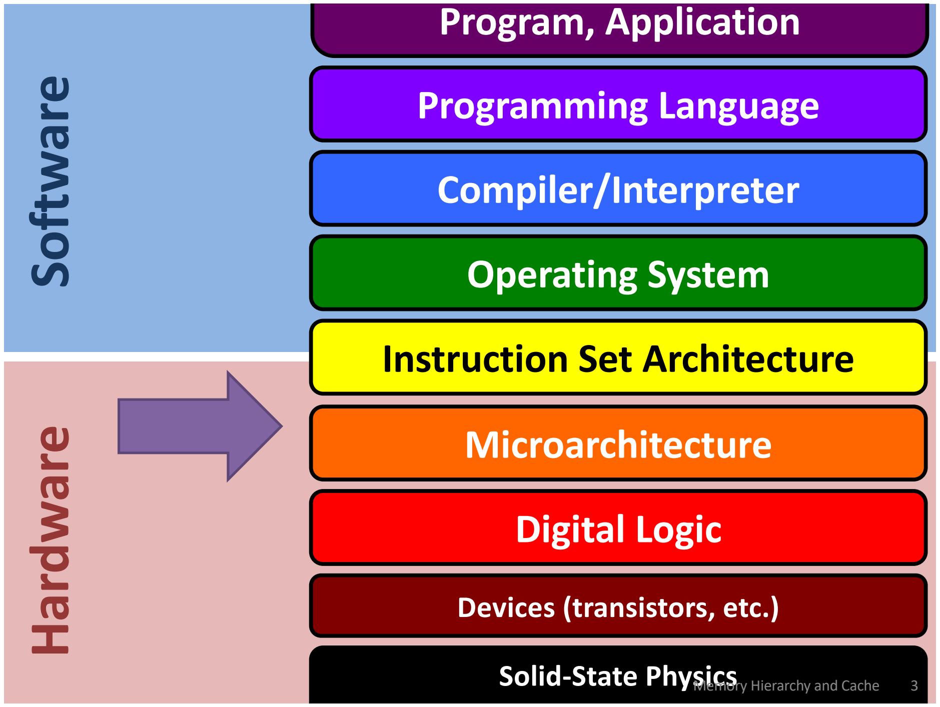
CS 240
Foundations of Computer Systems

WELLESLEY

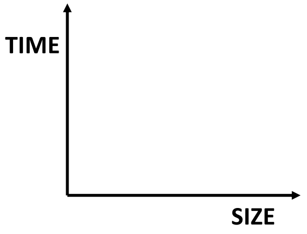
Memory Hierarchy and Cache

Memory hierarchy
Cache basics
Locality
Cache organization
Cache-aware programming

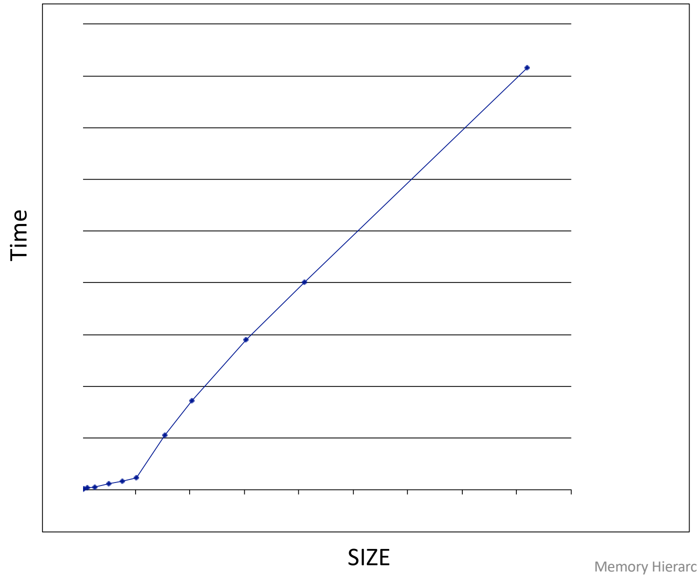


How does execution time grow with SIZE?

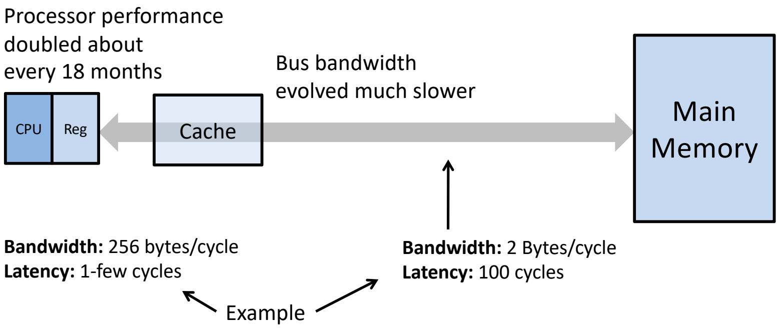
```
int array[SIZE];
fillArrayRandomly(array);
int s = 0;

for (int i = 0; i < 200000; i++) {
  for (int j = 0; j < SIZE; j++) {
    s += array[j];
  }
}
```



Reality



Processor-memory bottleneck



Processor performance doubled about every 18 months

Bus bandwidth evolved much slower

Bandwidth: 256 bytes/cycle
Latency: 1-few cycles

Bandwidth: 2 Bytes/cycle
Latency: 100 cycles

Example

Solution: caches

Cache

English:

n. a hidden storage space for provisions, weapons, or treasures
v. to store away in hiding for future use

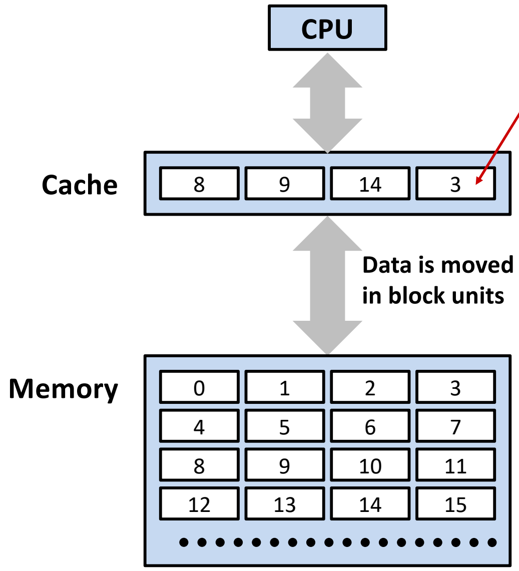
Computer Science:

n. a computer memory with short access time used to store frequently or recently used instructions or data

v. to store [data/instructions] temporarily for later quick retrieval

Also used more broadly in CS: software caches, file caches, etc.

