CS 240
Foundations of Computer Systems

WELLESLEY

Representing Data with Bits

bits, bytes, numbers, and notation

positional number representation

base = 10 (*decimal*)

2	4	0	
100	10	1	*weight*
10^2	10^1	10^0	
2	1	0	*position*

$= 2 \times 10^2 + 4 \times 10^1 + 0 \times 10^0$

Base determines:

Maximum digit (base – 1). Minimum digit is 0.

Weight of each position.

Each position holds a digit.

Represented value = sum of all position values

position value = digit value × base^{position}

binary = base 2

Binary digits are called ***bits***: 0, 1

base = 2 (*binary*)

1	0	1	1	
8	4	2	1	*weight*
2^3	2^2	2^1	2^0	
3	2	1	0	*position*

$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

When ambiguous, subscript with base:

101_{10} Dalmatians (movie)

101_2-Second Rule (folk wisdom for food safety)

Powers of 2:
memorize up to ≥ 2^{10} (in base ten)

ex

Power: $2^?$	Decimal value
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

conversion from binary to decimal

ex

$$1\,0\,1\,1\,0\,1_2 = ?_{10}$$

Interpret the positional representation according to the base:
sum the place weights where 1 appears (in either direction).

conversion from decimal to binary

ex

$$19_{10} = ?_2$$

Divide-by-2 Approach (Right to Left) | 64 32 16 8 4 2 1

Quotient *Remainder?*

Powers-of-2 Approach (Left to Right) | 64 32 16 8 4 2 1

Value *Power that fits?*

binary arithmetic

ex

$110_2 + 1011_2 = ?_2$

$1101_2 - 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

conversion and arithmetic

ex

$19_{10} = ?_2$

$1001_2 = ?_{10}$

$240_{10} = ?_2$

$11010011_2 = ?_{10}$

$101_2 + 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

byte = 8 bits

a.k.a. octet

What do you call 4 bits?

Smallest unit of data

used by a typical modern computer

Binary 00000000_2 -- 11111111_2

Decimal 000_{10} -- 255_{10}

Hexadecimal 00_{16} -- FF_{16}

Byte = 2 hex digits!

Programmer's hex notation (C, etc.):

`0xB4` = $B4_{16}$

Octal (base 8) also useful.

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

`char`: representing characters

A C-style string is represented by a series of bytes (`char`s).

- One-byte ASCII codes for each character.
- ASCII = American Standard Code for Information Interchange

32	space	48	0	64	@	80	P	96	`	112	p
33	!	49	1	65	A	81	Q	97	a	113	q
34	”	50	2	66	B	82	R	98	b	114	r
35	#	51	3	67	C	83	S	99	c	115	s
36	$	52	4	68	D	84	T	100	d	116	t
37	%	53	5	69	E	85	U	101	e	117	u
38	&	54	6	70	F	86	V	102	f	118	v
39	’	55	7	71	G	87	W	103	g	119	w
40	(	56	8	72	H	88	X	104	h	120	x
41	)	57	9	73	I	89	Y	105	I	121	y
42	*	58	:	74	J	90	Z	106	j	122	z
43	+	59	;	75	K	91	[	107	k	123	{
44	,	60	<	76	L	92	\	108	l	124	|
45	-	61	=	77	M	93	]	109	m	125	}
46	.	62	>	78	N	94	^	110	n	126	~
47	/	63	?	79	O	95	_	111	o	127	del

word |wərd|, n.

Natural unit of data used by processor.

Fixed size (e.g. 32 bits, 64 bits)

Defined by ISA: Instruction Set Architecture

machine instruction operands

word size = register size = address size

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	1	0	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Java/C int = 4 bytes: 11,501,584

MSB: most significant bit

LSB: least significant bit

fixed-size data representations

(size in bytes)

Java Data Type	C Data Type	[word = 32 bits]	[word = 64 bits]
boolean		1	1
byte	char	1	1
char		2	2
short	short int	2	2
int	int	4	4
float	float	4	4
	long int	4	8
double	double	8	8
long	long long	8	8
	long double	8	16

Depends on word size!

bitwise operators

bit = Boolean
0 = false
1 = true

Bitwise operators on fixed-width **bit vectors**.

AND &   OR | XOR ^   NOT ~

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001
```

```
  01010101
^ 01010101
  --------
```

ex

Laws of Boolean algebra apply bitwise.

e.g., DeMorgan's Law: ~(A | B) = ~A & ~B

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bitwise operators in C

`& | ^ ~` apply to any *integral* data type
`long, int, short, char, unsigned`

Examples (`char`)

ex

```
~0x41 =

~0x00 =

0x69 & 0x55 =

0x69 | 0x55 =
```

Many bit-twiddling puzzles in upcoming assignment

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basics of C (vs. Java)

		Java	C
Similar	*Interaction*	compile, then run	
	Variable declaration; arithmetic	`int a = 5;` `a = a * 5;`	
	Loops	`for (int i = 0; i < N; i++) {…}`	
Different	*Data*	Objects/references	Pointers/structs/allocated memory
	Management	Garbage collection	Manual memory management
	Functions	`public static int max(int a, int b)`	`int max(int a, int b)`
	Main, printing		

