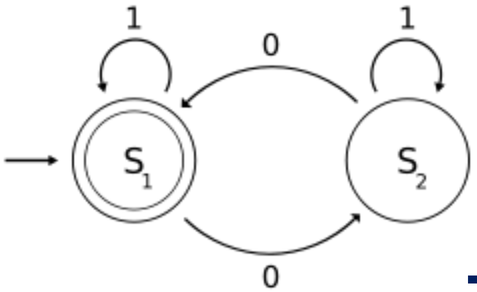
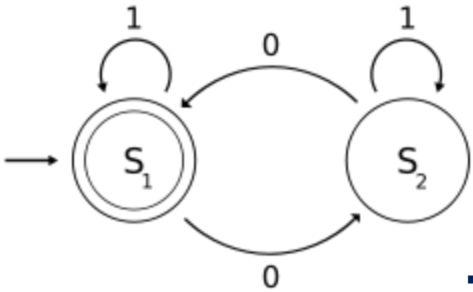


Context-Free Languages

Sipser: Section 2.1 pages 101 - 111

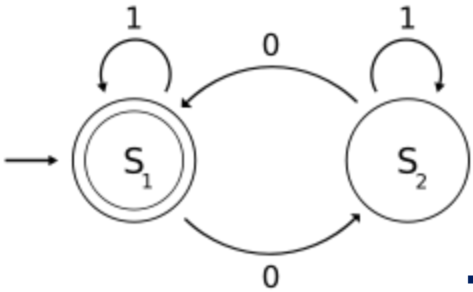


Extending Our Reach

Finite automata and people are *language recognition devices.*

Regular expressions and people are *language generating devices.*

Finite automata *recognize* and regular expressions *generate* an important but limited class of languages.



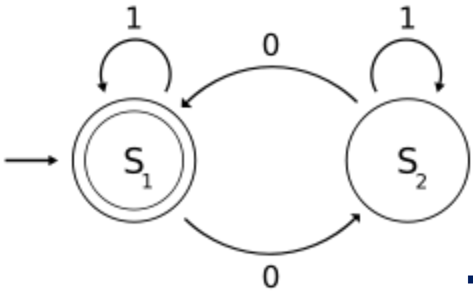
People Languages

A grammar for the English language tells us whether a particular sentence is well formed or not.

A typically English grammar is "a sentence can consist of a noun phrase followed by a predicate."

More concisely, we write

<sentence> → *<noun_phrase>* *<predicate>*



So What's a Noun Phrase?

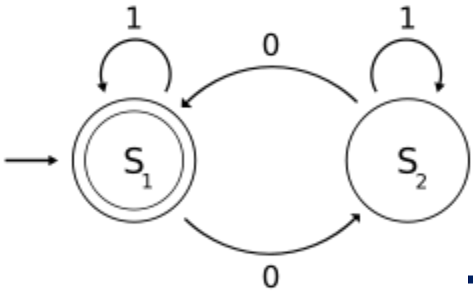
A sentence is

<*sentence*> → <*noun_phrase*> <*predicate*>

We must also provide definitions for the newly introduced constructs <*noun_phrase*> and <*predicate*>.

<*noun_phrase*> → <article> <noun>

<*predicate*> → <*verb*>



Generating Well Formed Sentences

Grammar rules so far:

<sentence> → *<noun_phrase>* *<predicate>*

<noun_phrase> → <article> <noun>

<predicate> → *<verb>*

To complete our simple grammar, we associate actual words with the terms *<article>*, *<noun>*, and *<verb>*.

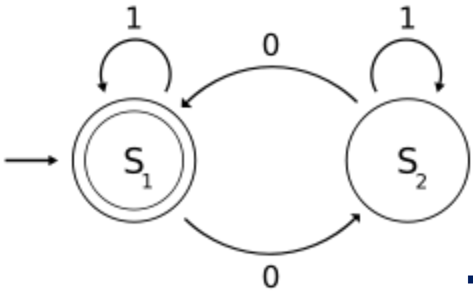
<article> → a

<article> → the

<noun> → student

<verb> → relaxes

<verb> → studies



Context-Free Grammars

A *context-free grammar* G is a quadruple (V, Σ, R, S), where

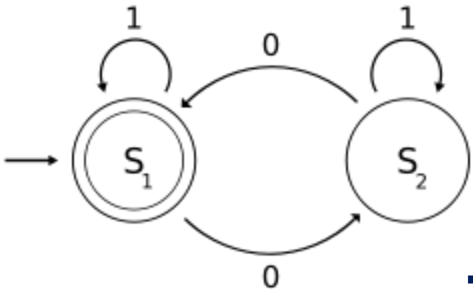
V is a finite set called ***variables***,

Σ is a finite set, disjoint from V, called the ***terminals***

R is a finite subset of $V \times (V \cup \Sigma)^*$ called ***rules***, and

S (the ***start symbol***) is an element of V.

