CS 240
Foundations of Computer Systems

WELLESLEY

Combinational Logic

Karnaugh maps

Building blocks: encoders, decoders, multiplexers

Abstraction!

https://cs.wellesley.edu/~cs240/

But first…

Recall: *sum of products*

logical sum (OR)
of products (AND)
of inputs or their complements (NOT).

A	B	C	M
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

Construct with:

- 1 code detector per 1-valued output row
- 1 large OR of all code detector outputs

Is it minimal?

Gray Codes = reflected binary codes

Alternate binary encoding
designed for electromechanical switches and counting.

00	01	11	10
0	1	2	3

000	001	011	010	110	111	101	100
0	1	2	3	4	5	6	7

How many bits change when incrementing?

Karnaugh Maps: find (minimal) sums of products

ex

A	B	C	D	F(A, B, C, D)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

gray code order

CD

AB	00	01	11	10
00	0	0	0	0
01	0	0	0	1
11	1	1	0	1
10	1	1	1	1

1. Cover exactly the 1s by drawing a (minimum) number of maximally sized rectangles whose dimensions (in cells) are powers of 2. (They may overlap or wrap around!)
2. For each rectangle, make a *product* of the inputs (or complements) that are 1 for all cells in the rectangle. (*minterms*)
3. Take the *sum* of these products.

Karnaugh Maps and Wrapping

ex

Blocks of 1s in Karnaugh maps can wrap around sides and even 4 corners.

Give the minimal sum-of-products for the Karnaugh map to the left.

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	1	0	0	1
10	1	0	0	1

The grouping and ordering of variables in a Karnaugh map doesn't matter, but the **AB/CD** ordering is easier to read from a truth table.

Convince yourself that the **AC/DB** table is equivalent to the **AB/CD** table and has the Same sum-of-products expression. In this particular AC/DB table, no wrapping is required for the rectangles!

AC \ DB	00	01	11	10
00	1			
01	1			
11	1	1		
10	1	1		

Karnaugh Maps and Ambiguity

ex

The minimal sum-of-products expression for a Karnaugh map may not be unique.

Ambiguity is introduced when an arbitrary choice needs to be made.

An example of ambiguity is this Karnaugh map. Give four different minimal sum-of-product expressions for this map

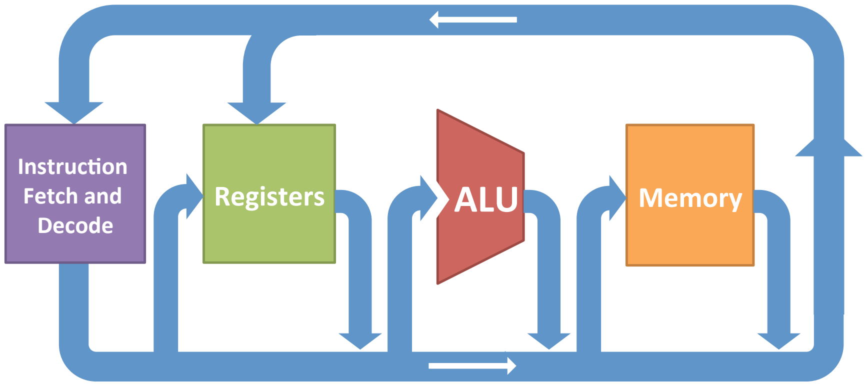
AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	1	1	1
10	0	0	0	0

Voting again with Karnaugh Maps

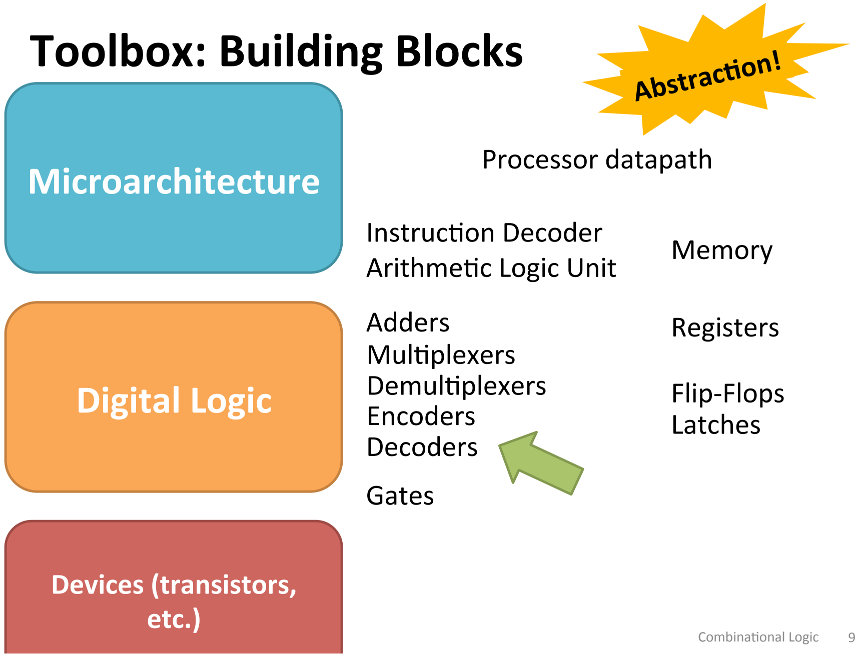
ex

A	B	C	M
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Goal for next 2 weeks: **Simple Processor**



Toolbox: Building Blocks



Abstraction!