General cache mechanics



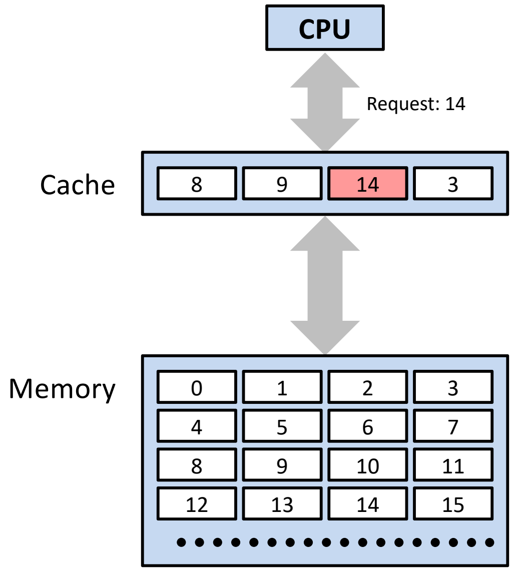
Block: unit of data in cache and memory.
(a.k.a. line)

Smaller, faster, more expensive.
Stores **subset of memory blocks**.
(lines)

Data is moved in block units

Larger, slower, cheaper.
Partitioned into blocks (lines).

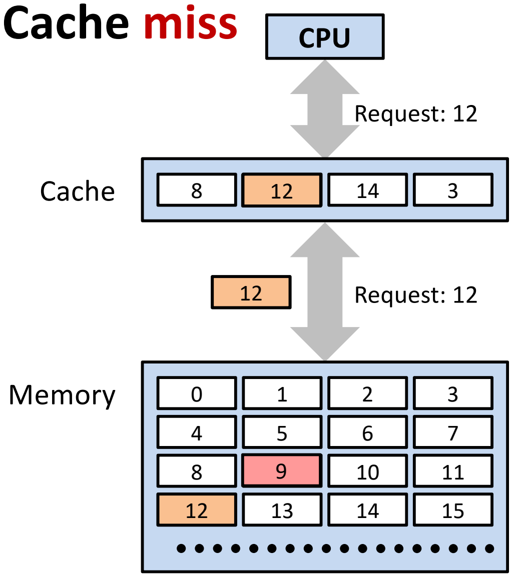
Cache hit



1. Request** data in block **b.

2. Cache hit:
Block b is in cache.

Cache **miss**



1. Request** data in block **b.

2. Cache miss:
block is not in cache

3. Cache eviction:
Evict a block to make room, maybe store to memory.

4. Cache fill:
Fetch block from memory, store in cache.

Placement Policy:
where to put block in cache

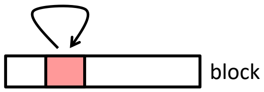
Replacement Policy:
which block to evict

Locality: why caches work

Programs tend to use data and instructions at addresses near or equal to those they have used recently.

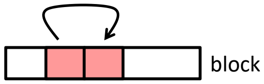
Temporal locality:

Recently referenced items are ***likely*** to be referenced again in the near future.



Spatial locality:

Items with nearby addresses are ***likely*** to be referenced close together in time.



How do caches exploit temporal and spatial locality?

Locality #1

```
int sum = 0;
for (int i = 0; i < n; i++) {
  sum += a[i];
}
return sum;
```

What is stored in memory?

Data:

Instructions:

Locality #2

row-major M x N 2D array in C

```
int sum_array_rows(int a[M][N]) {
    int sum = 0;

    for (int i = 0; i < M; i++) {
        for (int j = 0; j < N; j++) {
            sum += a[i][j];
        }
    }
    return sum;
}
```

a[0][0]	a[0][1]	a[0][2]	a[0][3]
a[1][0]	a[1][1]	a[1][2]	a[1][3]
a[2][0]	a[2][1]	a[2][2]	a[2][3]

Locality #3

row-major M x N 2D array in C

```
int sum_array_cols(int a[M][N]) {
    int sum = 0;

    for (int j = 0; j < N; j++) {
        for (int i = 0; i < M; i++) {
            sum += a[i][j];
        }
    }
    return sum;
}
```

a[0][0]	a[0][1]	a[0][2]	a[0][3]	
a[1][0]	a[1][1]	a[1][2]	a[1][3]	…
a[2][0]	a[2][1]	a[2][2]	a[2][3]	
	…			

Locality #4

```
int sum_array_3d(int a[M][N][N]) {
    int sum = 0;

    for (int i = 0; i < N; i++) {
        for (int j = 0; j < N; j++) {
            for (int k = 0; k < M; k++) {
                sum += a[k][i][j];
            }
        }
    }
    return sum;
}
```

What is "wrong" with this code?

How can it be fixed?

Cost of cache misses

Miss cost could be 100 × hit cost.

99% hits could be twice as good as 97%. How?

Assume cache hit time of 1 cycle, miss penalty of 100 cycles

Mean access time:

97% hits: 1 cycle + 0.03 * 100 cycles = 4 cycles

99% hits: 1 cycle + 0.01 * 100 cycles = 2 cycles

hit/miss rates

Cache performance metrics

Miss Rate

Fraction of memory accesses to data not in cache (misses / accesses)

Typically: 3% - 10% for L1; maybe < 1% for L2, depending on size, etc.

Hit Time

Time to find and deliver a block in the cache to the processor.

