(4-bit) **unsigned integer representation**

1	0	1	1
8	4	2	1
2^3	2^2	2^1	2^0
3	2	1	0

$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

weight

position

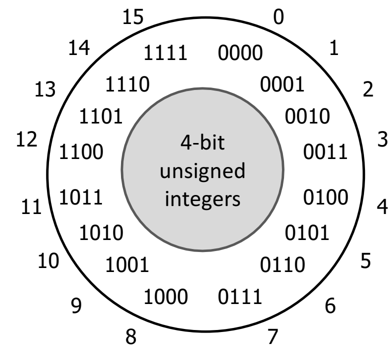
***n*-bit unsigned integers:**

minimum =

maximum =

modular arithmetic, overflow

11	1011
+ 2	+ 0010



13	1101
+ 5	+ 0101

`x+y` in *n*-bit unsigned arithmetic is in math

unsigned overflow =

=

Unsigned addition *overflows* if and only if

sign-magnitude

!!!

Most-significant bit (MSB) is ***sign bit***

0 means non-negative   1 means negative

Remaining bits are an unsigned magnitude

8-bit sign-magnitude:

00000000 represents _____

01111111 represents _____

10000101 represents _____

10000000 represents _____

Anything weird here?

Arithmetic?

Example:

4 - 3 != 4 + (-3)

```
 00000100
+10000011
```

Zero?

ex

(4-bit) **two's complement signed integer representation**

compare to unsigned

1	0	1	1
-2^3	2^2	2^1	2^0

$= 1 \times -2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

***4*-bit two's complement integers:**

minimum =

maximum =

two's complement vs. **unsigned**

_	_	… _	_	_
2^{n-1}	2^{n-2}	… 2^2	2^1	2^0
-2^{n-1}	2^{n-2}	… 2^2	2^1	2^0

unsigned places

two's complement places

What's the difference?

n-bit minimum =

n-bit maximum =

8-bit representations

ex

0 0 0 0 1 0 0 1

1 0 0 0 0 0 0 1

1 1 1 1 1 1 1 1

0 0 1 0 0 1 1 1

n-bit two's complement numbers:

minimum =

maximum =

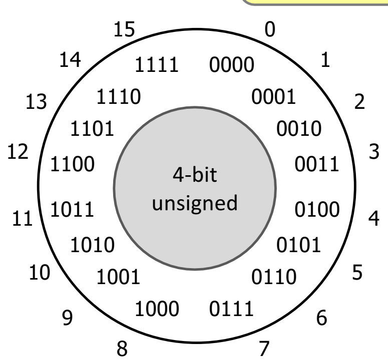
4-bit **unsigned** vs. 4-bit **two's complement**

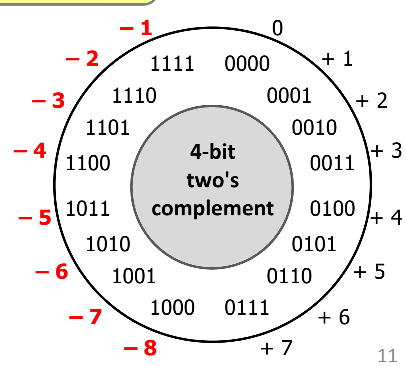
1 0 1 1

$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$1 \times -2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

11 ← **difference = ___ = 2—** → -5

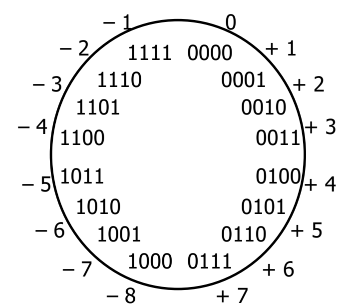




two's complement **addition**

2	0010	-2	1110
+ 3	+ 0011	+ -3	+ 1101

-2	1110	2	0010
+ 3	+ 0011	+ -3	+ 1101

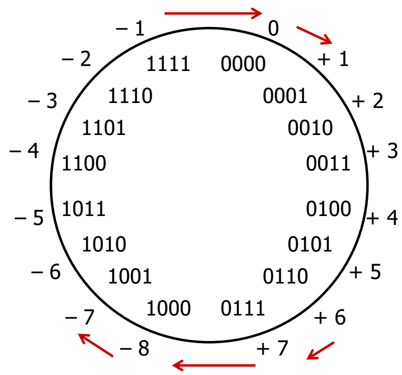


Modular Arithmetic

two's complement ***overflow***

Addition *overflows*
if and only if
if and only if

-1	1111
+ 2	+ 0010
6	0110
+ 3	+ 0011



Modular Arithmetic

Some CPUs/languages raise exceptions on overflow.
C and Java cruise along silently... Feature? Oops?

A few reasons two's complement is awesome

Addition, subtraction, hardware

Sign

Negative one

Complement rules

Another derivation

ex

How should we represent 8-bit negatives?

- For all positive integers ***x***,
  we want the representations of ***x*** and ***–x*** to sum to zero.
- We want to use the standard addition algorithm.

00000001	00000010	00000011
+	+	+
00000000	00000000	00000000

- Find a rule to represent –x where that works...

***Convert/cast** signed number to **larger** type.*

0 0 0 0 0 0 1 0	8-bit 2
_ _ _ _ _ _ _ _ 0 0 0 0 0 0 1 0	16-bit 2
1 1 1 1 1 1 0 0	8-bit -4
_ _ _ _ _ _ _ _ 1 1 1 1 1 1 0 0	16-bit -4

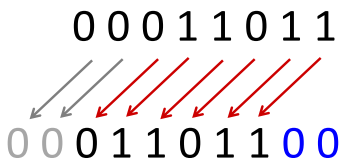
Rule/name?

unsigned **shifting** and **arithmetic**

unsigned
x = 27;

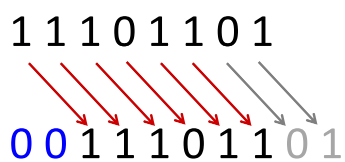
y = x << 2;

y == 108



logical shift left

logical shift right



unsigned
x = 237;

y = x >> 2;

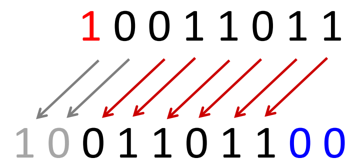
y == 59

two's complement **shifting** and **arithmetic**

signed
x = -101;

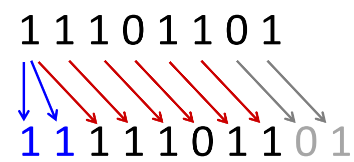
y = x << 2;

y == 108



logical shift left

arithmetic shift right



signed
x = -19;

y = x >> 2;

y == -5

shift**-and-**add

ex

Available operations

`x << k` implements `x *` 2^k

`x + y`

Implement `y = x * 24` using only <<, +, and integer literals

What does this function compute?

ex

```
unsigned puzzle(unsigned x, unsigned y) {
  unsigned result = 0;
  for (unsigned i = 0; i < 32; i++){
    if (y & (1 << i)) {
      result = result + (x << i);
    }
  }
  return result;
}
```