# Representing Data with Bits

bits, bytes, numbers, and notation

# positional number representation

| 2 | 4 | 0 | |
|---|---|---|---|
| 100 | 10 | 1 | ← *weight* |
| $10^2$ | $10^1$ | $10^0$ | ← *weight* |
| 2 | 1 | 0 | ← *position* |

$= 2 \times 10^2 + 4 \times 10^1 + 0 \times 10^0$

- **Base** determines:
  - Maximum digit (base – 1). Minimum digit is 0.
  - Weight of each position.
- Each position holds a digit.
- Represented value = sum of all position values
  - Position value = digit value x $\text{base}^{\text{position}}$

# binary = base 2

| 1 | 0 | 1 | 1 |
|---|---|---|---|
| 8 | 4 | 2 | 1 |
| $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| 3 | 2 | 1 | 0 |

$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

*weight*

*position*

When ambiguous, subscript with base:

$101_{10}$ Dalmatians (movie)

$101_2$-Second Rule (folk wisdom for food safety)

irony

# Powers of 2:
# memorize up to ≥ $2^{10}$ (in base ten)

ex

Show powers, strategies.

ex

# conversion and arithmetic

$19_{10} = ?_2$

$1001_2 = ?_{10}$

$240_{10} = ?_2$

$11010011_2 = ?_{10}$

$101_2 + 1011_2 = ?_2$

$1001011_2 \text{ x } 2_{10} = ?_2$

# **numbers** and **wires**

One wire carries one bit.

How many wires to represent a given number?

1 0 0 1

1 0 0 0 1 0 0 1

What if I want to build a computer (and not change the hardware later)?

# ***byte*** = 8 bits

a.k.a. octet

## Smallest unit of data

*used by a typical modern computer*

**Binary** $00000000_2$ -- $11111111_2$

**Decimal** $000_{10}$ -- $255_{10}$

**Hexadecimal** $00_{16}$ -- $FF_{16}$

**Byte = 2 hex digits!**

Programmer's hex notation (C, etc.):

**`0xB4`** = $B4_{16}$

Octal (base 8) also useful.

Why do 240 students often confuse Halloween and Christmas?

| Hex | Decimal | Binary |
|---|---|---|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

# Hex encoding practice

ex

# `char`: representing characters

A C-style string is represented by a series of bytes (`char`s).

- One-byte ASCII codes for each character.
- ASCII = American Standard Code for Information Interchange

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 32 | space | 48 | 0 | 64 | @ | 80 | P | 96 | ` | 112 | p |
| 33 | ! | 49 | 1 | 65 | A | 81 | Q | 97 | a | 113 | q |
| 34 | ” | 50 | 2 | 66 | B | 82 | R | 98 | b | 114 | r |
| 35 | # | 51 | 3 | 67 | C | 83 | S | 99 | c | 115 | s |
| 36 | $ | 52 | 4 | 68 | D | 84 | T | 100 | d | 116 | t |
| 37 | % | 53 | 5 | 69 | E | 85 | U | 101 | e | 117 | u |
| 38 | & | 54 | 6 | 70 | F | 86 | V | 102 | f | 118 | v |
| 39 | ’ | 55 | 7 | 71 | G | 87 | W | 103 | g | 119 | w |
| 40 | ( | 56 | 8 | 72 | H | 88 | X | 104 | h | 120 | x |
| 41 | ) | 57 | 9 | 73 | I | 89 | Y | 105 | I | 121 | y |
| 42 | * | 58 | : | 74 | J | 90 | Z | 106 | j | 122 | z |
| 43 | + | 59 | ; | 75 | K | 91 | [ | 107 | k | 123 | { |
| 44 | , | 60 | < | 76 | L | 92 | \ | 108 | l | 124 | \| |
| 45 | - | 61 | = | 77 | M | 93 | ] | 109 | m | 125 | } |
| 46 | . | 62 | > | 78 | N | 94 | ^ | 110 | n | 126 | ~ |
| 47 | / | 63 | ? | 79 | O | 95 | _ | 111 | o | 127 | del |

# ***word*** *|wərd|,* n.

**Natural unit of data used by processor.**

- **Fixed size** (e.g. 32 bits, 64 bits)
  - Defined by ISA: Instruction Set Architecture
- machine instruction operands
- word size = register size = address size

| 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

**Java/C int = 4 bytes:** 11,501,584

**MSB: most significant bit**

**LSB: least significant bit**

# fixed-size data representations

| Java Data Type | C Data Type | (size in bytes) 32-bit | 64-bit |
|---|---|---|---|
| boolean | | 1 | 1 |
| byte | char | 1 | 1 |
| char | | 2 | 2 |
| short | short int | 2 | 2 |
| int | int | 4 | 4 |
| float | float | 4 | 4 |
| | long int | 4 | 8 |
| double | double | 8 | 8 |
| long | long long | 8 | 8 |
| | long double | 8 | 16 |