Typically: **1 - 2 clock cycles** for L1; **5 - 20 clock cycles** for L2

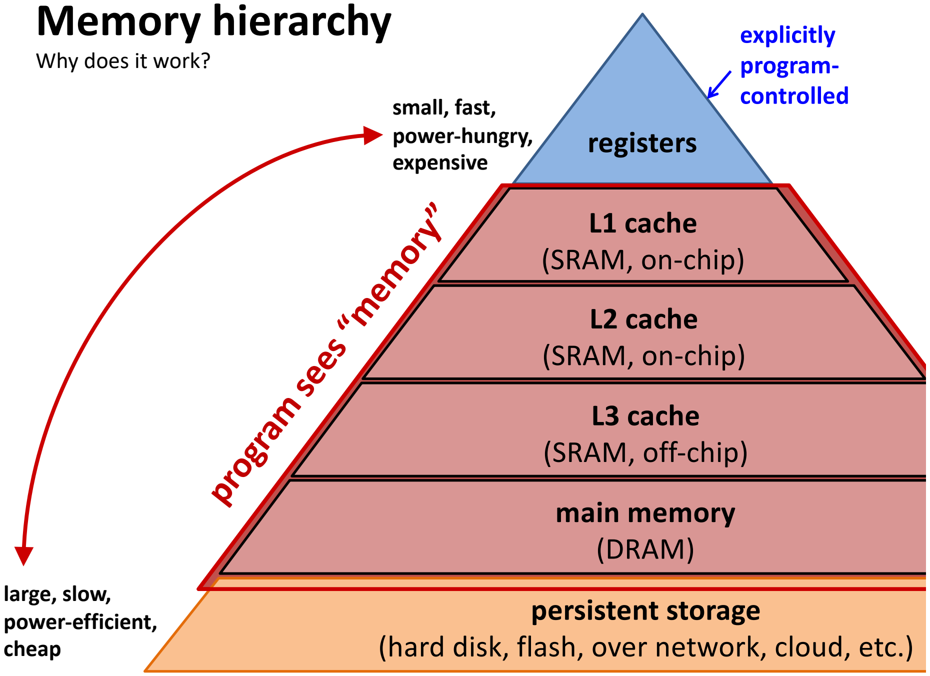
Miss Penalty

Additional time required on cache miss = main memory access time

Typically **50 - 200 cycles** for L2 *(trend: increasing!)*

Memory hierarchy

Why does it work?



explicitly program-controlled

registers

small, fast, power-hungry, expensive

program sees "memory"

L1 cache (SRAM, on-chip)

L2 cache (SRAM, on-chip)

L3 cache (SRAM, off-chip)

main memory (DRAM)

large, slow, power-efficient, cheap

persistent storage (hard disk, flash, over network, cloud, etc.)

Cache organization

Block

Fixed-size unit of data in memory/cache

Placement Policy

Where in the cache should a given block be stored?

- direct-mapped, set associative

Replacement Policy

What if there is no room in the cache for requested data?

- least recently used, most recently used

Write Policy

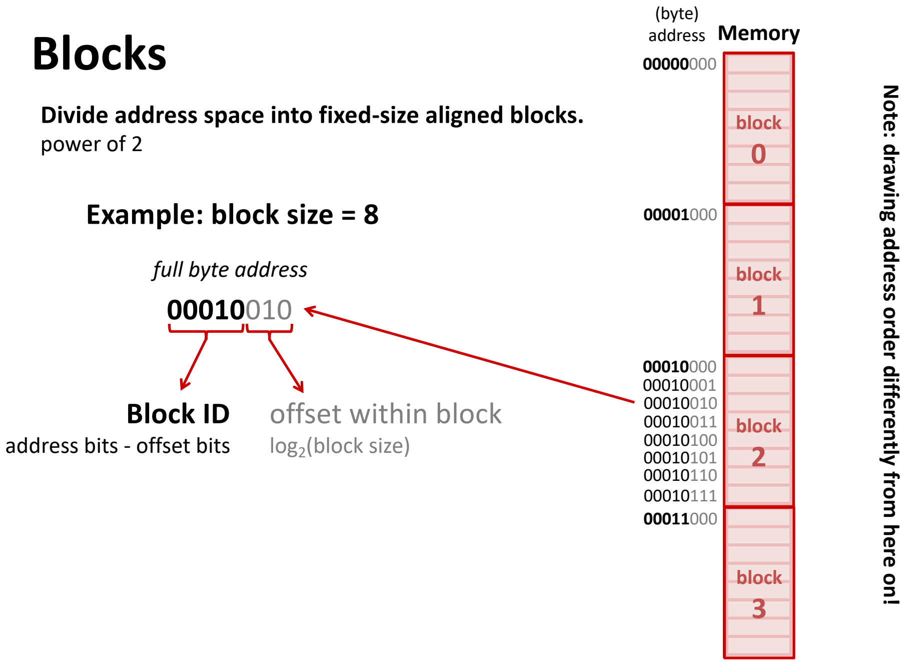
When should writes update lower levels of memory hierarchy?

- write back, write through, write allocate, no write allocate

Blocks

Divide address space into fixed-size aligned blocks.

power of 2



Example: block size = 8

full byte address

00010010

Block ID

address bits - offset bits

offset within block

$\log_2$(block size)

(byte) address

Memory

00000000

00001000

00010000

00010001

00010010

00010011

00010100

00010101

00010110

00010111

00011000

block 0

block 1

block 2

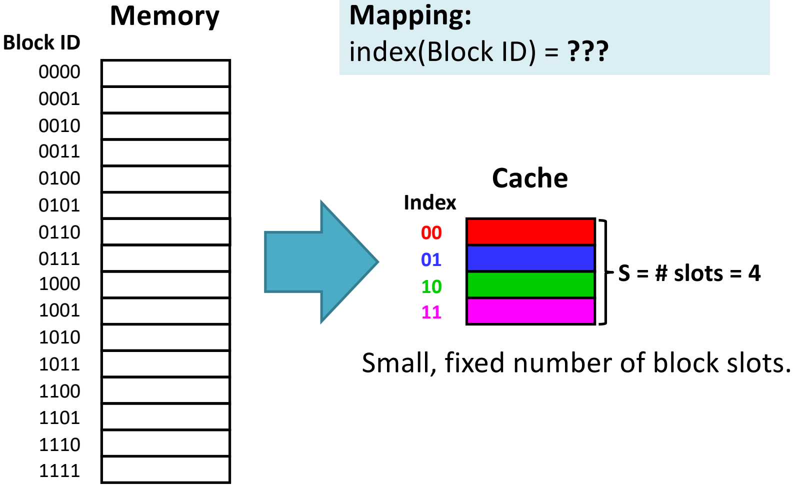
block 3

Note: drawing address order differently from here on!