For any $A \in V$ and $u \in (V \cup \Sigma)^*$, we write $A \rightarrow u$ whenever $(A, u) \in R$.



The Language of a Grammar

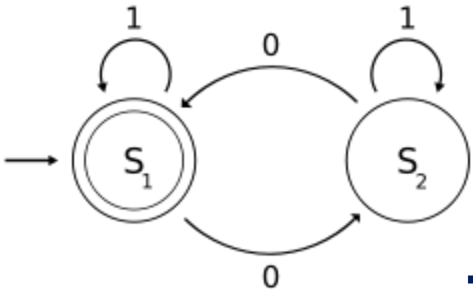
If $u, v, w \in (V \cup \Sigma)^*$ and $A \rightarrow w$ is a rule, then we say uAv ***yields*** uwv and write $uAv \Rightarrow uwv$.

Write $u \overset{*}{\Rightarrow} v$ (and read u derives v) if

$$u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v.$$

The ***language of the grammar*** G is

$$L(G) = \{ w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w \}.$$



For Example

Consider $G = (V, \Sigma, R, S)$, where

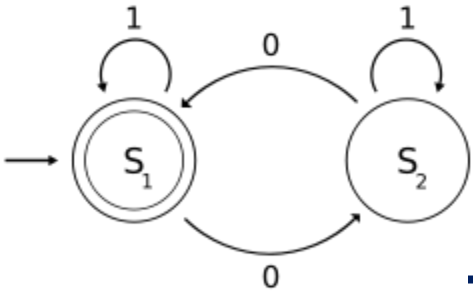
$V = \{S\}$,

$\Sigma = \{a, b\}$, and

$R = \{\ S \rightarrow aSa \mid bSb \mid aSb \mid bSa \mid \varepsilon\ \}$.

Is there a grammar whose language is

$PAL = \{\ w \in \Sigma^* \mid w = reverse(w)\ \}$?



Arithmetic Expressions & Parse Trees

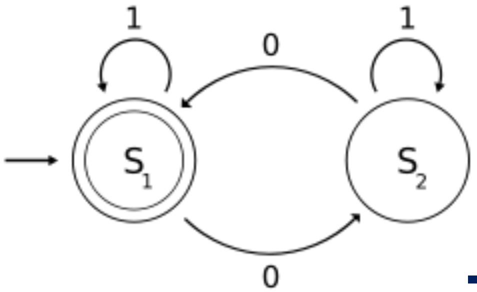
Consider $G = (V, \Sigma, R, S)$, where

$V = \{\langle EXPR \rangle, \langle TERM \rangle, \langle FACTOR \rangle\}$,

$\Sigma = \{ a, +, \times, (,) \}$,

$R = \{ \langle EXPR \rangle \rightarrow \langle EXPR \rangle + \langle TERM \rangle \mid \langle TERM \rangle,$
$\langle TERM \rangle \rightarrow \langle TERM \rangle \times \langle FACTOR \rangle \mid \langle FACTOR \rangle,$
$\langle FACTOR \rangle \rightarrow (\langle EXPR \rangle) \mid a \}$,

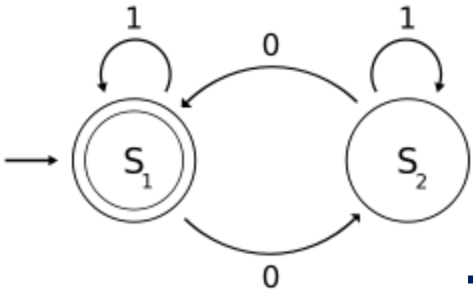
$S = \langle EXPR \rangle$.



Arithmetic Expressions & Parse Trees

Let's do the parse tree for a+axa.

Let's do the parse tree for (a+a)xa.

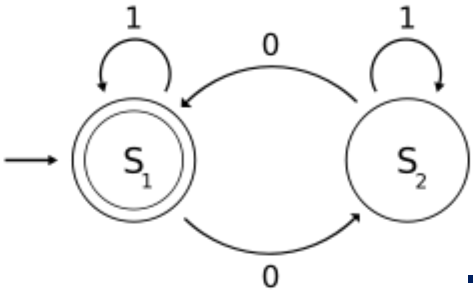


Needlessly Complicated?

