The Equivalence of NFAs and DFAs
**Theorem.** A language is regular if and only if it is accepted by a nondeterministic finite automaton.

**Proof.** $(\Rightarrow)$ Let $A$ be a regular language ...
A finite automaton is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\), where

1. \(Q\) is a finite set called the **states**,
2. \(\Sigma\) is a finite set called the **alphabet**,
3. \(\delta: Q \times \Sigma \to Q\) is the **transition function**,
4. \(q_0 \in Q\) is the **start state**, and
5. \(F \subseteq Q\) is the **set of accept states**.
A nondeterministic finite automaton is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\), where

1. \(Q\) is a finite set called the states,
2. \(\Sigma\) is a finite set called the alphabet,
3. \(\delta: Q \times \Sigma \rightarrow \mathcal{P}(Q)\) is the transition function,
4. \(q_0 \in Q\) is the start state, and
5. \(F \subseteq Q\) is the set of accept states.
Theorem. A language is regular if and only if it is accepted by a nondeterministic finite automaton.

Proof. (⇐) Suppose $A$ is accepted by a nondeterministic finite automaton ...
Equivalent Languages

**Definition.** Two machines are *equivalent* if they recognize the same language.

**Theorem.** Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

**Proof.**
You Only Go Around Once

$N_1$

Input 010110

Symbol read

0

1

0

1

0
A Simpler Example
Removing Choice

**Proof.** Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA recognizing language $A$.

We construct a DFA $M$ recognizing $A$.

1. $Q' = \mathcal{P}(Q)$.

2. For $R \in Q'$ and $a \in \Sigma$, let $\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$.

3. $q_0' = \{q_0\}$.

4. $F' = \{R \in Q' \mid R \text{ contains an accept state of } N\}.$
What About $\varepsilon$ Arrows?
Modifying Our Construction

Proof. Let \( N = (Q, \Sigma, \delta, q_0, F) \) be an NFA recognizing language \( A \).

We construct a DFA \( M \) recognizing \( A \).

1. \( Q' = \mathcal{P}(Q) \).

2. For \( R \in Q' \) and \( a \in \Sigma \), let \( \delta'(R, a) = \bigcup_{r \in R} E(\delta(r, a)) \).
   where \( E(R) = \{ q \mid q \text{ can be reached from } R \text{ along } 0 \text{ or more } \varepsilon \text{ arrows} \} \).

3. \( q_0' = E(\{q_0\}) \).

4. \( F' = \{ R \in Q' \mid R \text{ contains an accept state of } N \} \).
Equivalent DFA?