Digital Logic

Gateway to computer science

Digital logic values

A digital circuit has only two logical values are present.

- 3.3V, true, asserted
- 2.8V
- 0.5V
- 0.0V, false, unasserted

*Exact voltage levels and their interpretation varies with technology.

Transistors

Very fast binary switches.

- Base
- Collector
- Emitter
- (Circuit Ground)
- +Vcc (Supply Voltage)
Logic Gates
Tiny electronic devices that compute functions on two-valued signals.

NOT

NAND

Logic Gates

Integrated Circuits (invented 1950s)
Gates are manufactured in units called integrated circuits.
From SSI (tens) to VLSI (hundreds of thousands to billions)

Five basic gates

AND

OR

*Gates with more than two inputs are possible.

Boolean Algebra, revisited

AND = Boolean product

OR = Boolean sum

NOT = inverse or complement

 Gate = identity
Booleans & Circuit Equivalence

\[ A + B = A \cdot B \]

\[ A \cdot B = \overline{A} + \overline{B} \]

Name that rule

\[ A + (B + C) = (A + B) + C \]

\[ A \cdot (B \cdot C) = (A \cdot B) \cdot C \]

Demorgan’s Law (double bubble, toil and trouble, in Randy’s words...)

\[ \overline{A + B} = \overline{A} \cdot \overline{B} \]

\[ \overline{A \cdot B} = \overline{A} + \overline{B} \]
**Take care**

\[
A + 1 = \begin{cases} 1 & \text{if } A = 0 \\ A & \text{if } A \neq 0 \end{cases}
\]

\[
A + AB = A + B
\]

**A universal gate**

*Are there others?*

**Circuit simplification**

*All other things being equal, smaller circuits are cheaper, faster, cooler, and easier to design.*

Can we find a simpler circuit that performs the same function?*

Start with an equivalent Boolean expression

\[ F(A, B, C) = \]

**Code detectors**

A four input AND gate recognizes exactly one input code.

Design a code detector that recognizes 1001.

Design a code detector that recognizes either 1001 or 1111.
Voting machines

A majority circuit outputs 1 whenever a majority of its inputs equal 1. We design a simple majority circuit for three inputs.

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*Waxing theorems, could it be better?*