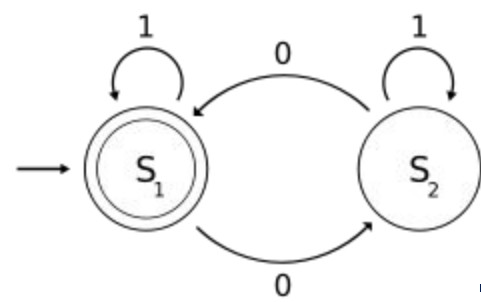
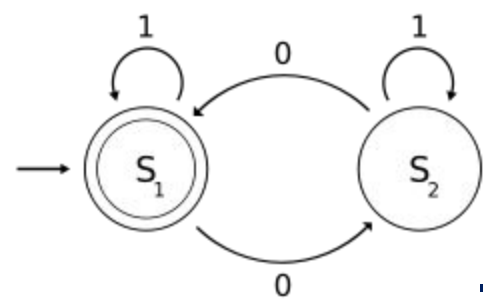




Regular Expressions

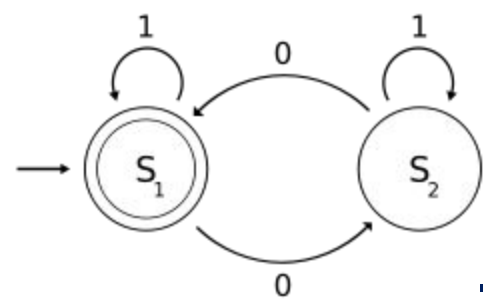


Today's adventures



- Defining regular expressions
- Identities and properties
- A language is regular iff a regular expression generates it (“ $\leq$ ”)

