Nondeterminism
Let \( A = \{ w \mid w \text{ is a string of } 0\text{s and } 1\text{s containing an odd number of } 1\text{s} \} \) and let \( B = \{ w \mid w \text{ is a string of } 0\text{s and } 1\text{s containing an even number of } 1\text{s} \} \).

Construct a (nondeterministic) machine \( N \) that recognizes language \( A \circ B \).
Something's Fishy

\[ \epsilon \]

\[ q_1 \]
\[ q_2 \]
\[ \hat{q}_1 \]
\[ \hat{q}_2 \]
Relaxing the Rules

**Deterministic (DFA)**

**Nondeterministic (NFA)**
How Does That Compute?

Deterministic computation

- start
- ...
- accept or reject

Nondeterministic computation

- reject
- ...
- accept
For Example

\( N_1 \)

Input 010110

Symbol read

0 \( \rightarrow \) \( q_1 \)

1 \( \rightarrow \) \( q_2 \)

0, \( \varepsilon \) \( \rightarrow \) \( q_3 \)

1 \( \rightarrow \) \( q_4 \)
Another Example
A nondeterministic finite automaton is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\), where

1. \(Q\) is a finite set called the states,
2. \(\Sigma\) is a finite set called the alphabet,
3. \(\delta : Q \times \Sigma \epsilon \rightarrow \mathcal{P}(Q)\) is the transition function,
4. \(q_0 \in Q\) is the start state, and
5. \(F \subseteq Q\) is the set of accept states.
Let \( N = (Q, \Sigma, \delta, q_0, F) \) be an NFA and \( w \) a string over the alphabet \( \Sigma \). Then \( N \) accepts \( w \) if we can write \( w = y_1 y_2 \cdots y_m \), where each \( y_i \) is a member of \( \Sigma_\varepsilon \) and a sequence of states \( r_0, r_1, \ldots, r_m \) exists in \( Q \) with three conditions:

1. \( r_0 = q_0 \),
2. \( r_{i+1} \in \delta(r_i, y_{i+1}) \), for \( i = 0, \ldots, m-1 \), and
3. \( r_m \in F \).
Build an NFA that recognizes the language $B = \{ w \mid w$ is a string of $a$s and $b$s that starts and ends with the same symbol and contains at least two symbols $\}$. 
When You Can't Prove What You Want...

**Theorem.** The class of regular languages is closed under concatenation.
Theorem. The class of regular languages recognized by NFAs is closed under concatenation.

Proof.
Kleene Star

**Theorem.** The class of regular languages recognized by NFAs is closed under Kleene star.

**Proof.**
Suppose ...

... somebody showed that the class of languages accepted by NFAs and the class of languages accepted by DFAs were equal ...

**Theorem.** A language is regular if and only if it is accepted by a nondeterministic finite automata.
We Would Have ...

**Corollary.** The class of regular languages is closed under concatenation.

**Corollary.** The class of regular languages is closed under Kleene star.
We Also Have a Nifty Proof of Closure Under Unions*

Theorem. The class of regular languages is closed under union.

Proof.

*Which turns out to help construct machines for recognizing regular languages.