**Depends on word size!**

ex

# *bitwise* operators

**Bitwise operators** on fixed-width **bit vectors**.

AND &    OR |    XOR ^    NOT ~

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  ________    __________    __________    __________
  01000001

                                            01010101
                                          ^ 01010101
                                          __________
```

Laws of Boolean algebra apply bitwise.

*e.g.,* DeMorgan's Law: ~(A | B) = ~A & ~B

# *Aside: sets as bit vectors*

ex

**Representation:** *n*-bit vector gives subset of {0, …, *n*–1}.

$a_i = 1 \equiv i \in A$

```
01101001      { 0, 3, 5, 6 }
76543210

01010101      { 0, 2, 4, 6 }
76543210
```

**Bitwise Operations**

| | | |
|---|---|---|
| & | 01000001 | { 0, 6 } |
| \| | 01111101 | { 0, 2, 3, 4, 5, 6 } |
| ^ | 00111100 | { 2, 3, 4, 5 } |
| ~ | 10101010 | { 1, 3, 5, 7 } |

**Set Operations?**

# *bitwise* operators in C

ex

`& | ^ ~` apply to any *integral* data type

**`long, int, short, char, unsigned`**

Examples (**`char`**)

```
~0x41 =

~0x00 =

0x69 & 0x55 =

0x69 | 0x55 =
```

Many bit-twiddling puzzles in upcoming assignment

ex

# *logical* operations in C

`&&`   `||`   `!`   apply to any "integral" data type
**`long, int, short, char, unsigned`**

**0** is **false**   **nonzero** is **true**   **result** always **0 or 1**

**early termination**  a.k.a.  **short-circuit evaluation**

Examples (**`char`**)

