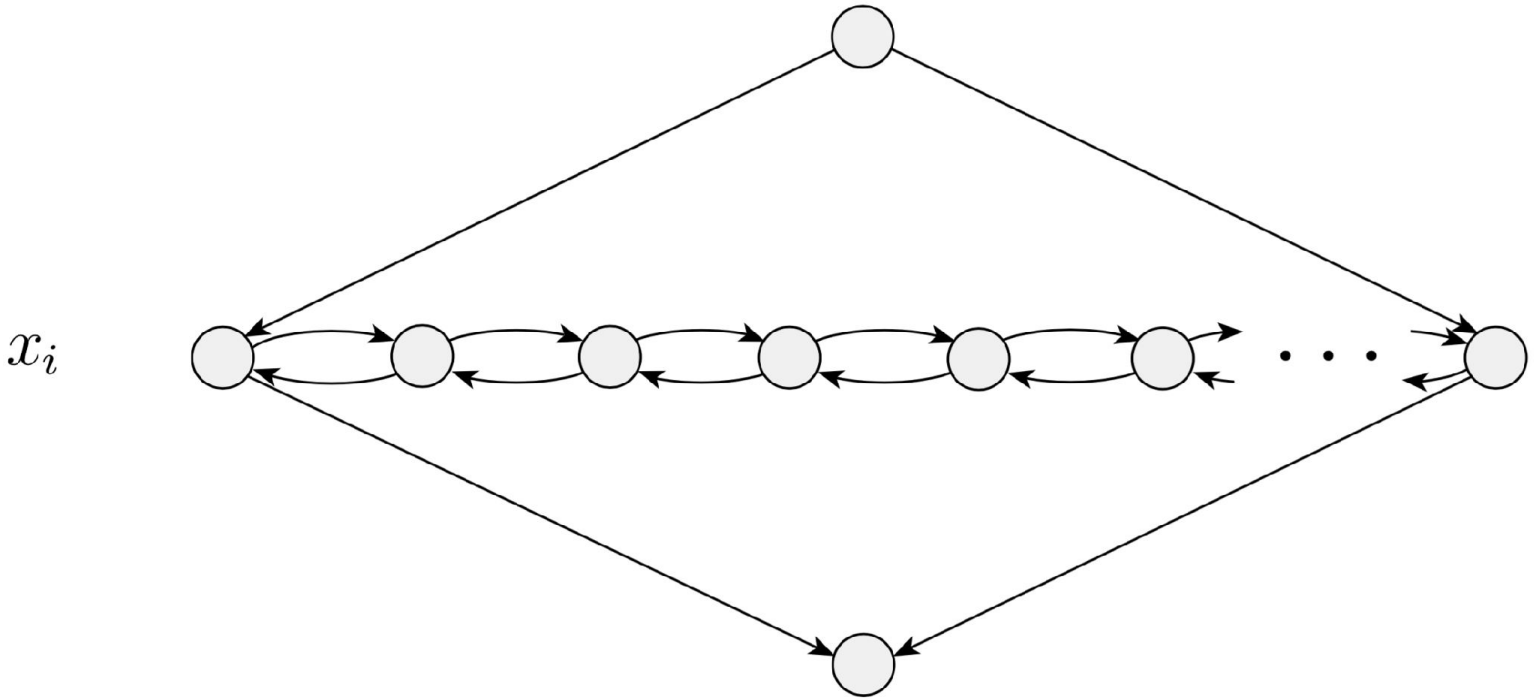
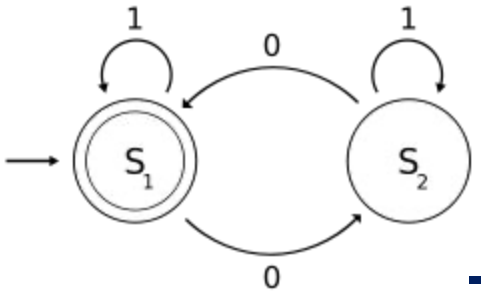


