Fixed-Point Representation

Implied binary point.

$b_7\ b_6\ b_5\ b_4\ b_3$ [.] $b_2\ b_1\ b_0$

$b_7\ b_6\ b_5\ b_4\ b_3\ b_2\ b_1\ b_0$ [.]

range: difference between largest and smallest representable numbers

precision: smallest difference between any two representable numbers

fixed point = fixed range, fixed precision

IEEE **Floating Point** Standard 754

IEEE = Institute of Electrical and Electronics Engineers

Numerical form:

$$V_{10} = (-1)^{s} * M * 2^{E}$$

Sign bit ***s*** determines whether number is negative or positive

Significand (mantissa) ***M*** usually a fractional value in range [1.0,2.0)

Exponent ***E*** weights value by a (-/+) power of two

Analogous to scientific notation

Representation:

MSB `s` = sign bit ***s***

`exp` field encodes ***E*** (but is *not equal* to E)

`frac` field encodes ***M*** (but is *not equal* to M)

s	exp	frac

Numerically well-behaved, but hard to make fast in hardware

Three kinds of values

$V = (-1)^{s} * M * 2^{E}$

s	exp	frac

1. Normalized: M = 1.xxxxx...

As in scientific notation: $0.011 \times 2^5 = 1.1 \times 2^3$

Representation advantage?

2. Denormalized, near zero: M = 0.xxxxx..., smallest E

Evenly space near zero.

3. Special values:

0.0:	`s` = 0	`exp` = 00...0	`frac` = 00...0
+inf, -inf: division by 0.0		`exp` = 11...1	`frac` = 00...0
NaN ("Not a Number"): sqrt(-1), $\infty - \infty$, $\infty * 0$, etc.		`exp` = 11...1	`frac` $\neq$ 00...0

Normalized values, with `float` example

$V = (-1)^{s} * M * 2^{E}$

s	exp (k=8)	frac (n=23)

Value: `float f = 12345.0;`

$12345_{10} = 11000000111001_2$

$= 1.1000000111001_2 \times 2^{13}$ (normalized form)

Significand:

M = `1.1000000111001`$_2$

`frac=` `10000001110010000000000`$_2$

Exponent: E = `exp` – Bias → `exp` = E + Bias

E = 13

Bias = 127 = $2^7 - 1$ = $\mathbf{2^{k-1} - 1}$ Splits exponents roughly -/+

`exp` = 140 = `10001100`$_2$

Result:

s	exp	frac
0	10001100	10000001110010000000000

2. Denormalized Values: near zero

"Near zero": **exp** = 000…0

Exponent:

E = **1 + exp** – Bias = 1 - Bias   not: **exp** – Bias

Significand: leading zero

M = 0.xxx…x$_2$

frac = xxx…x

Cases:

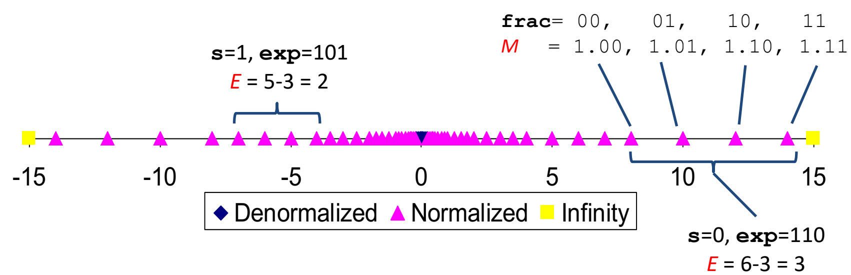
- **exp** = 000…0, **frac** = 000…0   0.0, -0.0
- **exp** = 000…0, **frac** ≠ 000…0

Value distribution example

6-bit IEEE-like format

Bias = $2^{3-1} - 1 = 3$

s	exp	frac
1	3	2

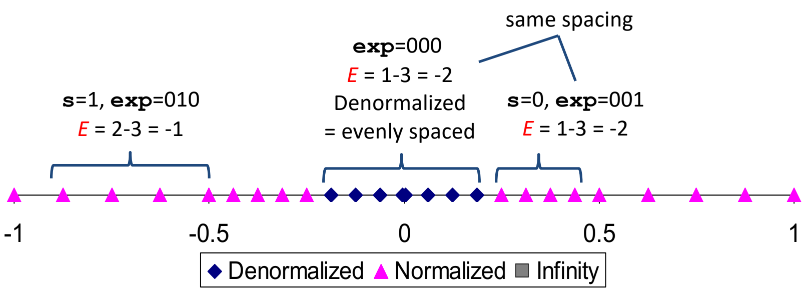


Value distribution example (zoom in on 0)

6-bit IEEE-like format

Bias = $2^{3-1} - 1 = 3$

s	exp	frac
1	3	2



Try to represent 3.14, 6-bit example

6-bit IEEE-like format

Bias = $2^{3-1} - 1 = 3$

s	exp	frac
1	3	2

Value: `3.14;`

3.14 = 11.0010 0011 1101 0111 0000 1010 000…

= 1.1001 0001 1110 1011 1000 0101 0000… $_2 \times 2^1$ (normalized form)

Significand:

M = 1.100100011110101110111000010100000… $_2$

frac= $\underline{10}_2$

Exponent:

E = 1   Bias = 3   **exp** = 4 = 100$_2$

Result:

0 100 10 = $1.10_2 \times 2^1 = 3$ next highest?