```
 !0x41 =
 !0x00 =
!!0x41 =

0x69 && 0x55 =
0x69 || 0x55 =
```

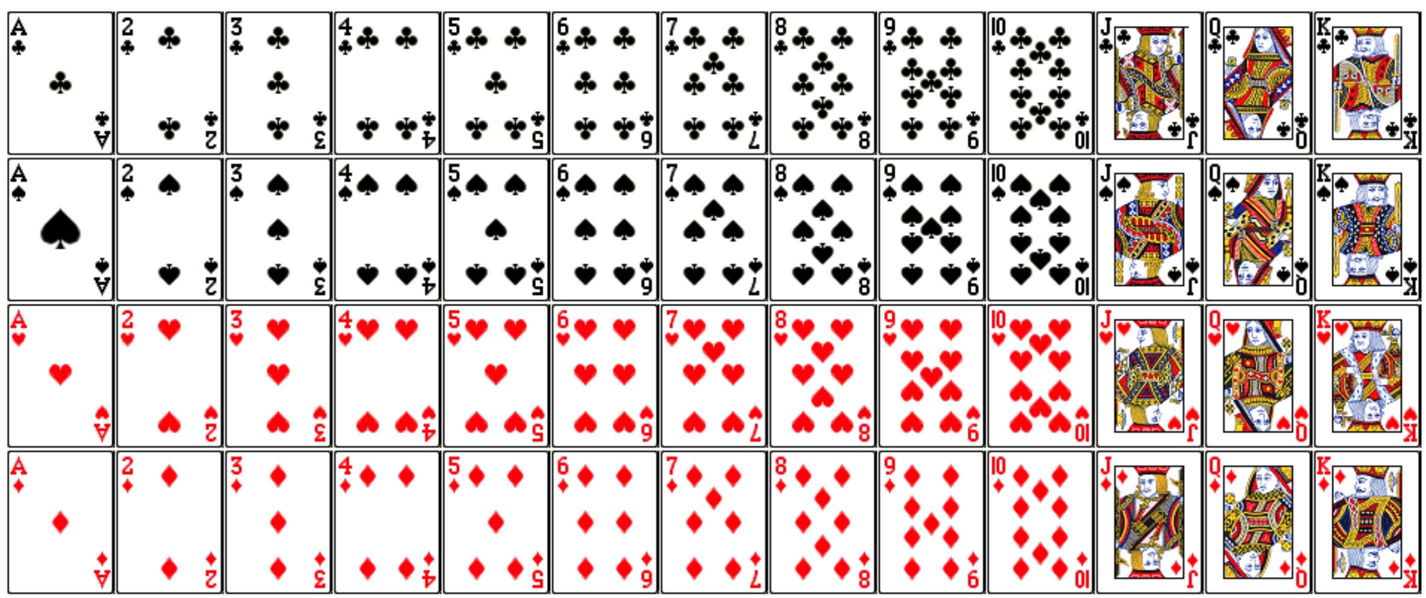
# Encode playing cards.

## 52 cards in 4 suits

- How do we encode suits, face cards?

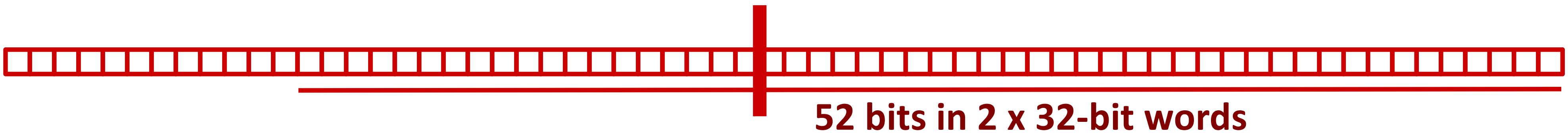
## What operations should be easy to implement?

- Get and compare rank
- Get and compare suit



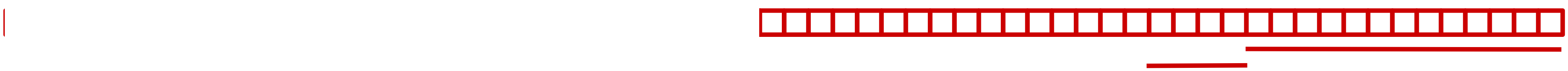
# Two possible representations

**52 cards – 52 bits with bit corresponding to card set to 1**




- **"One-hot" encoding**
  - Hard to compare values and suits independently
  - Not space efficient

**4 bits for suit, 13 bits for card value – 17 bits with two set to 1**



- **Pair of one-hot encoded values**
  - Easier to compare suits and values independently
  - Smaller, but still not space efficient

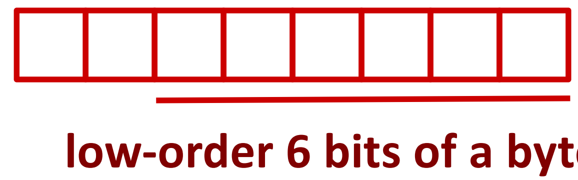
# Two better representations

## Binary encoding of all 52 cards – only 6 bits needed

Number cards uniquely from 0

Smaller than one-hot encodings.

Hard to compare value and suit



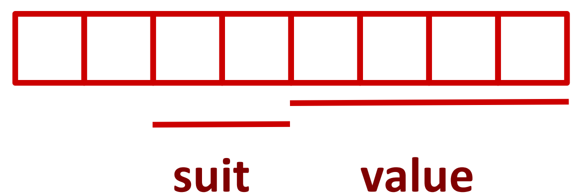

## Binary encoding of suit (2 bits) and value (4 bits) separately

Number each suit uniquely

Number each value uniquely

Still small

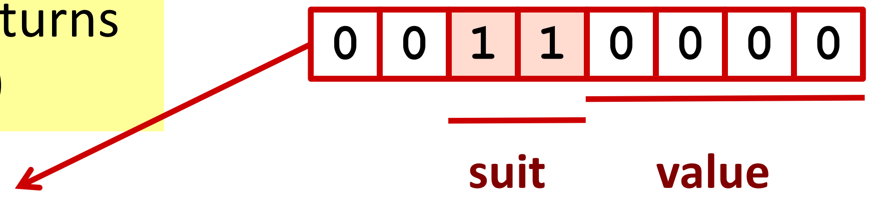
Easy suit, value comparisons



# Compare Card Suits

**mask:** a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0

```
#define SUIT_MASK 0x30
```



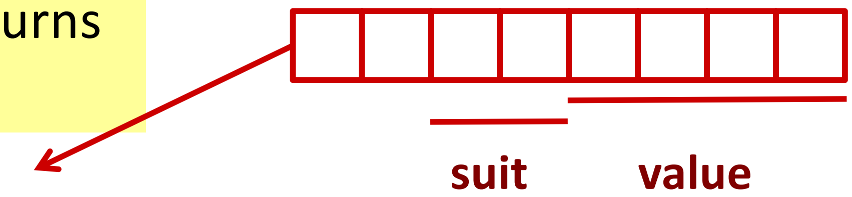

```
int sameSuit(char card1, char card2) {
  return !((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK));