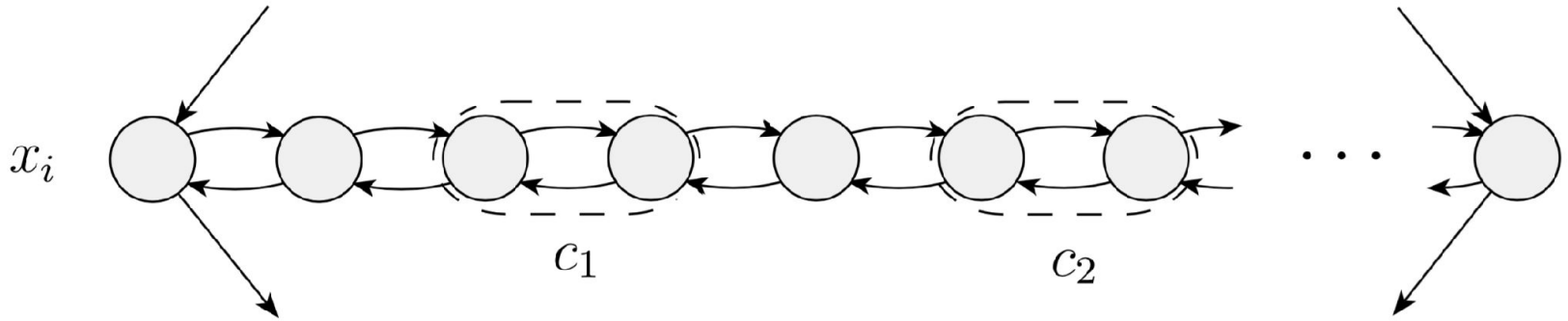
Modeling Variable x_i*

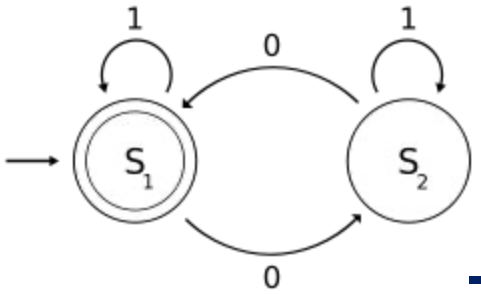


* The choice gadget.



The Consistency Gadget





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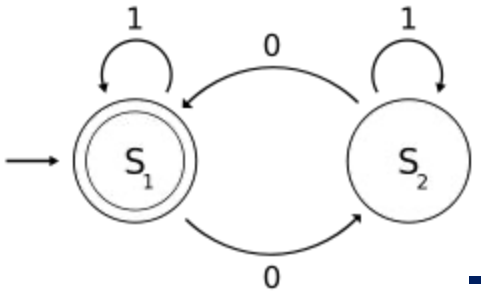
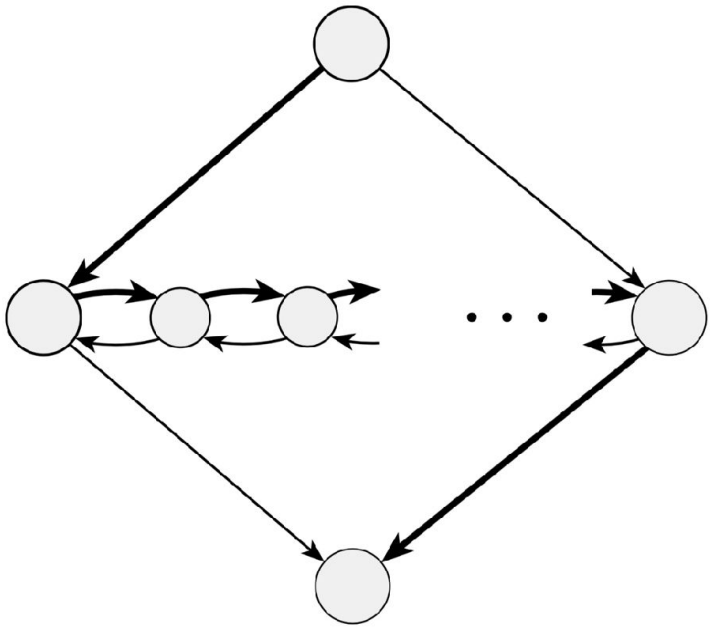
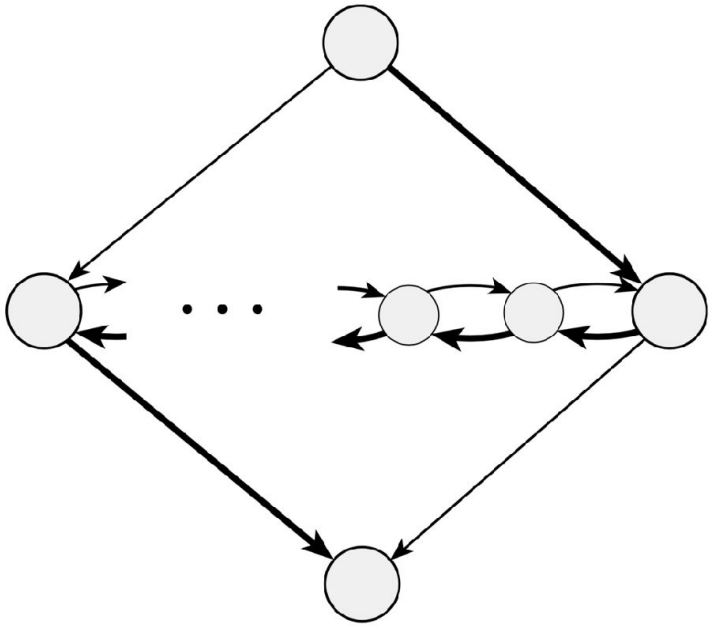