Weekly tip: The kind of practice matters



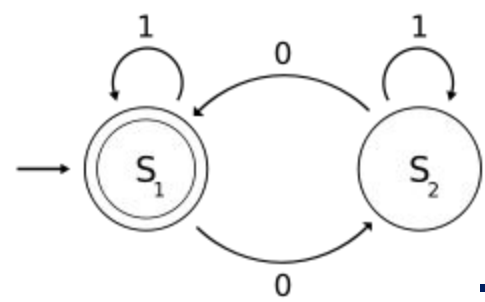
Regular Expressions

$$(0 \cup 1)0^*$$

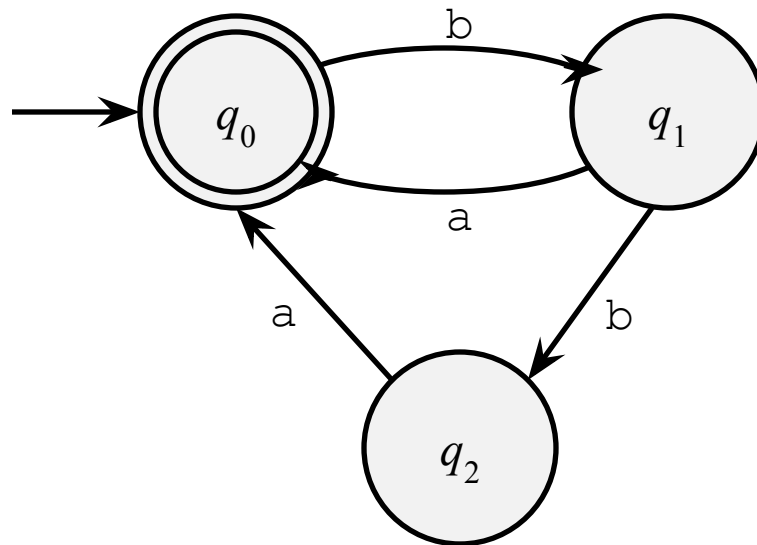
$$(0 \cup 1)^*$$

$$\Sigma^*1$$

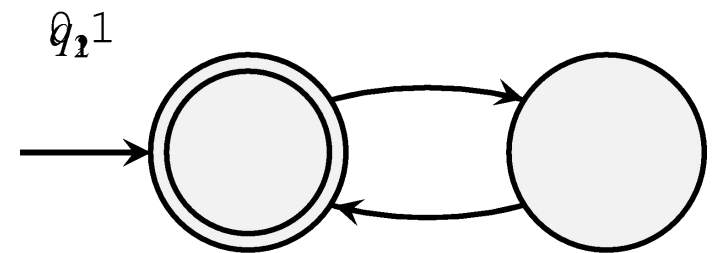
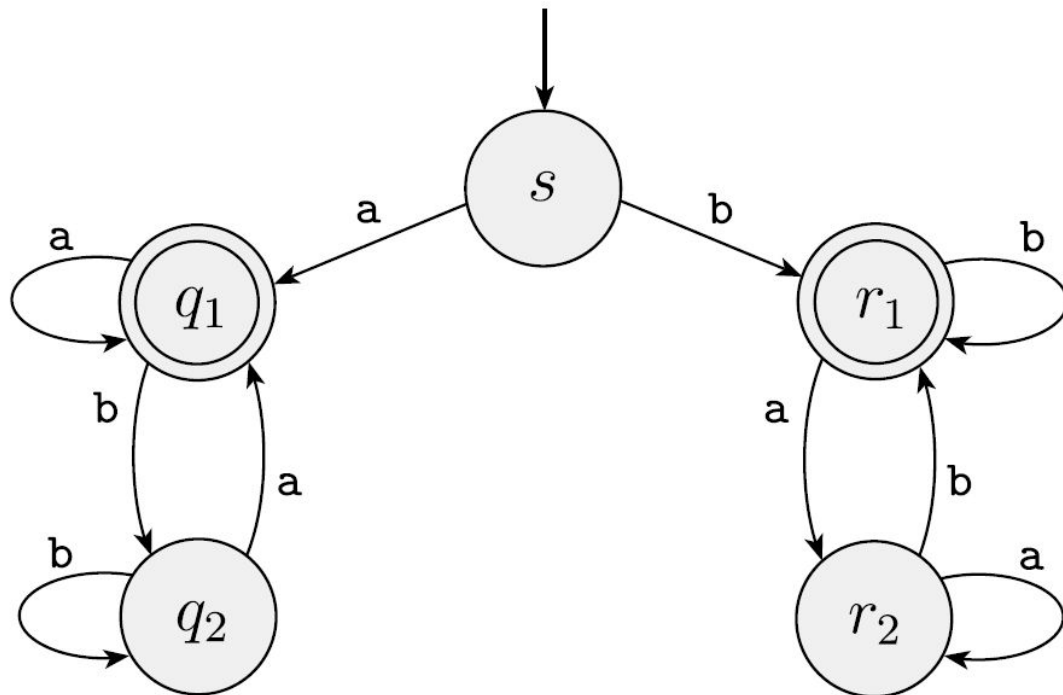
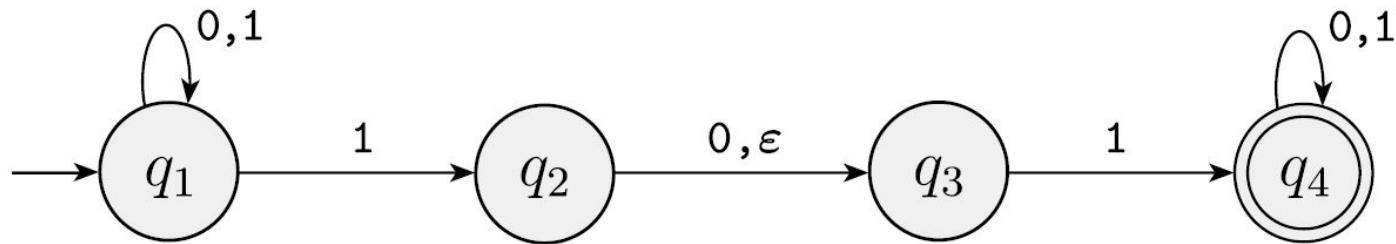
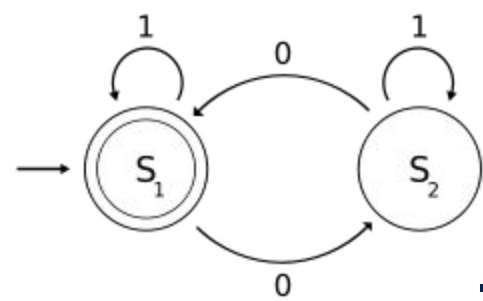
$$R^+$$

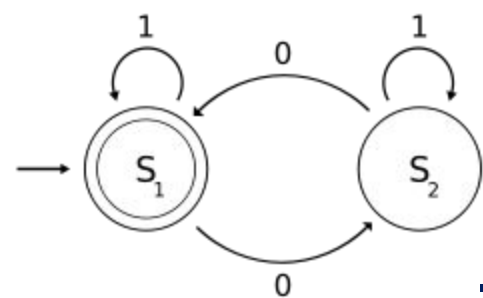


Long Ago in a Place Not Far Away



Old Home Week

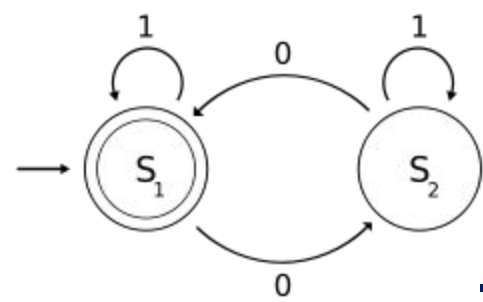




Regular Expressions

Definition. Say that R is a ***regular expression*** if R is

1. a for some a in the alphabet Σ ,
2. ε ,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions,
6. $(R_1)^*$, where R_1 is a regular expression.