  //same as (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
...
if ( sameSuit(hand[0], hand[1]) ) { ... }
```

# Compare Card Values

ex

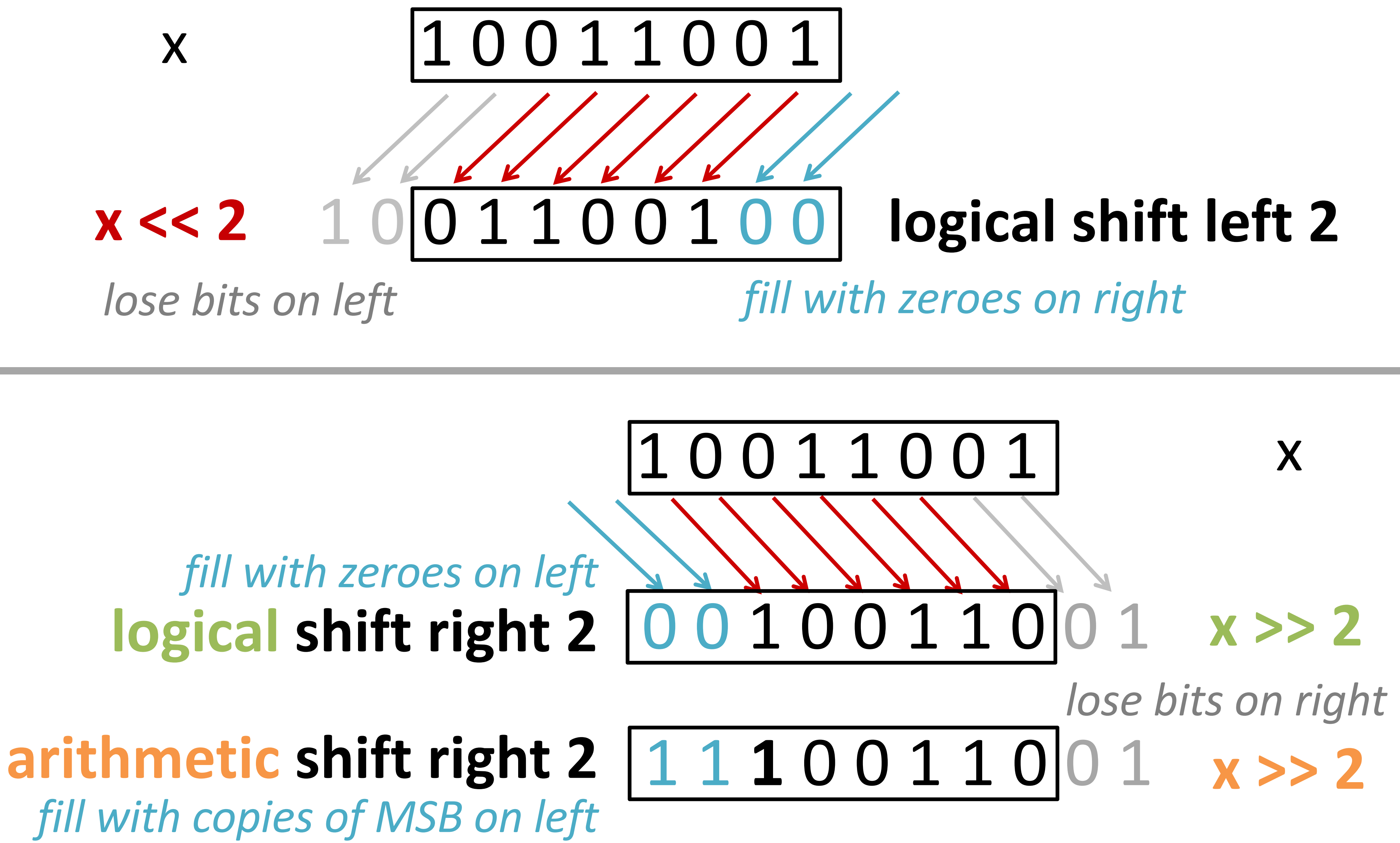
**mask:** a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0




```
#define VALUE_MASK

int greaterValue(char card1, char card2) {




}
```

```
char hand[5];          // represents a 5-card hand
char card1, card2;   // two cards to compare
...
if ( greaterValue(hand[0], hand[1]) ) { ... }
```

# Bit shifting



# Shift gotchas

**Logical or arithmetic shift right: how do we tell?**

**C:** compiler chooses

Usually based on type: rain check!

**Java:** >> is arithmetic, >>> is logical

**Shift an *n*-bit type by at least 0 and no more than n-1.**

**C:** other shift distances are undefined.

*anything* could happen

**Java:** shift distance is used modulo number of bits in shifted type

Given int x: x << 34 == x << 2

# **Shift and Mask:** extract a bit field

ex

Write C code:

extract $2^{nd}$ *most significant byte* from a 32-bit integer.

given $\mathrm{x} =$ 01100001 01100010 01100011 01100100

should return: 00000000 00000000 00000000 01100010

All other bits are zero.

Desired bits in least significant byte.