Zig-zagging and Zag-zigging

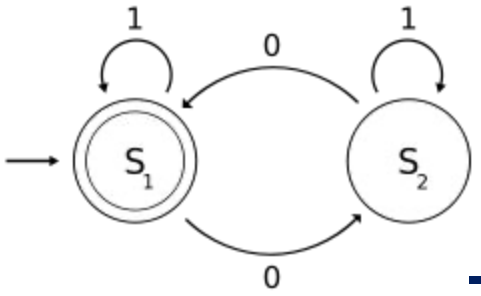


zig-zag



zag-zig

* The consistency gadget in action.



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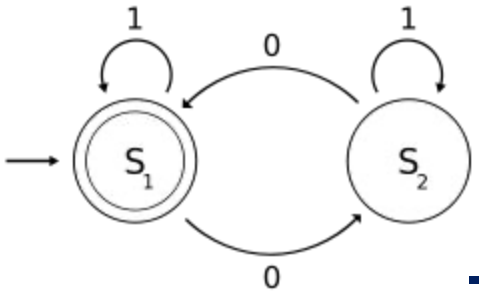
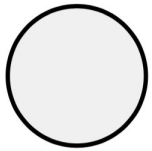
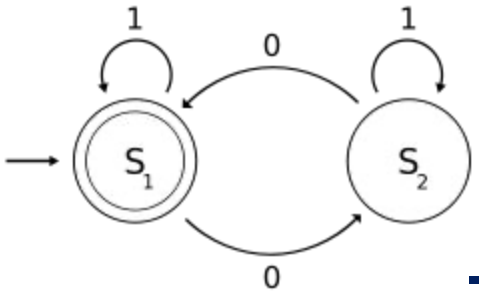


