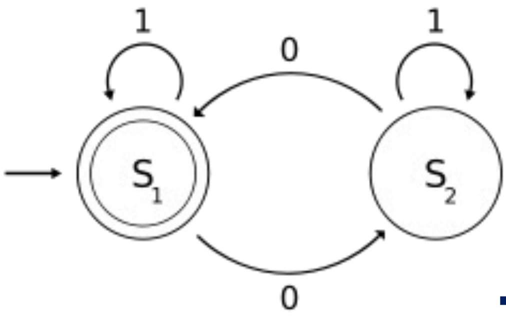
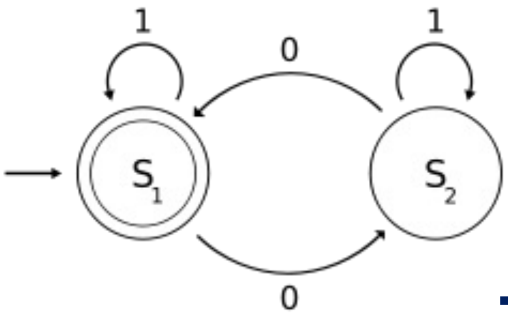


The Equivalence of NFAs and DFAs

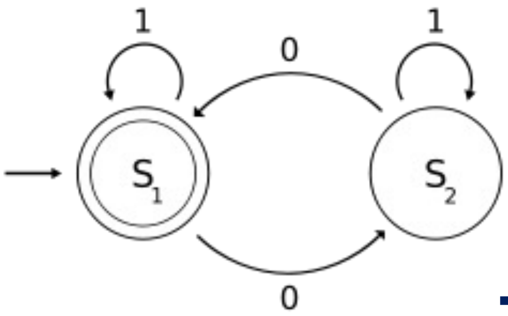
Sipser: Section 1.2 pages 54 - 58



Unfinished Business

Theorem. *A* language is regular if and only if it is accepted by a nondeterministic finite automaton.

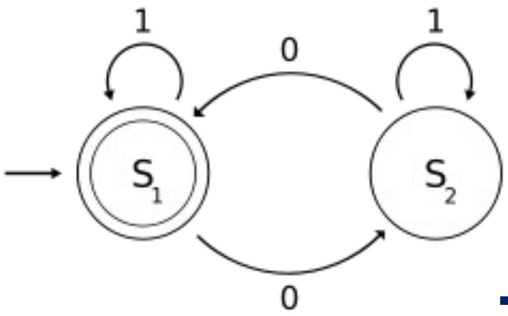
Proof. (⇒) Let *A* be a regular language …



DFA

A **finite automaton** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

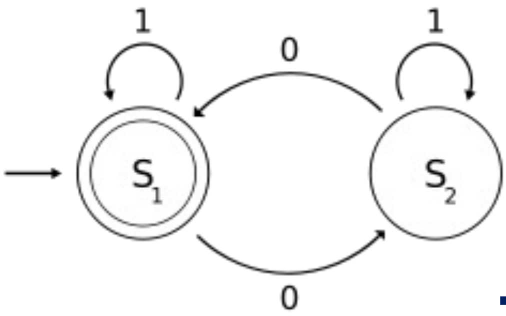
1. Q is a finite set called the **states**,
2. Σ is a finite set called the **alphabet**,
3. $\delta: Q \times \Sigma \rightarrow Q$ is the **transition function**,
4. $q_0 \in Q$ is the **start state**, and
5. $F \subseteq Q$ is the **set of accept states**.



NFA

A **nondeterministic finite automaton** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

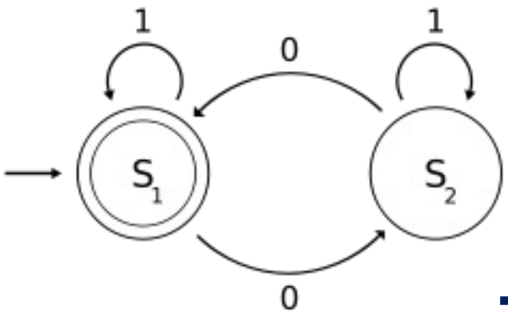
1. Q is a finite set called the **states**,
2. Σ is a finite set called the **alphabet**,
3. $\delta: Q \times \Sigma_\varepsilon \rightarrow P(Q)$ is the **transition function**,
4. $q_0 \in Q$ is the **start state**, and
5. $F \subseteq Q$ is the **set of accept states**.



Going the Other Way

Theorem. *A* language is regular if and only if it is accepted by a nondeterministic finite automaton.

Proof. (⇐) Suppose *A* is accepted by a nondeterministic finite automaton …

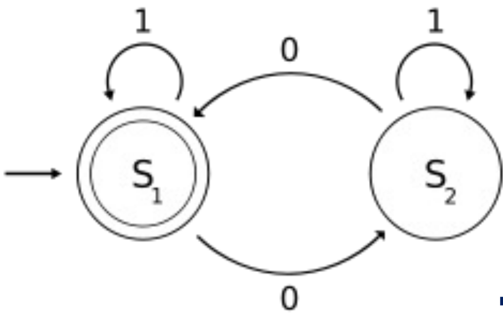


Equivalent Languages

Definition. Two machines are *equivalent* if they recognize the same language.