Working with Regular Expressions

$$0^*10^* = \{ w \mid \quad \quad \quad \}$$

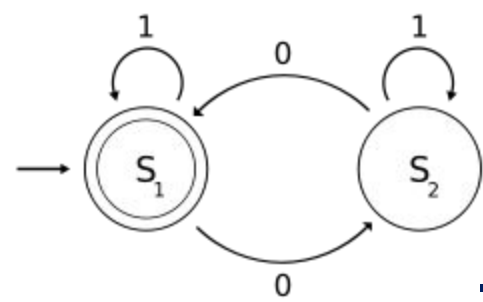
$$= \{ w \mid w \text{ is a string of odd length} \}$$

$$(0 \cup \varepsilon)(1 \cup \varepsilon) =$$

$$(01)^* \emptyset =$$

$$(+ \cup - \cup \varepsilon)(DD^* \cup DD^*.D^* \cup D^*.DD^*) =$$

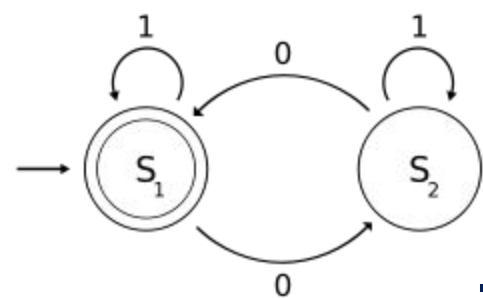
where $D = \{0,1,2,3,4,5,6,7,8,9\}$



Identities

Let R be a regular expression.

- $R \cup \emptyset =$
- $R \circ \varepsilon =$
- $R \cup \varepsilon =$
- $R \circ \emptyset =$

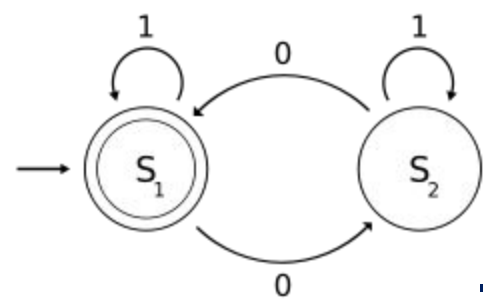


Regular Expressions and NFAs

Theorem. A language is regular if and only if some regular expression describes it.

Proof. ($\Leftarrow$)

1. If $a \in \Sigma$, then a is regular.
2. ε is regular.
3. $\emptyset$ is regular.
4. If R_1 and R_2 are regular, then $(R_1 \cup R_2)$ is regular.
5. If R_1 and R_2 are regular, then $(R_1 \circ R_2)$ is regular.
6. If R_1 is regular, then $(R_1)^*$ is regular.

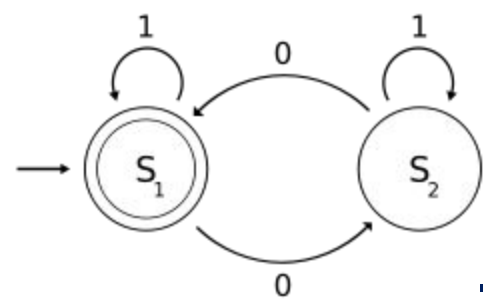


Regular Expressions and NFAs

Theorem. A language is regular if and only if some regular expression describes it.

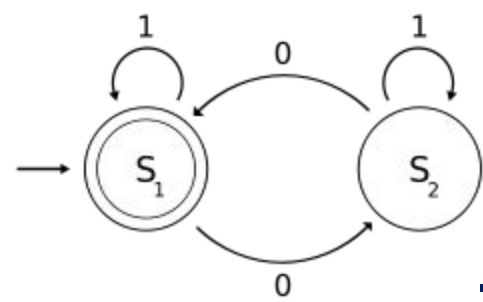
Proof. ($\Leftarrow$)

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.



Proof in Action

Build an NFA that recognizes the regular expression: $(ab \cup a)^*$



Proof in Action

Build an NFA that recognizes the regular expression:

$$a(a \cup b)^*a$$