Digital Logic

*Gate*way to computer science

Digital data/computation = Boolean

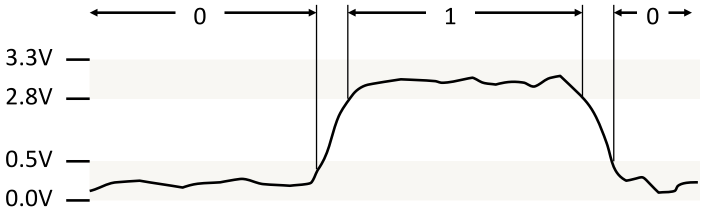
Abstraction!

Boolean value (*bit*): 0 or 1

Boolean functions (AND, OR, NOT, …)

Electronically:

bit = high voltage vs. low voltage

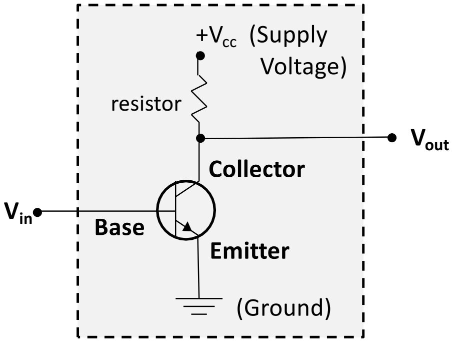


Boolean functions = logic gates, built from transistors

Transistors (more in lab)

If *Base* voltage is high:
Current may flow freely from *Collector* to *Emitter*.

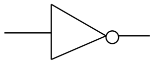
If *Base* voltage is low:
Current may not flow from *Collector* to *Emitter*.



Truth table

V_{in}	V_{out}		in	out		in	out
low	high	=	0	1	=	F	T
high	low		1	0		T	F

NOT gate



Abstraction!

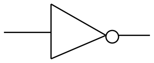
Digital Logic Gates

Abstraction!

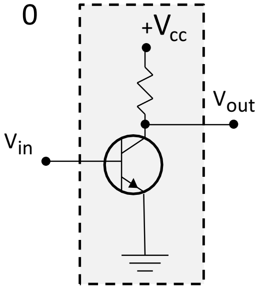
ex

Tiny electronic devices that compute basic Boolean functions.

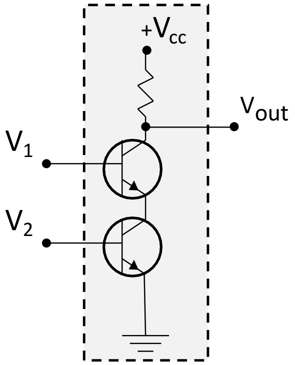
NOT



V_{in}	V_{out}
0	1
1	0

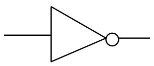


		V_2	
		0	1
V_1	0		
	1		

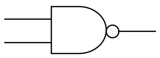


ex

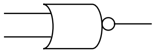
Five basic gates: define with truth tables



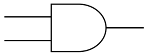
NOT	
0	1
1	0



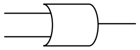
NAND	0	1
0	1	1
1	1	0



NOR	0	1
0		
1		



AND	0	1
0		
1		

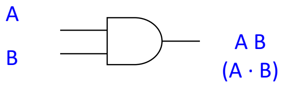


OR	0	1
0		
1		

Boolean Algebra

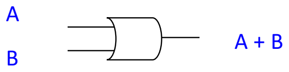
for combinational logic

inputs	=	*variables*
wires	=	*expressions*
gates	=	*operators/functions*
circuits	=	*functions*



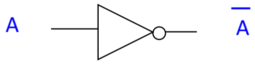
AND = Boolean product

·	0	1
0	0	0
1	0	1



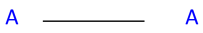
OR = Boolean sum

+	**0**	**1**
0	0	1
1	1	1



NOT = inverse or complement

0	1
1	0



wire = identity

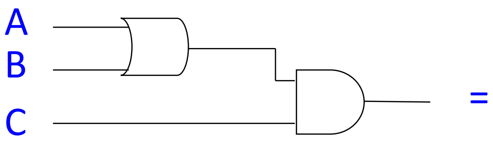
0	0
1	1

ex

Circuits

Connect inputs and outputs of gates with wires.

Crossed wires touch ***only if*** there is a dot.



What is the output if A=1, B=0, C=1?

What is the truth table of this circuit?

What is an equivalent Boolean expression?

ex

Translation

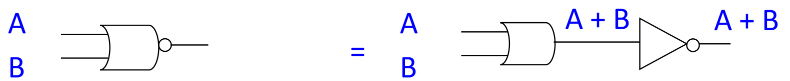
Connect gates to implement these functions. Check with truth tables.

Use a direct translation -- it is straightforward and bidirectional.