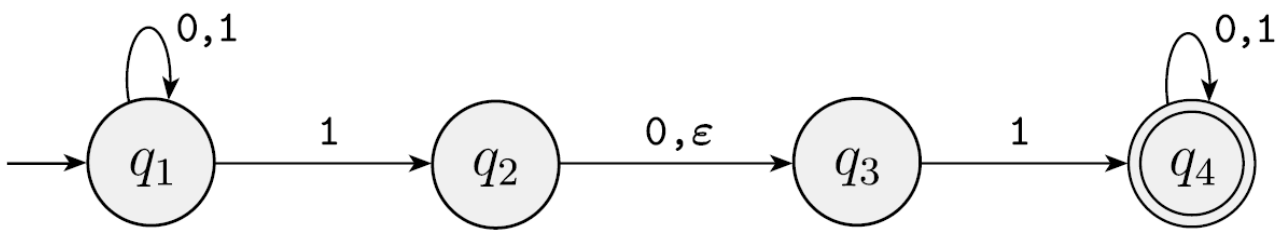
Theorem. Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Proof.

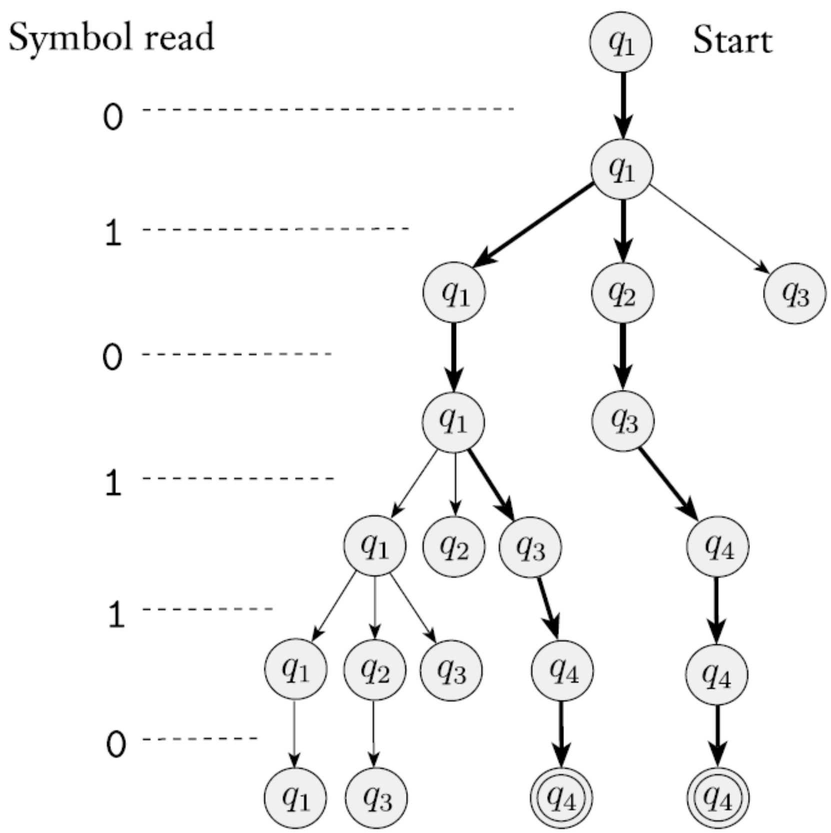


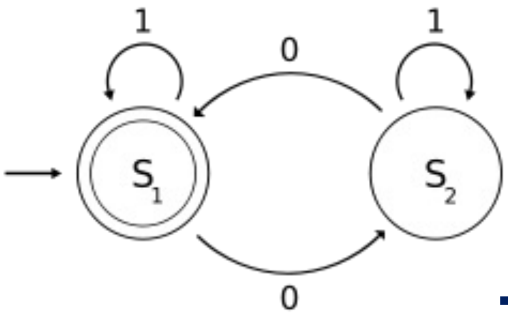
You Only Go Around Once

N_1

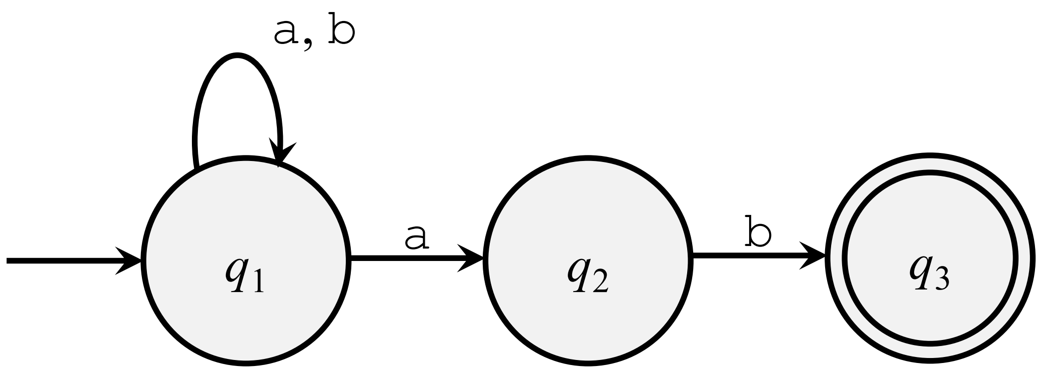


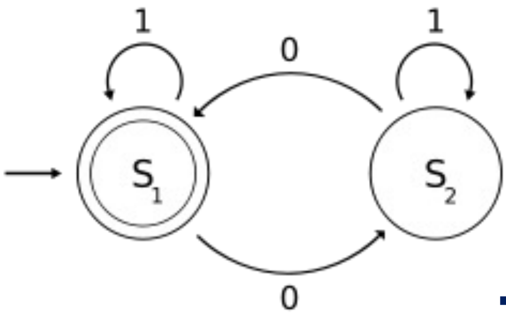
Input 010110





A Simpler Example



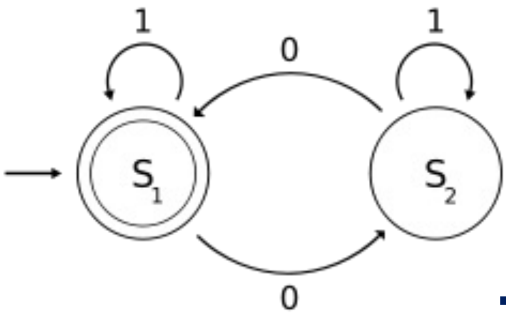


Removing Choice

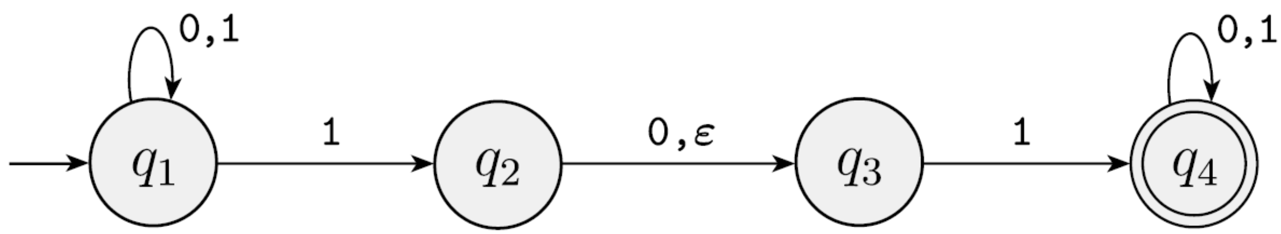
Proof. Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA recognizing language A.

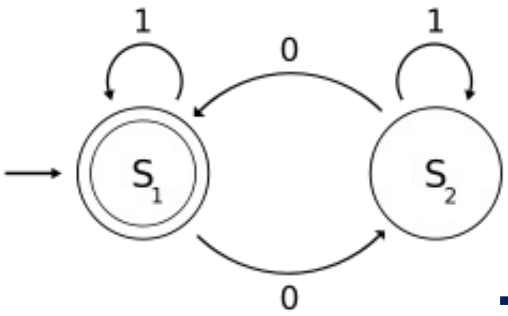
We construct a DFA M recognizing A.

1. $Q' = P(Q)$.
2. For $R \in Q'$ and $a \in \Sigma$, let $\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$.
3. $q_0' = \{q_0\}$.
4. $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$.



What About ε Arrows?





Modifying Our Construction

Proof. Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA recognizing language A.

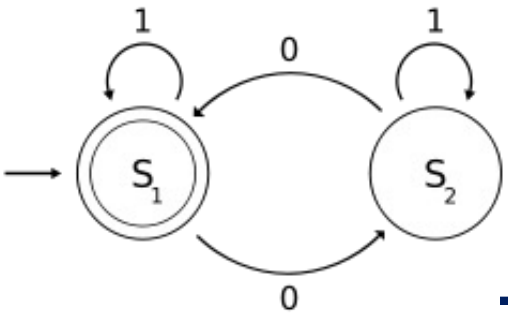
We construct a DFA M recognizing A.

1. $Q' = P(Q)$.

2. For $R \in Q'$ and $a \in \Sigma$, let $\delta'(R, a) = \bigcup_{r \in R} E(\delta(r, a))$.
   where $E(R) = \{q \mid q$ can be reached from R along 0 or more ε arrows $\}$.

3. $q_0' = E(\{q_0\})$.

4. $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$.



Equivalent DFA?