Placement policy

Mapping:

index(Block ID) = **???**



Memory

Block ID

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Cache

Index

00
01
10
11

S = # slots = 4

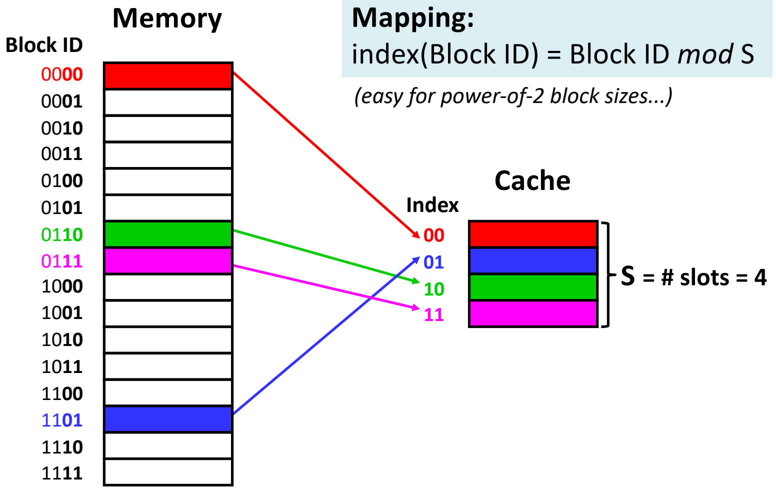
Small, fixed number of block slots.

Large, fixed number of block slots.

Placement: *direct-mapped*

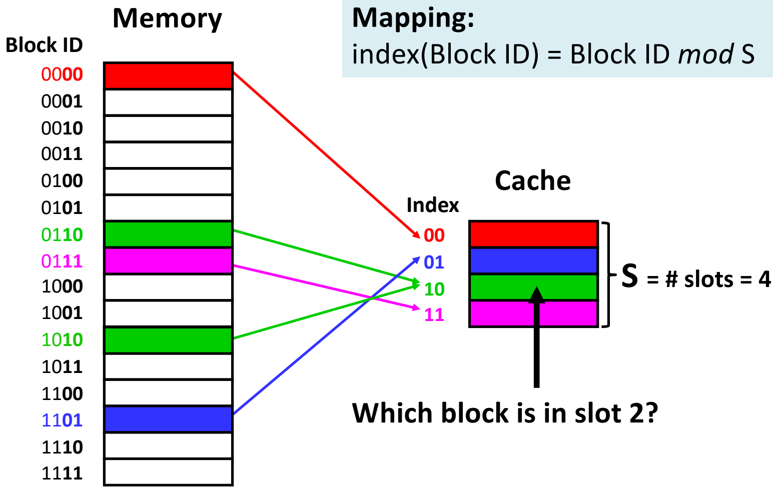
Mapping:
index(Block ID) = Block ID *mod* S

(easy for power-of-2 block sizes...)



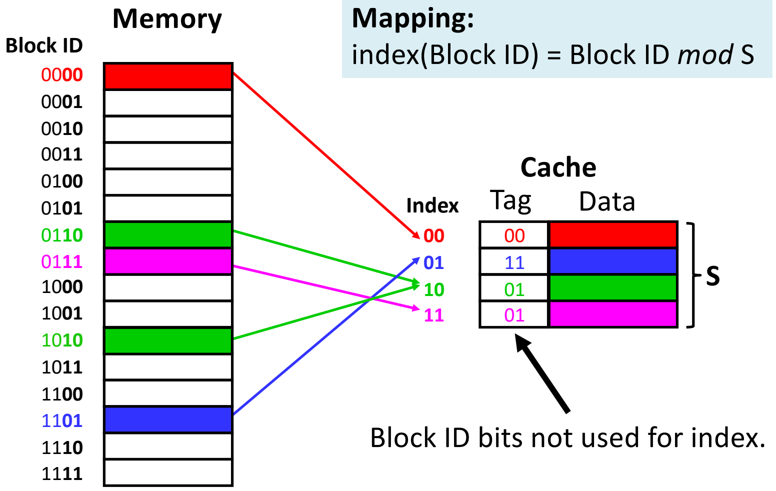
Placement: mapping ambiguity?

Mapping:
index(Block ID) = Block ID *mod* S

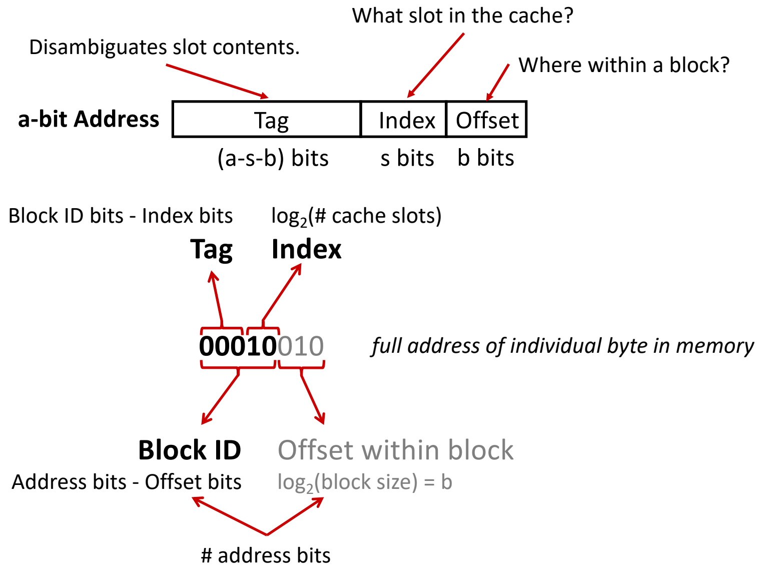


Placement: tags resolve ambiguity

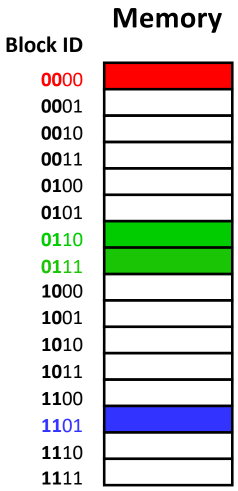
Mapping:
index(Block ID) = Block ID *mod* S



Address = tag, index, offset

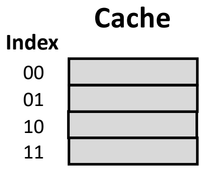


Placement: ~~direct-mapped~~



Why not this mapping?
index(Block ID) = Block ID / S

(still easy for power-of-2 block sizes...)