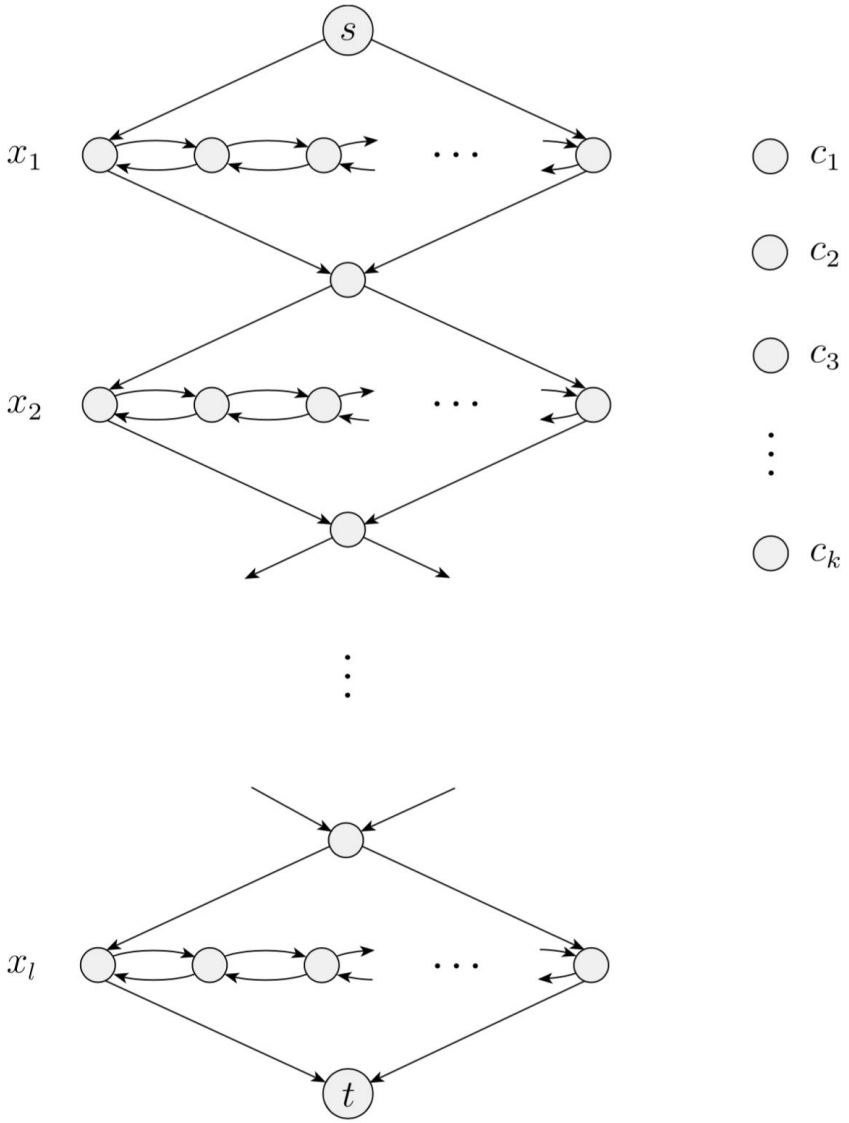
Modeling Clause c_j

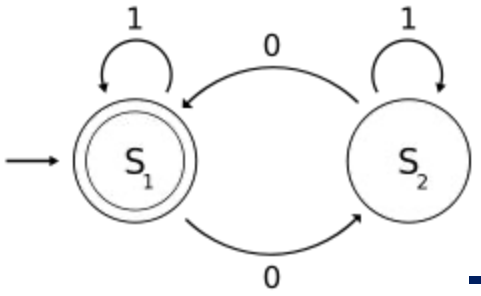


c_j

