CS240 Laboratory 2
Digital Logic

- **Circuit equivalence**
- **Boolean Algebra/Universal gates**
- **Exclusive OR**
- **Linux, C, Emacs**
- **Bit Puzzles**
- **Bitbucket, Mercurial**

Circuit Equivalence

Two boolean functions with same truth table = **equivalent**

When there is an equivalent circuit that uses fewer gates, transistors, or chips, it is preferable to use that circuit in the design

Example:

Given: F = A'B' + A'B   Q = A’ + A’B + A’B’

A	B	A’B’	A’B	F
0	0	1	0	1
0	1	0	1	1
1	0	0	0	0
1	1	0	0	0

A	B	A’	A’B	A’ B’	Q
0	0	1	0	1	1
0	1	1	0	0	1
1	0	0	0	0	0
1	1	0	0	0	0

F and Q are equivalent because they have the same truth table.

Identities of Boolean Algebra

- Identity law	$1A = A$	$0 + A = A$
- Null law	$0A = 0$	$1 + A = 1$
- Idempotent law	$AA = A$	$A + A = A$
- Inverse law	$AA' = 0$	$A + A' = 1$
- Commutative law	$AB = BA$	$A + B = B + A$
- Associative law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
- Distributive law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
- Absorption law	$A(A + B) = A$	$A + AB = A$
- De Morgan's law	$(AB)' = A' + B'$	$(A + B)' = A'B'$

Example:

$F = A'B' + A'B$
$= A'(B' + B)$ distributive
$= A'(1)$ inverse
$= A'$ identity

$Q = A' + A'B + A'B'$
$= A' + A'B'$ absorption
$= A'$ absorption

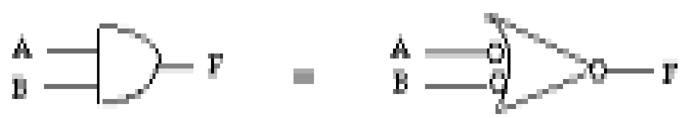
Universal Gates

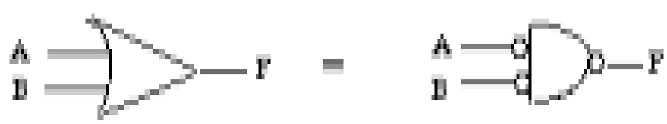
Any Boolean function can be constructed with NOT, AND, and OR gates

NAND and NOR = **universal gates**

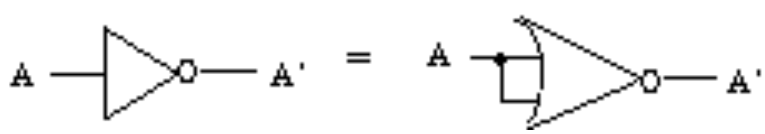
DeMorgan's Law shows how to make **AND** from NOR (and vice-versa)

AB = (A' + B')' (**AND** from NOR)
A + B = (A'B')' (**OR** from NAND)

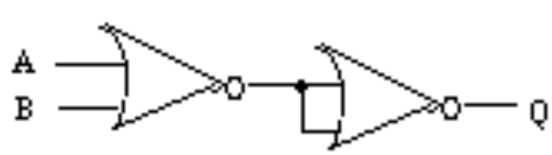




NOT from a NOR



OR from a NOR



To implement a function using only NOR gates:

- apply DeMorgan's Law to each AND in the expression until all ANDs are converted to NORs
- use a NOR gate for any NOT gates, as well.
- remove any redundant gates (NOT NOT, may remove both)

Implementing the circuit using only NAND gates is similar.

Example: $Q = (AB)'B'$

$= (A' + B')B'$

$= ((A'+B')' + B)'$ NOTE: you can use a NOR gate to produce A' and you can do the same for B'

Simplifying Circuits or Proving Equivalency

General rule to simplify circuits or prove equivalency:

1. Distribute if possible, and if you can't, apply DeMorgan's Law so that you can.
2. Apply other identities to remove terms, and repeat step 1.

EXAMPLE: Is $(A'B)'(AB)' + A'B'$ equivalent to $(AB)'$?

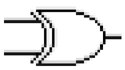
$F = (A'B)'(AB)' + A'B'$	-- can't distribute
$= (A + B')(A' + B') + A'B'$	DeMorgan's
$= AA' + AB' + A'B + B'B' + A'B'$	distributive
$= 0 + AB' + A'B + B' + A'B'$	inverse and idempotent
$= AB' + A'B + A'B'$	identity
$= B'(A + A') + A'B$	distributive
$= B'(1) + A'B$	inverse
$= B' + A'B$	identity
$= B' + (A + B')'$	DeMorgan's
$= (B(A + B'))'$	DeMorgan's
$= (AB + BB')'$	distributive
$= (AB + 1)'$	inverse
$= (AB)'$	identity

Exclusive OR (XOR)

$F = AB' + A'B = A \oplus B$

A	**B**	**F**
0	0	0
0	1	1
1	0	1
1	1	0

Available on IC as a gate, useful for comparison problems



Example: Even parity $F = A \oplus B \oplus C$

A	**B**	**C**	**F**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1