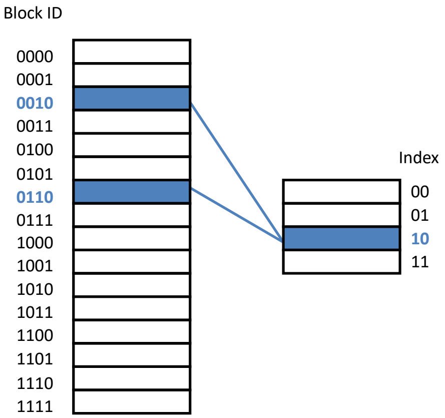
Puzzle #1

Cache starts *empty.*
Access (address, hit/miss) stream:

(0xA, miss), (0xB, hit), (0xC, miss)

What could the block size be?

Placement: direct-mapping conflicts



What happens when accessing in repeated pattern:
0010, 0110, 0010, 0110, 0010...?

cache conflict
Every access suffers a miss, evicts cache line needed by next access.

Placement: *set-associative*

S = # ~~slots~~ sets in cache

One index per ***set*** of block slots.
Store block in ***any*** slot within set.

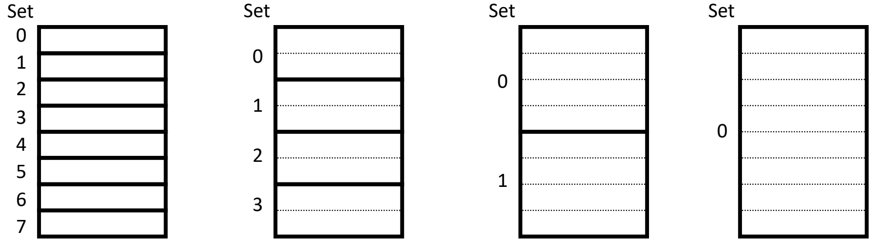
Mapping:
index(Block ID) = Block ID *mod* S

1-way
8 sets,
1 block each

2-way
4 sets,
2 blocks each

4-way
2 sets,
4 blocks each

8-way
1 set,
8 blocks



direct mapped

fully associative

Replacement policy: if set is full, what block should be replaced?
Common: **least recently used (LRU)**
but hardware may implement "not most recently used"

Example: tag, index, offset? #1

4-bit Address | Tag | Index | Offset

Direct-mapped
4 slots
2-byte blocks

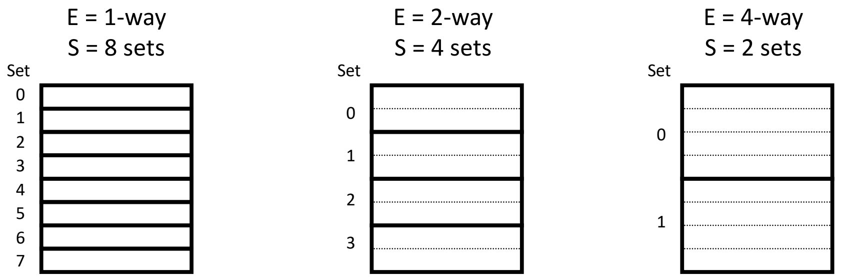
tag bits ____
set index bits ____
block offset bits____

index(1101) = ____

Example: tag, index, offset? #2

E-way set-associative
S slots
16-byte blocks

16-bit Address | Tag | Index | Offset



E = 1-way		E = 2-way		E = 4-way	
tag bits	____	tag bits	____	tag bits	____
set index bits	____	set index bits	____	set index bits	____
block offset bits	____	block offset bits	____	block offset bits	____
index(0x1833)	____	index(0x1833)	____	index(0x1833)	____

Replacement policy

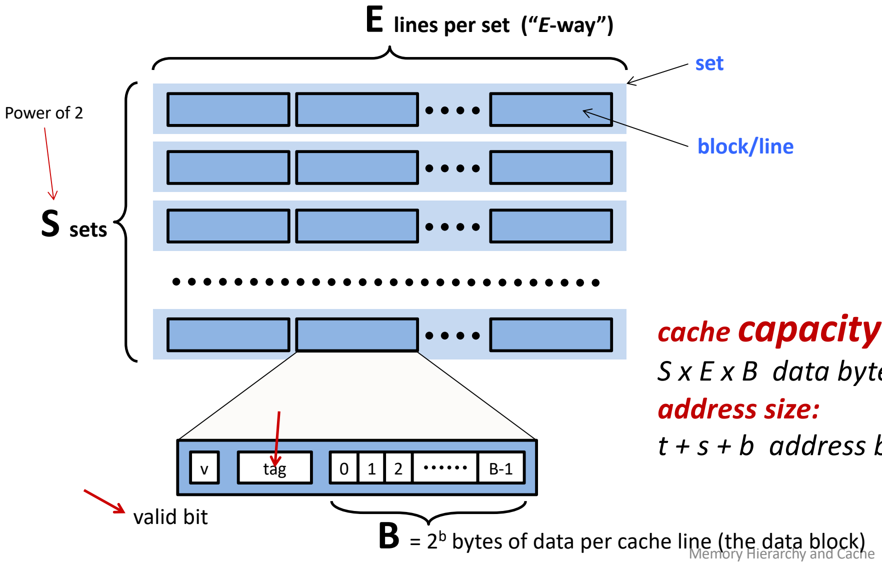
If set is full, what block should be replaced?

Common: **least recently used (LRU)**
(but hardware usually implements "not most recently used"

Another puzzle: Cache starts *empty*, uses LRU.
Access (address, hit/miss) stream:
(0xA, miss); (0xB, miss); (0xA, miss)

associativity of cache?