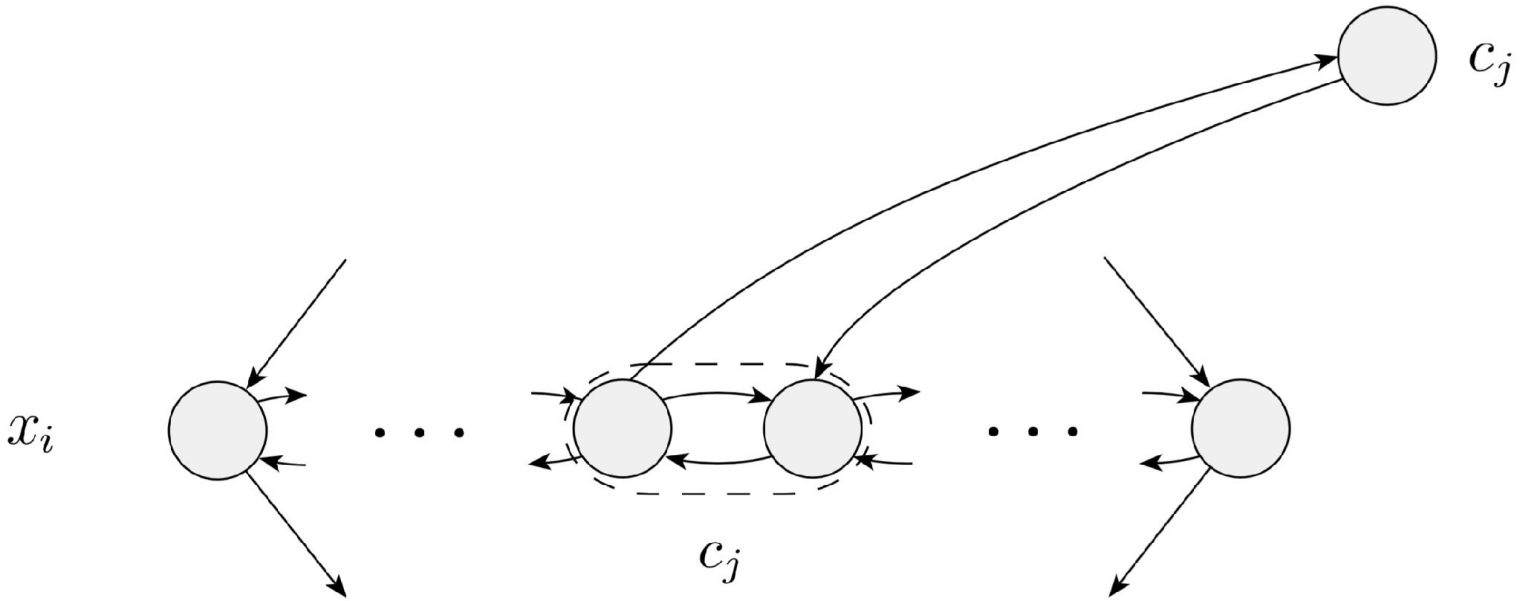


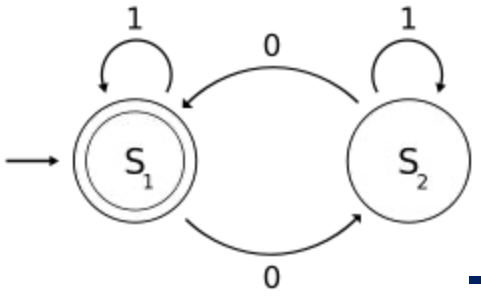
The Big Picture