```
public class HelloWorld {
   public static void main(String[] args) {
      System.out.println("Hello");
   }
}
```

```
#include<stdio.h>
int main(void) {
   printf("Hello\n");
   return 0;
}
```

https://introcs.cs.princeton.edu/java/faq/c2java.html 15

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Representation Example 1: Sets as Bit Vectors

Representation: *n*-bit vector gives subset of {0, …, *n*–1}.

$a_i = 1 \equiv i \in A$

a = 0b**01101001**   A = { 0, 3, 5, 6 }
76543210

b = 0b**01010101**   B = { 0, 2, 4, 6 }
76543210

Bitwise Operations		**Set Operations**
`a & b = 0b01000001`	{0, 6}	Intersection
`a	b = 0b01111101`	{0, 2, 3, 4, 5, 6}	Union
`a ^ b = 0b00111100`	{2, 3, 4, 5}	Symmetric difference
`~ b = 0b10101010`	{1, 3, 5, 7}	Complement

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logical operations in C

&& || ! apply to any "integral" data type
`long, int, short, char, unsigned`

0 is **false** **nonzero** is **true** result always **0 or 1**

early termination a.k.a. **short-circuit evaluation**

Examples (`char`)

```
 !0x41 =
 !0x00 =
!!0x41 =

0x69 && 0x55 =
0x69 || 0x55 =
```

ex

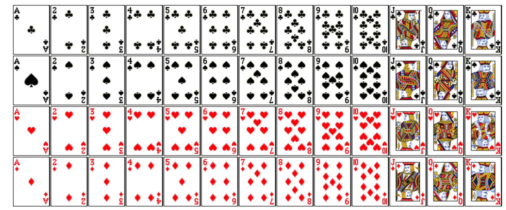
Representation Example 2:
Playing Cards

52 cards in 4 suits

- How do we encode suits, face cards?

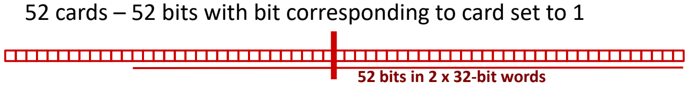
What operations should be easy to implement?

- Get and compare rank
- Get and compare suit



Two possible representations

52 cards – 52 bits with bit corresponding to card set to 1



"One-hot" encoding

- Hard to compare values and suits independently
- Not space efficient

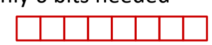
4 bits for suit, 13 bits for card value – 17 bits with two set to 1

Pair of one-hot encoded values

- Easier to compare suits and values independently
- Smaller, but still not space efficient

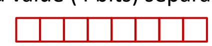
Two better representations

Binary encoding of all 52 cards – only 6 bits needed



- Number cards uniquely from 0
- Smaller than one-hot encodings.
- Hard to compare value and suit

Binary encoding of suit (2 bits) and value (4 bits) separately



- Number each suit uniquely
- Number each value uniquely
- Still small
- Easy suit, value comparisons

Compare Card **Suits**

mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0

0	0	1	1	0	0	0	0

suit value

```
#define SUIT_MASK 0x30

int sameSuit(char card1, char card2) {
  return !((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK));

  //same as (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}

char hand[5];        // represents a 5-card hand
...
if (sameSuit(hand[0], hand[1])) { ... }
```


Compare Card **Values**

ex

mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0

suit value

```
#define VALUE_MASK

int greaterValue(char card1, char card2) {



}

char hand[5];        // represents a 5-card hand
...
if (greaterValue(hand[0], hand[1])) { ... }
```


Bit shifting

logical shift left 2

x << 2

1 0 0 1 1 0 0 1 x

1 0 | 0 1 1 0 0 1 0 0

lose bits on left

fill with zeroes on right

1 0 0 1 1 0 0 1 x

fill with zeroes on left

logical shift right 2 **x >> 2**

0 0 1 0 0 1 1 0 | 0 1

lose bits on right

arithmetic shift right 2 **x >> 2**

1 1 1 0 0 1 1 0 | 0 1

fill with copies of MSB on left

Shift gotchas

!!!

Logical or arithmetic shift right: how do we tell?

C: compiler chooses

Usually based on type: rain check!

Java: >> is arithmetic, >>> is logical

Shift an *n*-bit type by at least `0` and no more than `(n-1)`.

C: other shift distances are undefined.

anything could happen

Java: shift distance is used modulo number of bits in shifted type

Given `int x`: `x << 34 == x << 2`

Shift and mask: extract a bit field

ex

Write a C function that
extracts the *2nd most significant byte*
from its 32-bit integer argument.

Example behavior:

argument: `0b 01100001 01100010 01100011 01100100`

expected result: `0b 00000000 00000000 00000000 01100010`

All other bits are zero. Desired bits in least significant byte.

```
int get2ndMSB(int x) {
```

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