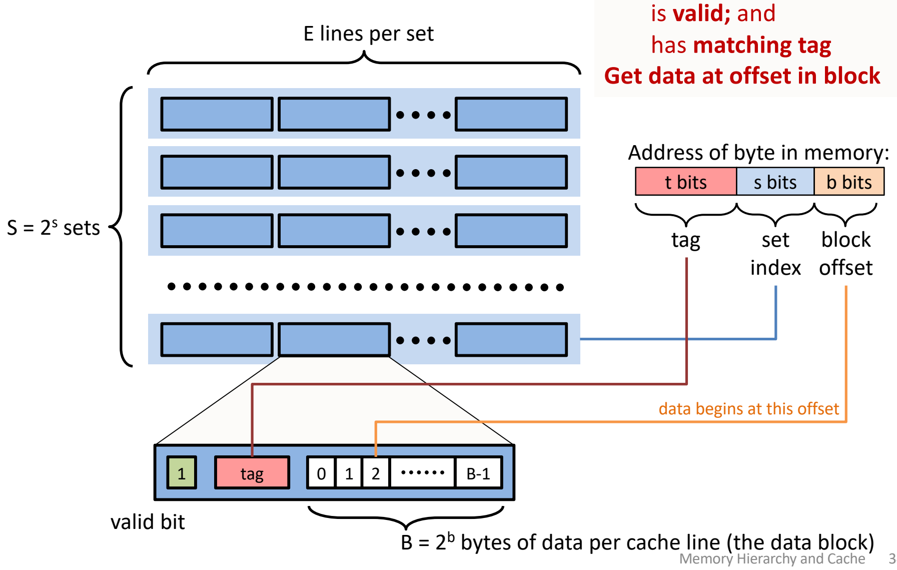
General cache organization (S, E, B)


E lines per set ("E-way")
set
block/line
Power of 2
S sets
v
tag
0
1
2
B-1
valid bit
B = 2^b bytes of data per cache line (the data block)

cache capacity:
S x E x B data bytes
address size:
t + s + b address bits

Cache read

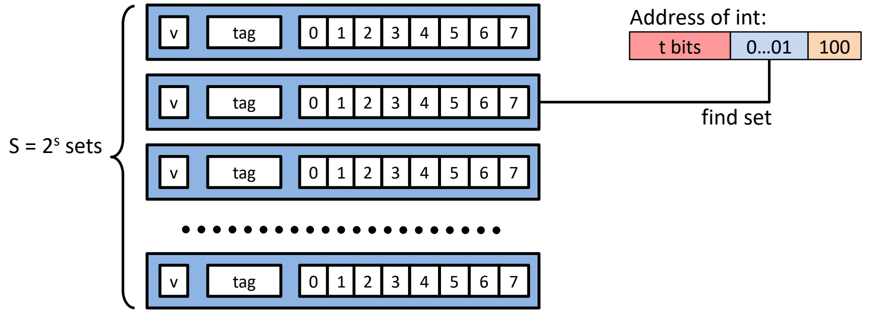
Locate set by index
Hit if any block in set:
is **valid**; and
has **matching tag**
Get data at offset in block


E lines per set
S = 2^s sets
Address of byte in memory:
t bits
s bits
b bits
tag
set index
block offset
data begins at this offset
1
tag
0
1
2
B-1
valid bit
B = 2^b bytes of data per cache line (the data block)

Cache read: direct-mapped (E = 1)

This cache:

- Block size: 8 bytes
- Associativity: 1 block per set (direct mapped)



Cache read: direct-mapped (E = 1)

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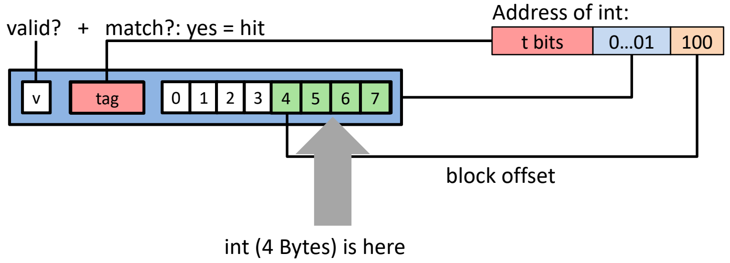


If no match: old line is evicted and replaced

Direct-mapped cache practice

12-bit address
16 lines, 4-byte block size
Direct mapped

Offset bits? Index bits? Tag bits?

	11	10	9	8	7	6	5	4	3	2	1	0
Access 0x354												
Access 0xA20												

Index	*Tag*	*Valid*	*B0*	*B1*	*B2*	*B3*
0	19	1	99	11	23	11
1	15	0	–	–	–	–
2	1B	1	00	02	04	08
3	36	0	–	–	–	–
4	32	1	43	6D	8F	09
5	0D	1	36	72	F0	1D
6	31	0	–	–	–	–
7	16	1	11	C2	DF	03

Index	*Tag*	*Valid*	*B0*	*B1*	*B2*	*B3*
8	24	1	3A	00	51	89
9	2D	0	–	–	–	–
A	2D	1	93	15	DA	3B
B	0B	0	–	–	–	–
C	12	0	–	–	–	–
D	16	1	04	96	34	15
E	13	1	83	77	1B	D3
F	14	0	–	–	–	–

Example #1 (E = 1)

Locals in registers.
Assume `a` *is aligned such that*
`&a[r][c]` ***is*** `aa...a rrrr cccc 000`