Microarchitecture

Processor datapath

Instruction Decoder
Arithmetic Logic Unit

Memory

Digital Logic

Adders
Multiplexers
Demultiplexers
Encoders
Decoders

Gates

Registers

Flip-Flops
Latches

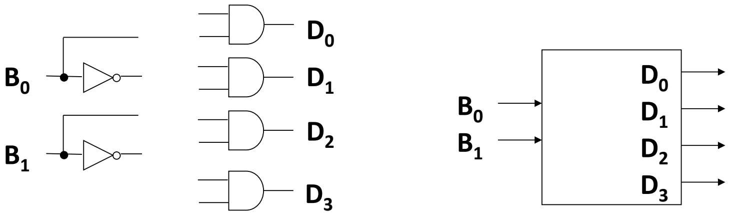
Devices (transistors, etc.)

Decoders

ex

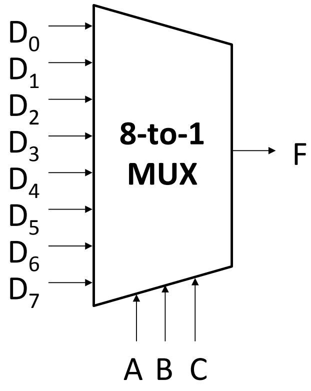
Decodes input number, asserts corresponding output.

- *n*-bit input (an unsigned number)
- 2^n outputs
- Built with code detectors.



Multiplexers

Select one of several inputs as output.



2^n data inputs

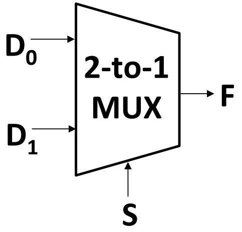
1 data output

***n* selector lines**

Build a 2-to-1 MUX from gates

ex

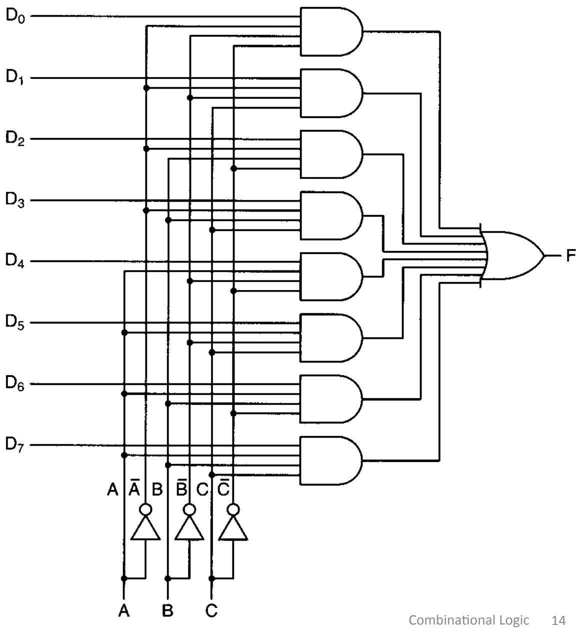
If S=0, then F=D_0.
If S=1, then F=D_1.



1. Construct the truth table.

2. Build the circuit.

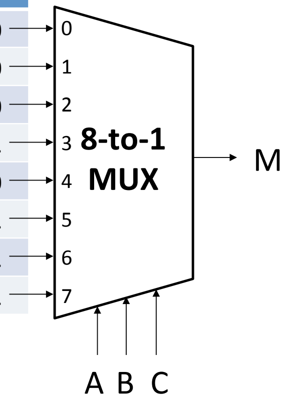
8-to-1 MUX



Costume idea: MUX OX

MUX + voltage source = truth table

A	B	C	M
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**



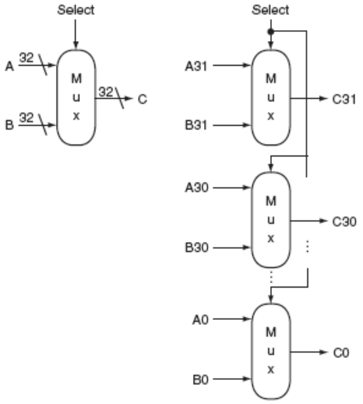
Buses and **Logic Arrays**

A bus is a collection of data lines treated as a single logical signal.

= fixed-width value

Array of logic elements applies same operation to each bit in a bus.

= bitwise operator


Select
A
32
B
32
M
u
x
32
C
Select
A31
B31
M
u
x
C31
A30
B30
M
u
x
C30
A0
B0
M
u
x
C0