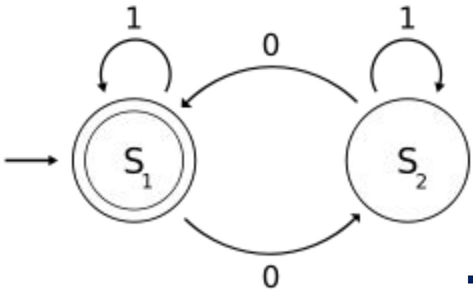
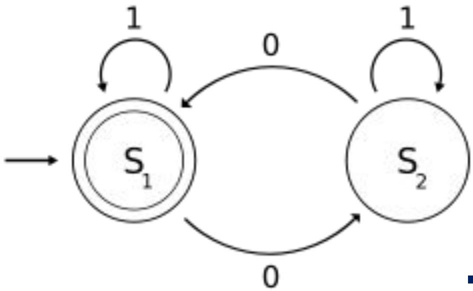


Context-Free Languages (part 1)

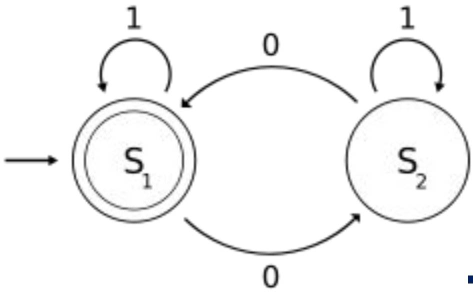
Sipser: Section 2.1 pages 101 - 111



Today's adventures

- Look beyond regular languages
- Explore the notion of a grammar
- Define context-free grammars
- Examples of context-free grammars – describing languages and coming up with our own grammars

Weekly tip: Build a support network

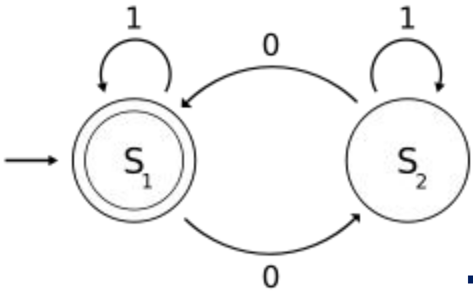


Extending Our Reach

Finite automata and people are *language recognition devices.*

Regular expressions and people are *language generating devices.*

Finite automata *recognize* and regular expressions *generate* an important but limited class of languages.



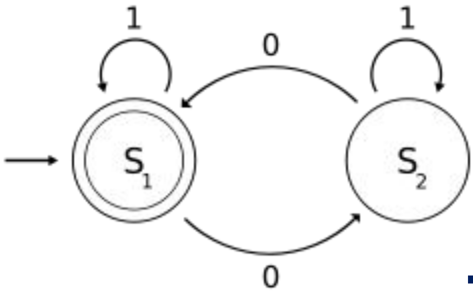
People Languages

A grammar for the English language tells us whether a particular sentence is well formed or not.

A typically English grammar is "a sentence can consist of a noun phrase followed by a predicate."

More concisely, we write

<sentence> → *<noun_phrase>* *<predicate>*



So What's a Noun Phrase?

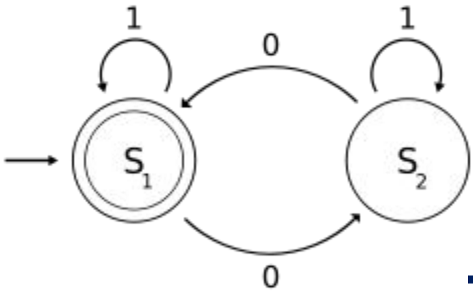
A sentence is

<sentence> → *<noun_phrase>* *<predicate>*

We must also provide definitions for the newly introduced constructs *<noun_phrase>* and *<predicate>*.

<noun_phrase> → <article> <noun>

<predicate> → *<verb>*



Generating Well Formed Sentences

Grammar rules so far:

<*sentence*> → <*noun_phrase*> <*predicate*>

<*noun_phrase*> → <article> <noun>

<*predicate*> → <*verb*>

To complete our simple grammar, we associate actual words with the terms <*article*>, <*noun*>, and <*verb*>.

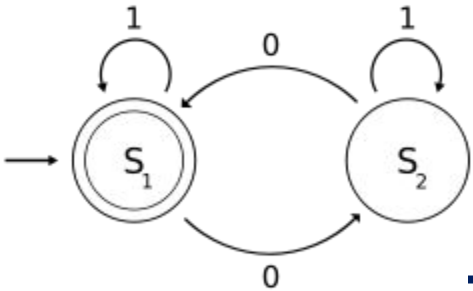
<*article*> → a

<*article*> → the

<*noun*> → student

<*verb*> → relaxes

<*verb*> → studies



Context-Free Grammars

A *context-free grammar* G is a quadruple (V, Σ, R, S), where

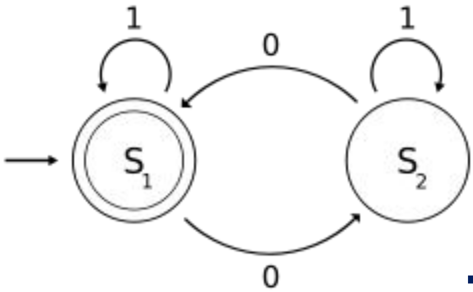
V is a finite set called ***variables*** ,

Σ is a finite set, disjoint from V, called the ***terminals***

R is a finite subset of $V \times (V \cup \Sigma)^*$ called ***rules*** , and

S (the ***start symbol***) is an element of V.

For any $A \in V$ and $u \in (V \cup \Sigma)^*$, we write $A \to u$ whenever $(A, u) \in R$.



The Language of a Grammar

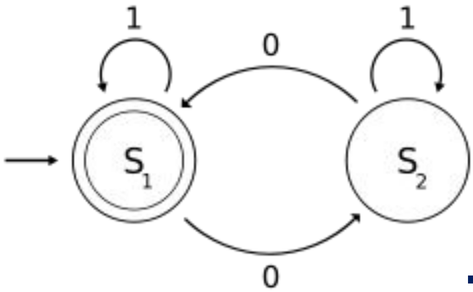
If u, v, $w \in (V \cup \Sigma)^*$ and $A \rightarrow w$ is a rule, then we say uAv ***yields*** uwv and write $uAv \Rightarrow uwv$.

Write $u \stackrel{*}{\Rightarrow} v$ (and read u derives v) if

$$u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v.$$

The ***language of the grammar*** G is

$$L(G) = \{\, w \in \Sigma^* \mid S \stackrel{*}{\Rightarrow} w \,\}.$$



Some Examples

1. Consider $G = (V, \Sigma, R, S)$, where

$$V = \{S\},$$

$$\Sigma = \{a, b\}, \text{ and}$$

$$R = \{\ S \to aSa \mid bSb \mid aSb \mid bSa \mid \varepsilon\ \}.$$

What language does this grammar generate?

2. Is there a grammar whose language is

$$PAL = \{\ w \in \Sigma^* \mid w = reverse(w)\ \}?$$