Assume: cold (empty) cache
3-bit set index, 5-bit offset

`aa...arrr rcc cc000`

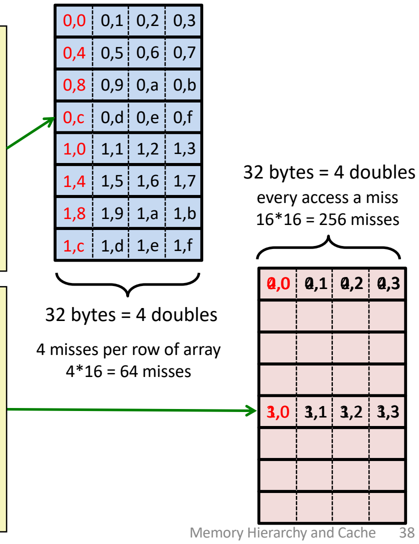
`0,0: aa...a000 000 00000`

```
int sum_array_rows(double a[16][16]){
    double sum = 0;

    for (int r = 0; r < 16; r++){
        for (int c = 0; c < 16; c++){
            sum += a[r][c];
        }
    }
    return sum;
}
```

```
int sum_array_cols(double a[16][16]){
    double sum = 0;

    for (int c = 0; c < 16; c++){
        for (int r = 0; r < 16; r++){
            sum += a[r][c];
        }
    }
    return sum;
}
```



0,0	0,1	0,2	0,3
0,4	0,5	0,6	0,7
0,8	0,9	0,a	0,b
0,c	0,d	0,e	0,f
1,0	1,1	1,2	1,3
1,4	1,5	1,6	1,7
1,8	1,9	1,a	1,b
1,c	1,d	1,e	1,f

32 bytes = 4 doubles

4 misses per row of array
4*16 = 64 misses

32 bytes = 4 doubles
every access a miss
16*16 = 256 misses

0,0	0,1	0,2	0,3
3,0	3,1	3,2	3,3

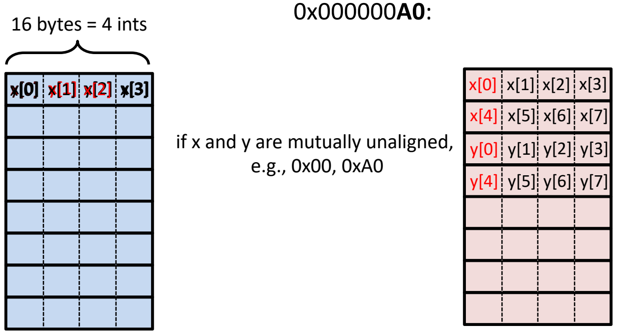
Example #2 (E = 1)

block = 16 bytes; 8 sets in cache
How many block offset bits?
How many set index bits?

```
int dotprod(int x[8], int y[8]) {
    int sum = 0;

    for (int i = 0; i < 8; i++) {
        sum += x[i]*y[i];
    }
    return sum;
}
```

Address bits:
B =
S =
Addresses as bits
0x000000**00**:
0x000000**80**:
0x000000**A0**:



16 bytes = 4 ints

if x and y are mutually aligned, e.g., 0x00, 0x80

x[0]	x[1]	x[2]	x[3]

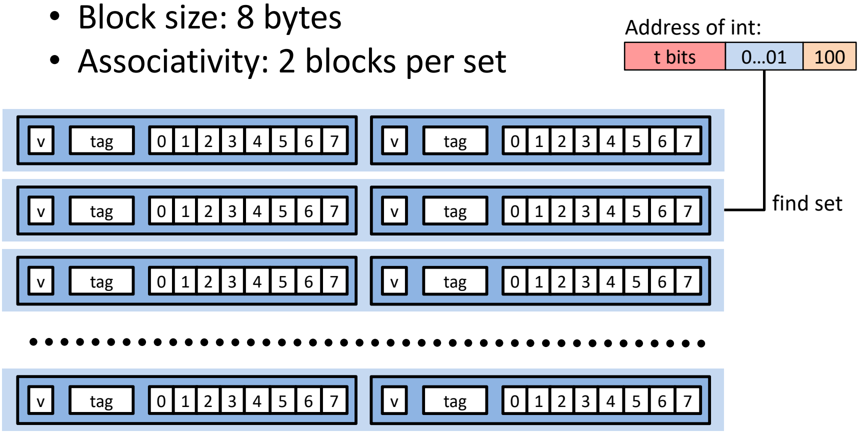
if x and y are mutually unaligned, e.g., 0x00, 0xA0

x[0]	x[1]	x[2]	x[3]
x[4]	x[5]	x[6]	x[7]
y[0]	y[1]	y[2]	y[3]
y[4]	y[5]	y[6]	y[7]

Cache read: set-associative (Example: E = 2)

This cache:

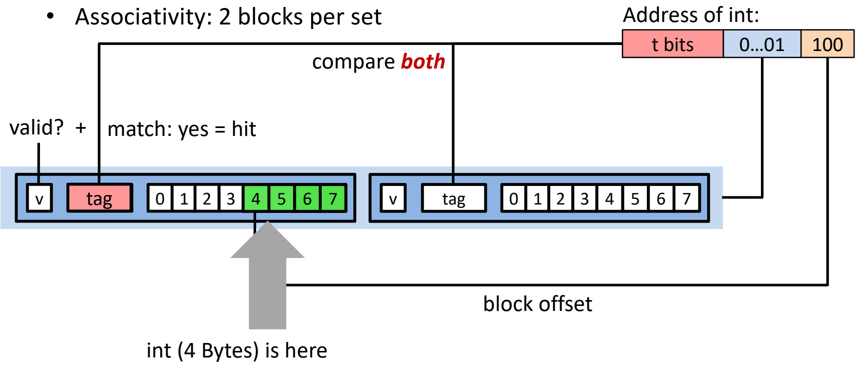
- Block size: 8 bytes
- Associativity: 2 blocks per set



Cache read: set-associative (Example: E = 2)

This cache:

- Block size: 8 bytes
- Associativity: 2 blocks per set



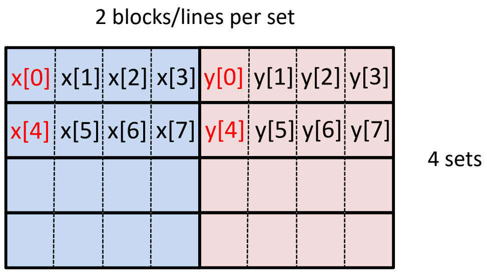
If no match: Evict and replace one line in set.