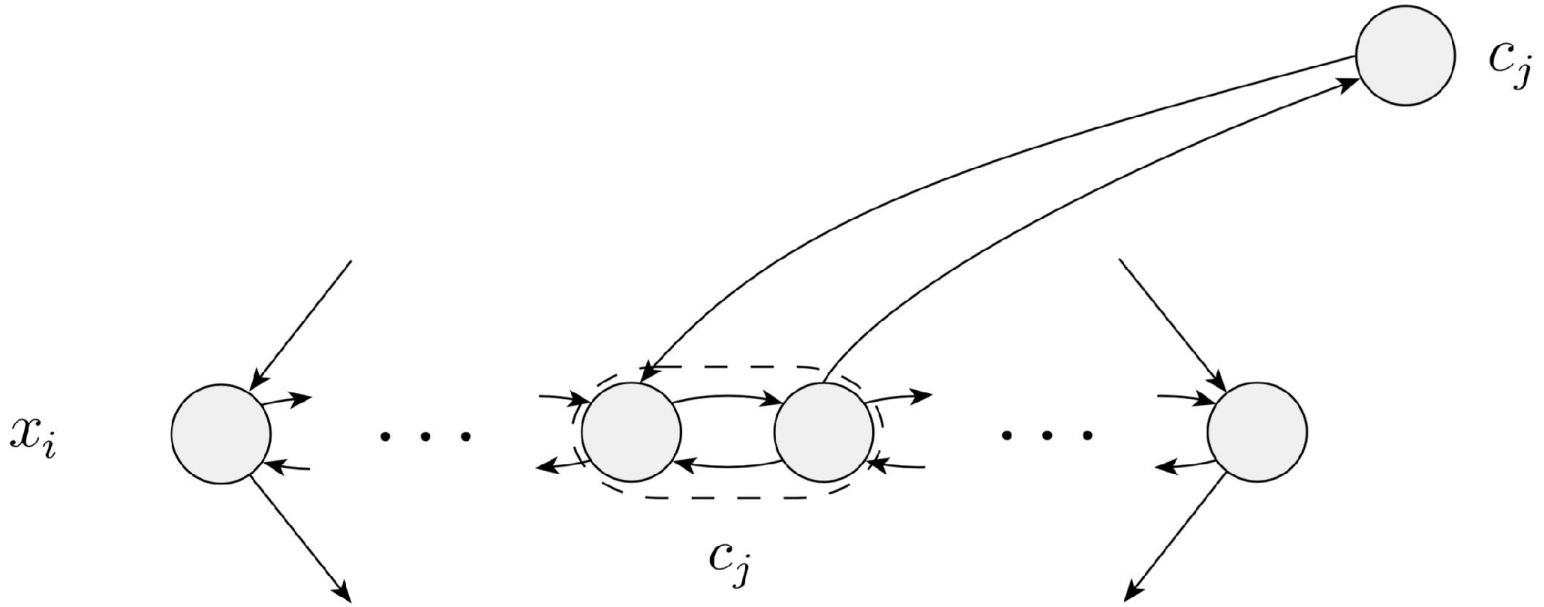


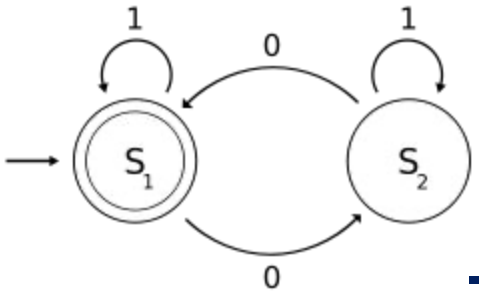
The Constraint Gadget when c_j Contains x_i



