## **Digital Logic**

Gateway to computer science



## **Digital data/computation = Boolean**

Boolean value (*bit*): 0 or 1 Boolean functions (AND, OR, NOT, ...) Electronically:

bit = high voltage vs. low voltage



Boolean functions = logic gates, built from transistors

Abstraction

### Transistors (more in lab)

**If Base voltage is high:** Current may flow freely from *Collector* to *Emitter*.

#### If *Base* voltage is low:

Vout

high

low

V<sub>in</sub>

low

high

Current may not flow from *Collector* to *Emitter*.

**Truth table** 

out

1

0

in

 $\mathbf{O}$ 

1





## **Digital Logic Gates**

Tiny electronic devices that compute basic Boolean functions.



## Integrated Circuits (1950s -

#### Early (first?) transistor



#### **Small integrated circuit**



#### Wafer



Chip



## Five basic gates: define with truth tables



## **Boolean Algebra**

for combinational logic



AND = Boolean product

•	0	1
0	0	0
1	0	1



1

0

inputs = variables
wires = expressions
gates = operators/functions
circuits = functions



OR = Boolean sum

+	0	1
0	0	1
1	1	1

Α \_\_\_\_\_ Α



## Circuits

Connect inputs and outputs of gates with wires. Crossed wires touch *only if* there is a dot.



What is the output if A=1, B=0, C=1? What is the truth table of this circuit? What is an equivalent Boolean expression?



## Translation



Connect gates to implement these functions. Check with truth tables.

Use a direct translation -- it is straightforward and bidirectional.

 $F = (A\overline{B} + C)D$ 

 $Z = \overline{W} + (X + \overline{WY})$ 



### Identity law, inverse law



### **Commutativity, Associativity**





### Idempotent law, Null/Zero law





### **DeMorgan's Law**

(double bubble, toil and trouble, in Randy's words...)





## **One law, Absorption law**



Write truth tables. Do they correspond to simpler circuits?







# NAND is *universal*.

All Boolean functions can be implemented using only NANDs. Build NOT, AND, OR, NOR, using only NAND gates.

### **XOR: Exclusive OR**





Output = 1 if exactly one input = 1.

Truth table:

Build from earlier gates:

Often used as a one-bit comparator.

### Larger gates



Build a 4-input AND gate using any number of 2-input gates.





# **Circuit simplification**

Is there a simpler circuit that performs the same function?



Start with an equivalent Boolean expression, then simplify with algebra. F(A, B, C) =

Check the answer with a truth table.



### Circuit derivation: code detectors

AND gate + NOT gates = code detector, recognizes exactly one input code.



Design a 4-input code detector to output 1 if ABCD = 1001, and 0 otherwise.



Design a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others. (accept = 1, reject = 0)



## Circuit derivation: *sum-of-products* form

logical sum (OR)

of products (AND)

of inputs or their complements (NOT)

**Draw the truth table** and **design a sum-of-products circuit** for a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others. **How are the truth table and the sum-of-products circuit related?** 

# Voting machines



A **majority circuit** outputs 1 if and only if a majority of its inputs equal 1. Design a majority circuit for three inputs. **Use a sum of products.** 

Α	В	С	Majority
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

#### **Triply redundant computers in spacecraft**

• Space program also hastened Integrated Circuits.



## Computers

- Manual calculations
- powered all early
  - US **space** missions.
- Facilitated transition to digital computers.

#### Mary Jackson



#### **Katherine Johnson**

 Supported Mercury, Apollo, Space Shuttle, ...

#### **Dorothy Vaughn**

- First black supervisor within NACA
- Early self-taught FORTRAN programmer for NASA move to digital computers.



## Early pioneers in reliable computing



#### **Katherine Johnson**

- Calculated first US human space flight trajectories
- Mercury, Apollo 11, Space Shuttle, ...
- Reputation for accuracy in manual calculations, verified early code
- Called to verify results of code for launch calculations for first US human in orbit
- Backup calculations helped save Apollo 13
- Presidential Medal of Freedom 2015

#### **Margaret Hamilton**

- Led software team for Apollo 11 Guidance Computer, averted mission abort on first moon landing.
- Coined "software engineering", developed techniques for correctness and reliability.
- Presidential Medal of Freedom 2016

#### Apollo 11 code print-out

