Reading. Section 1.3 and 1.4, Introduction to the Theory of Computation by M. Sipser.

Problem 1. Give regular expressions generating the following languages. Consider the alphabet $\Sigma = \{0, 1\}$.

(a) $\{w \mid w \text{ contains at least two } 1\text{s}\}$

(b) $\{w \mid w \text{ has length at least } 3 \text{ and its third symbol is a } 0\}$

(c) $\{w \mid w \text{ starts with a } 0 \text{ and has odd length, or starts with a } 1 \text{ and has even length}\}$

(d) $\{w \mid w \text{ does not contain the substring } 101\}$

Problem 2. Use the procedure described in Lemma 1.55 in Sipser to convert the following regular expressions to nondeterministic finite automata.

(a) $ab^*$

(b) $aa(a \cup b)^*$

Problem 3. Use the state-elimination algorithm covered in class to convert the following finite automaton to a regular expression. Show your work as you eliminate each state.

Problem 4. Observe the type of strings that are accepted by the following finite automaton and state the regular expression for the language it recognizes. State the regular expression directly, that is, no need to use the state-elimination algorithm.

If not already in this form, simplify your regular expression to an equivalent regular expression that uses the Kleene star operation exactly once.
Problem 5. Are the following statements true or false? Justify your answers with a brief explanation or a counterexample.

(a) All subsets of a regular language are regular.

(b) The class of regular languages is closed under set difference.

(c) The class of finite languages is closed under complement.

(d) \( L_1 = L_2 \) if and only if \( L_1^* = L_2^* \).

Problem 6. Consider \( \Sigma = \{0, 1\} \). Are the following languages regular? Prove your answer.

(a) \( \{0^{2n} \mid n \geq 0\} \)

(b) \( \{0^n1^m0^n \mid m, n \geq 0\} \)

(c) \( \{w \in \Sigma^* \mid w \text{ does not have three consecutives 0's}\} \)

(d) The complement of \( \{0^n1^n \mid n \geq 0\} \)

(e) \( \{w \in \Sigma^* \mid w = w^R\} \)