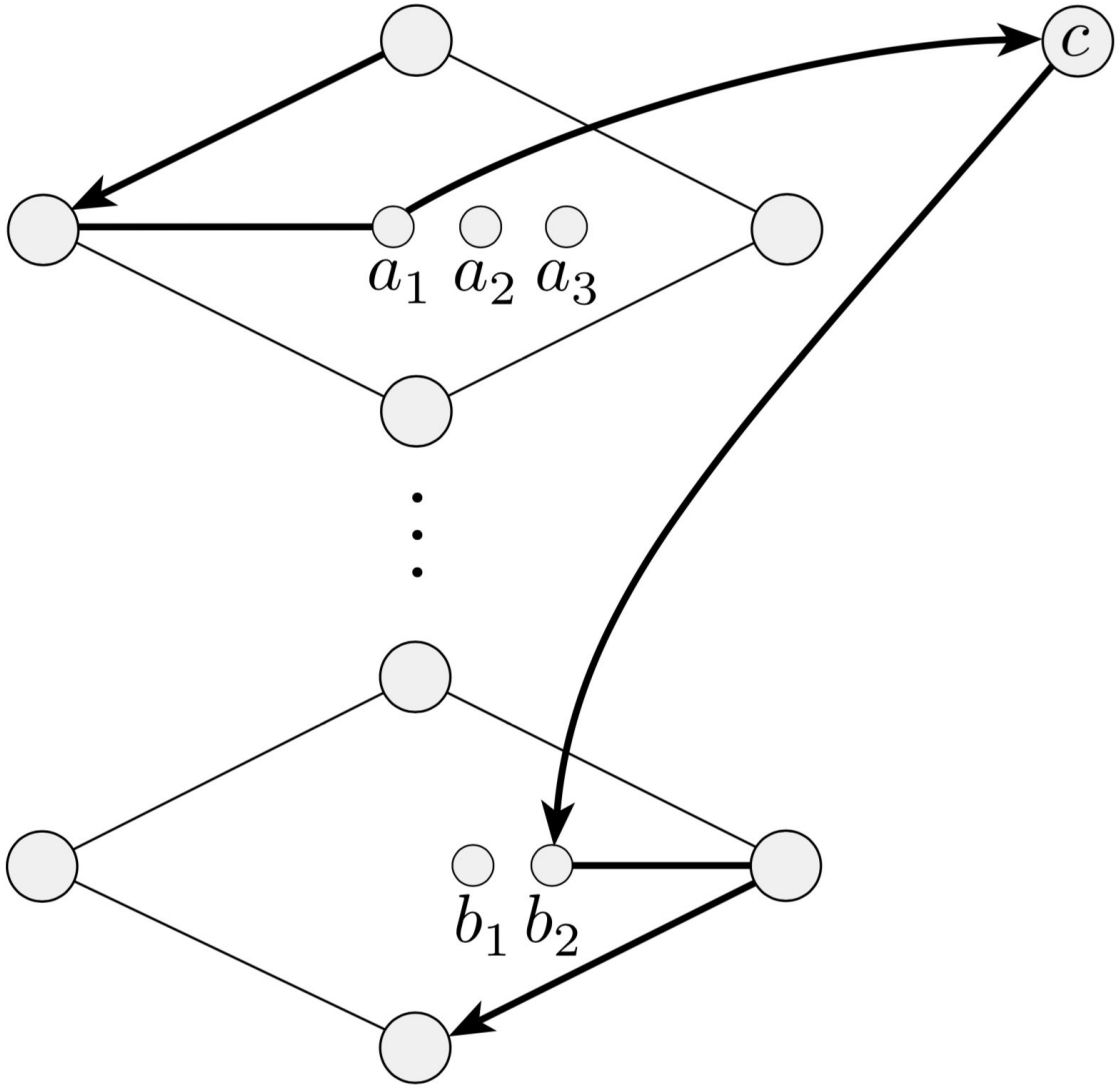


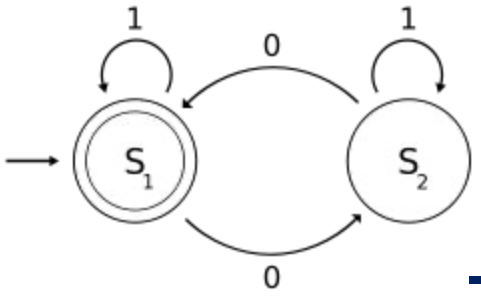
The Constraint Gadget when c_j Contains $\neg x_i$





A Situation that Cannot Occur

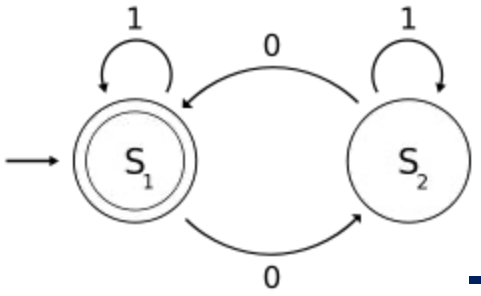




SUBSET-SUM is NP-Complete

SUBSET-SUM: Given a set of integers, does any subset sum to *t*?

45 4 -18 16 -21 201 115 -8 -12 -64 61 -17 14 94 -190 -89 51 -79 77 23 -106 48 57 -35 141 28 -219 81



SUBSET-SUM is NP-Complete

We show *3SAT* $\leq_p$ *SUBSET-SUM* as in the following example.

Given

$(x_1 \vee \neg x_2 \vee x_3) \wedge$

$(x_2 \vee x_3 \vee \ldots) \wedge$

$\ldots \wedge$

$(\neg x_3 \vee \ldots)$

we construct*:

	1	2	3	4	$\cdots$	l	c_1	c_2	$\cdots$	c_k
y_1	1	0	0	0	$\cdots$	0	1	0	$\cdots$	0
z_1	1	0	0	0	$\cdots$	0	0	0	$\cdots$	0
y_2		1	0	0	$\cdots$	0	0	1	$\cdots$	0
z_2		1	0	0	$\cdots$	0	1	0	$\cdots$	0
y_3			1	0	$\cdots$	0	1	1	$\cdots$	0
z_3			1	0	$\cdots$	0	0	0	$\cdots$	1
$\vdots$					$\ddots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
y_l						1	0	0	$\cdots$	0
z_l						1	0	0	$\cdots$	0
g_1							1	0	$\cdots$	0
h_1							1	0	$\cdots$	0
g_2								1	$\cdots$	0
h_2								1	$\cdots$	0
$\vdots$									$\ddots$	$\vdots$
g_k										1
h_k										1
t	1	1	1	1	$\cdots$	1	3	3	$\cdots$	3

*these are all decimal numbers