Example #3 (E = 2)

```
float dotprod(float x[8], float y[8]) {
    float sum = 0;

    for (int i = 0; i < 8; i++) {
        sum += x[i]*y[i];
    }
    return sum;
}
```

If x and y aligned,
e.g. &x[0] = 0, &y[0] = 128,
can still fit both because each set
has space for two blocks/lines



2 blocks/lines per set

x[0]	x[1]	x[2]	x[3]	y[0]	y[1]	y[2]	y[3]
x[4]	x[5]	x[6]	x[7]	y[4]	y[5]	y[6]	y[7]

4 sets

Types of Cache Misses

Cold (compulsory) miss

Conflict miss

Capacity miss

Which ones can we mitigate/eliminate? How?

Writing to cache

Multiple copies of data exist, must be kept in sync.

Write-hit policy

Write-through:

Write-back: needs a ***dirty bit***

Write-miss policy

Write-allocate:

No-write-allocate:

Typical caches:

Write-back + Write-allocate, usually

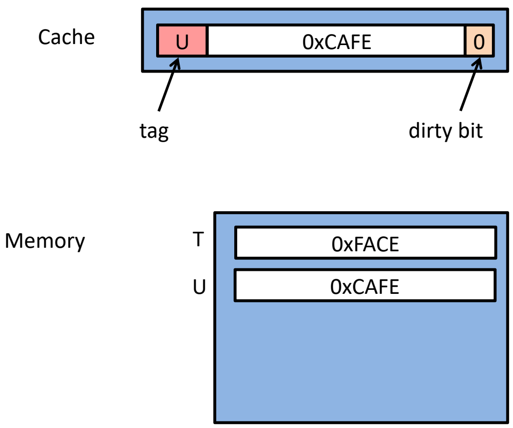
Write-through + No-write-allocate, occasionally

Write-back, write-allocate example

eax =
ecx = T
edx = U

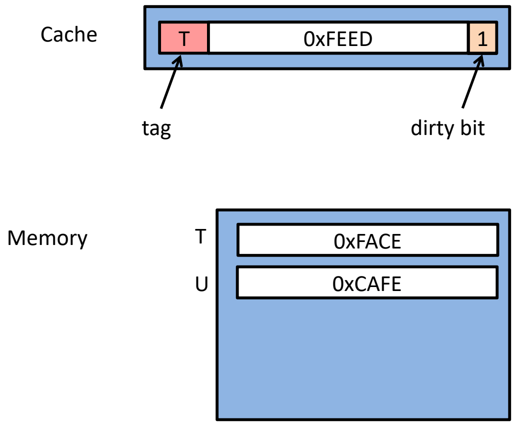
Cache/memory not involved

1. **mov $T, %ecx**
2. **mov $U, %edx**
3. **mov $0xFEED, (%ecx)**
   a. Miss on T.



Write-back, write-allocate example

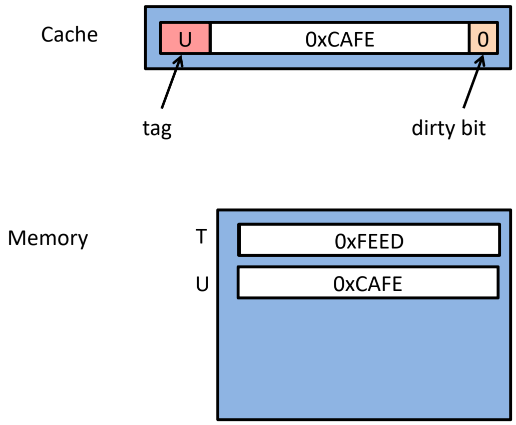
eax =
ecx = T
edx = U



1. **mov $T, %ecx**
2. **mov $U, %edx**
3. **mov $0xFEED, (%ecx)**
   a. Miss on T.
   b. Evict U (clean: discard).
   c. Fill T (write-allocate).
   d. Write T in cache (dirty).
4. **mov (%edx), %eax**
   a. Miss on U.

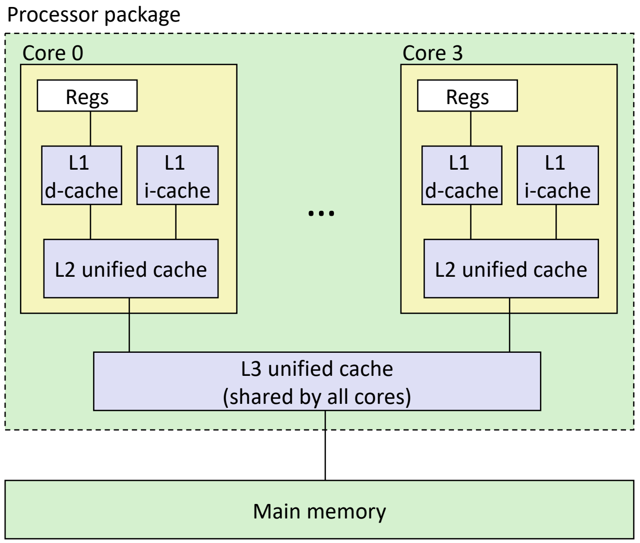
Write-back, write-allocate example

eax = 0xCAFE
ecx = T
edx = U



1. **mov $T, %ecx**
2. **mov $U, %edx**
3. **mov $0xFEED, (%ecx)**
   a. Miss on T.
   b. Evict U (clean: discard).
   c. Fill T (write-allocate).
   d. Write T in cache (dirty).
4. **mov (%edx), %eax**
   a. Miss on U.
   b. Evict T (dirty: write back).
   c. Fill U.
   d. Set %eax.
5. **DONE.**

Example memory hierarchy



Typical laptop/desktop processor
(c.a. 201_)

L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 11 cycles

L3 unified cache:
8 MB, 16-way,
Access: 30-40 cycles

Block size: 64 bytes for all caches.

slower, but more likely to hit

(Aside) **Software caches**

Examples

File system buffer caches, web browser caches, database caches, network CDN caches, etc.

Some design differences

Almost always fully-associative

Often use complex replacement policies

Not necessarily constrained to single “block” transfers

Cache-friendly code

- Locality, locality, locality.
- Programmer can optimize for cache performance
  - Data structure layout
  - Data access patterns
    - Nested loops
    - Blocking (see CSAPP 6.5)
- All systems favor “cache-friendly code”
  - Performance is hardware-specific
  - Generic rules capture most advantages
    - Keep working set small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code