How about just

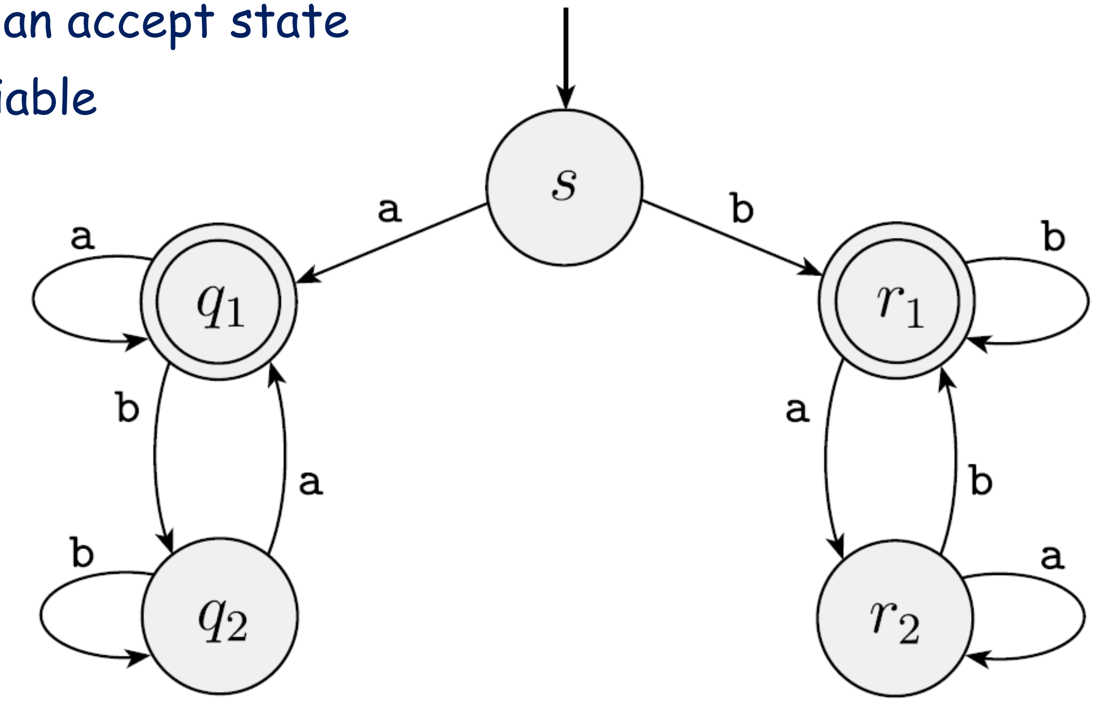
$$\langle EXPR\rangle \rightarrow \langle EXPR\rangle + \langle EXPR\rangle \mid$$
$$\langle EXPR\rangle \times \langle EXPR\rangle \mid$$
$$(\langle EXPR\rangle) \mid$$
$$a$$

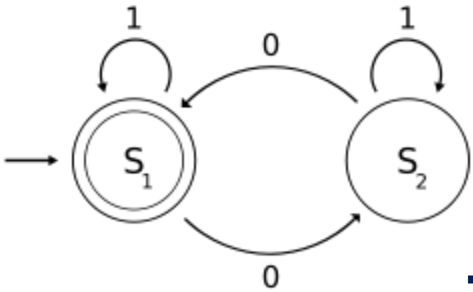
A grammar *G* is ***ambiguous*** if some string *w* has two or more different leftmost derivations.



Regular Languages are Context-Free

1. Make variable R_i for each state q_i.
2. Add rule R_i -> aR_j if there is a transition from q_i to q_j on symbol a
3. Add rule R_i -> ε if q_i is an accept state
4. Make R_0 the start variable





Chomsky Normal Form

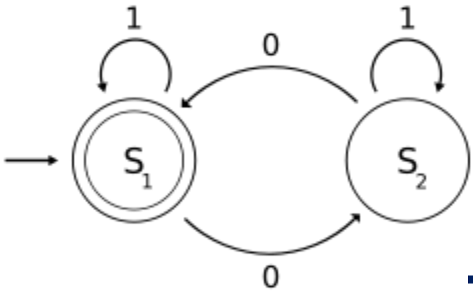
A context-free grammar G is in ***Chomsky normal form*** if every rule is of the form

$$S \to \varepsilon$$

$$A \to BC$$

$$A \to a$$

where $A, B, C \in V$, $B \neq S \neq C$, and $a \in \Sigma$.



Chomsky Normal Form

Theorem Any context-free language is generated by a context-free grammar in Chomsky normal form.

Proof

1. Make sure S appears only on the left.
2. Remove empty rules: $A \rightarrow \varepsilon$.
3. Handle unit rules: $A \rightarrow B$.
4. Fix all the rest.

$$S \rightarrow ASA \mid aA$$

$$A \rightarrow b \mid \varepsilon$$