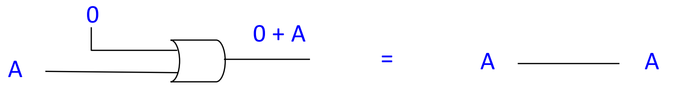
$F = (A\overline{B} + C)D$

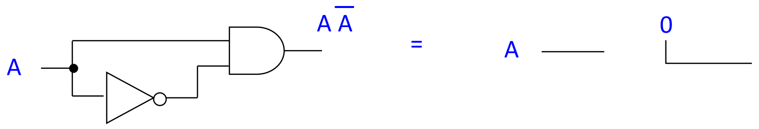
$Z = \overline{W} + (X + \overline{WY})$

Note on notation: bubble = inverse/complement

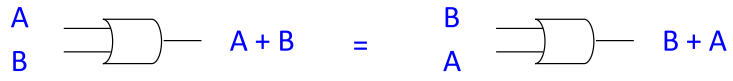


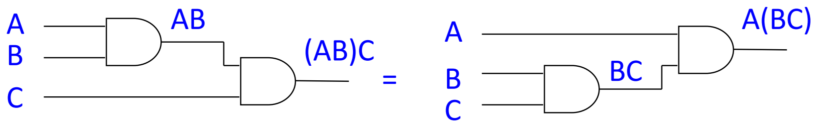
Identity law, inverse law



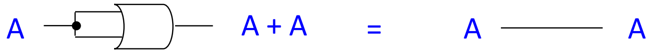


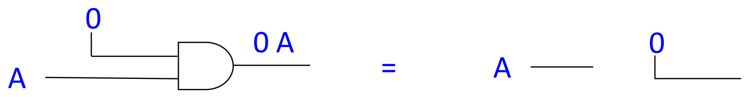
Commutativity, Associativity



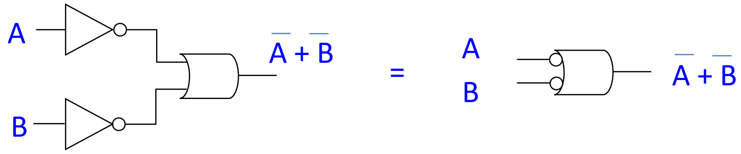


Idempotent law, Null/Zero law



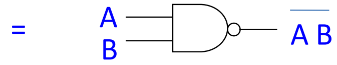


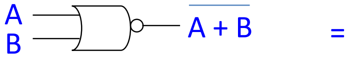
Note on notation: bubble = inverse/complement



DeMorgan's Law

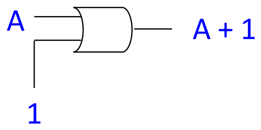
(**double bubble**, toil and trouble, in Randy's words...)





One law, Absorption law

Write truth tables. Do they correspond to simpler circuits?



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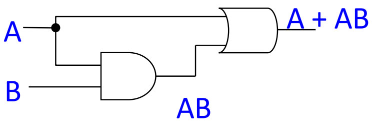

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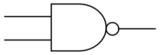
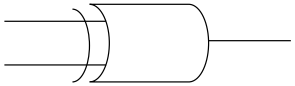
NAND is *universal*.



All Boolean functions can be implemented using only NANDs.
Build NOT, AND, OR, NOR, using only NAND gates.

XOR: Exclusive OR



Output = 1 if exactly one input = 1.

Truth table:

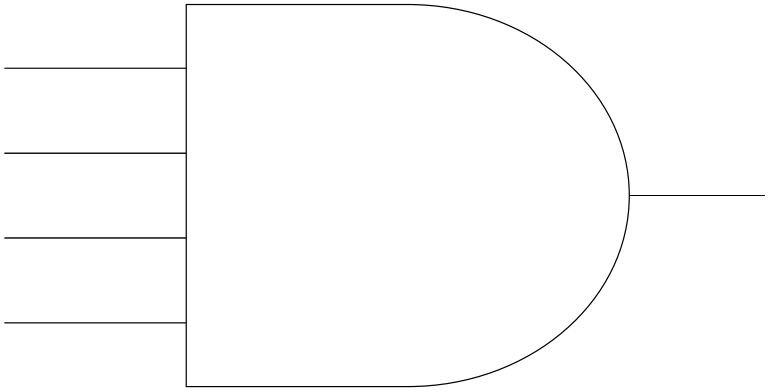
Build from earlier gates:

Often used as a one-bit comparator.

Video game designers, Halloween costumers extraordinaire, sci-fi/fantasy screenwriters, I have an idea…

Larger gates

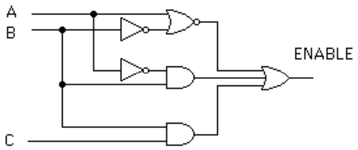
Build a 4-input AND gate using any number of 2-input gates.



Why simplify?

Circuit simplification

Is there a simpler circuit that performs the same function?



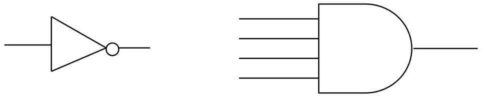
Start with an equivalent Boolean expression, then simplify with algebra.

F(A, B, C) =

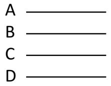
Check the answer with a truth table.

Circuit derivation: ***code detectors***

AND gate + NOT gates = code detector, recognizes exactly one input code.



Design a 4-input code detector to output 1 if ABCD = 1001, and 0 otherwise.



Design a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others. (accept = 1, reject = 0)

Circuit derivation: ***sum-of-products*** form

logical sum (OR)

of products (AND)

of inputs or their complements (NOT)

Draw the truth table and **design a sum-of-products circuit** for a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others.

How are the truth table and the sum-of-products circuit related?

Voting machines

A **majority circuit** outputs 1 if and only if a majority of its inputs equal 1.

Design a majority circuit for three inputs. **Use a sum of products.**

A	B	C	Majority
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Triply redundant computers in spacecraft

- Space program also hastened Integrated Circuits.