Assignment 8 (due 12/10/2019)

**Problem 1.** Show that $\mathsf{EQ}_{\mathsf{CFG}}$ is Turing-recognizable.

**Problem 2.** Show that $\mathsf{EQ}_{\mathsf{CFG}}$ is undecidable.

*Hint: Consider defining a reduction from language $\mathsf{ALL}_{\mathsf{CFG}} = \{ \langle G \rangle \mid G$ is a CFG and $L(G) = \Sigma^* \}$. $\mathsf{ALL}_{\mathsf{CFG}}$ is undecidable.*

**Problem 3.** Let $\mathsf{REV}_{\mathsf{TM}} = \{ \langle M \rangle \mid M$ is a TM such that $L(M) = (L(M))^R \}$. Recall that $L^R = \{ w^R \mid w \in L \}$. Show that $\mathsf{REV}_{\mathsf{TM}}$ is undecidable.

**Problem 4.** Mapping reducibility helps us relate Turing recognizable and Turing decidable languages to each other. In this question, we reason why we cannot use mapping reducibility to relate regular languages.

Find languages $L_a$ and $L_b$ such that $L_a \leq_m L_b$ and $L_b$ is regular, but $L_a$ is not regular. To show that $L_a \leq_m L_b$ you must give a mapping reduction from $L_a$ to $L_b$.

This shows that we cannot conclude that a language is regular by finding a mapping reduction from it to a known regular language.

**Problem 5.** A triangle in an undirected graph is a 3-clique. Show that $\mathsf{TRIANGLE}$ is in $\mathsf{P}$, where $\mathsf{TRIANGLE} = \{ \langle G \rangle \mid G$ contains a triangle}.  

**Problem 6.** Consider the following language:

$\mathsf{INDSET} = \{ \langle G, n \rangle \mid G$ is an undirected graph with an independent set of size at least $n \}$

An independent set in an undirected graph is a set of vertices that have no edges between them. Prove that $\mathsf{INDSET}$ is NP-complete by reducing $\mathsf{CLIQUE}$ to it.

*Hint: Consider the complement graph $\overline{G}$ which has the same vertices as $G$ and $(u, v)$ if an edge in $\overline{G}$ iff $(u, v)$ is not an edge in $G$.*

**Problem 7.** Suppose you are in charge of forming a student committee at Wellesley. Given a set of $n$ students and a compatibility function that maps every student to a subset of students they are compatible with, you must choose $k$ students to be in the committee such that each student in the committee is compatible with everyone else.

Show that determining whether it is possible to form such a committee is NP-complete by a reduction from that language.

*Hint: What NP-complete problem about graphs does this problem remind you off?