Closure Under Regular Operations
Warm Up

Design an NFA that recognizes the language
\[ \{ w \in \{a\}^* \mid |w| \text{ is divisible by 3 or 5} \}. \]
Theorem. The class of regular languages is closed under the union operation.

Proof.
The class of regular languages is closed under the union operation.

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**Theorem.** The class of regular languages is closed under the union operation.

**Proof.** Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$ and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize $A_2$.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1 \cup A_2$.

1. $Q = \{q_0\} \cup Q_1 \cup Q_2$, for a new state $q_0$.
2. $q_0$ is the start state of $N$.
3. $F = F_1 \cup F_2$.
4. For any $q \in Q$ and any $a \in \Sigma$, the transition function $\delta(q, a)$ is defined as follows:
   
   $\delta(q, a) = \begin{cases} 
   \delta_1(q, a) & q \in Q_1 \\
   \delta_2(q, a) & q \in Q_2 \\
   \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\
   \emptyset & q = q_0 \text{ and } a \neq \varepsilon 
   \end{cases}$
Closure of Regular Languages
Under Concatenation Using NFAs

**Theorem.** The class of regular languages is closed under the concatenation operation.

**Proof.**
Theorem. The class of regular languages is closed under the concatenation operation.

Proof. Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$ and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize $A_2$.

Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$. 

Closure of Regular Languages
Under Concatenation Using NFAs
Theorem. The class of regular languages is closed under the Kleene star operation.

Proof.
The class of regular languages is closed under the Kleene star operation.

Proof. Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$. Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1^*$. 
Prove that every NFA can be converted to an equivalent one that has a single accept state.