CS240 Laboratory 2
Digital Logic

- Circuit equivalence
- Boolean Algebra/Universal gates
- Exclusive OR
- Binary Numbers
- Integrated circuits
- Logic Diagrams vs. pin-outs
- LogicWorks demo

Circuit Equivalence

Two boolean functions with same truth table = **equivalent**

When there is an equivalent circuit which uses fewer gates, transistors, or chips, it is preferable to use that circuit in the design

Example:

Given: F = A'B' + A'B   Q = A’ + A’B + A’B’

A	B	A’B’	A’B	F
0	0	1	0	1
0	1	0	1	1
1	0	0	0	0
1	1	0	0	0

A	B	A’	A’B	A’ B’	Q
0	0	1	0	1	1
0	1	1	0	0	1
1	0	0	0	0	0
1	1	0	0	0	0

F and Q are equivalent because they have the same truth table.

Identities of Boolean Algebra

- Identity law $1A = A \quad 0 + A = A$
- Null law $0A = 0 \quad 1 + A = 1$
- Idempotent law $AA = A \quad A + A = A$
- Inverse law $AA' = 0 \quad A + A' = 1$
- Commutative law $AB = BA \quad A + B = B + A$
- Associative law $(AB)C = A(BC)$
  $(A + B) + C = A + (B + C)$
- Distributive law $A + BC = (A + B)(A + C)$
  $A(B + C) = AB + AC$
- Absorption law $A(A + B) = A$
  $A + AB = A$
- De Morgan's law $(AB)' = A' + B'$
  $(A + B)' = A'B'$

Example:

$F = A'B' + A'B$
$= A'(B' + B)$ distributive
$= A'(1)$ inverse
$= A'$ identity

$Q = A' + A'B + A'B'$
$= A' + A'B'$ absorption
$= A'$ absorption

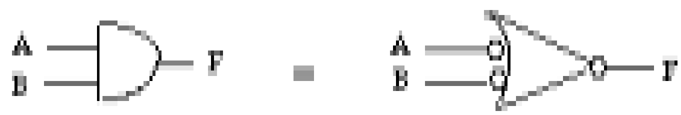
Universal Gates

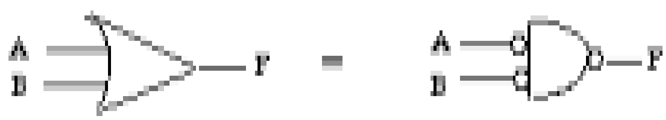
Any Boolean function can be constructed with NOT, AND, and OR gates

NAND and NOR = **universal gates**

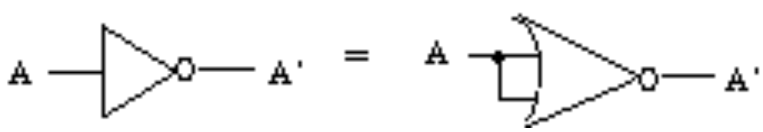
DeMorgan's Law shows how to make **AND** from NOR (and vice-versa)

$AB = (A' + B')'$ (**AND** from NOR)
$A + B = (A'B')'$ (**OR** from NAND)

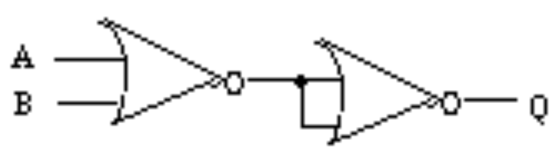




NOT from a NOR



OR from a NOR



To implement a function using only NOR gates:

- apply DeMorgan's Law to each AND in the expression until all ANDs are converted to NORs
- use a NOR gate for any NOT gates, as well.
- remove any redundant gates (NOT NOT, may remove both)

Implementing the circuit using only NAND gates is similar.

Example: $Q = (AB)'B'$

$= (A' + B')B'$

$= ((A'+B')' + B)'$ (NOR gates only, since NOR can be used as a NOT gate)

Simplifying Circuits or Proving Equivalency

General rule to simplify circuits or prove equivalency:

1. Distribute if possible, and if you can't, apply DeMorgan's Law so that you can.
2. Apply other identities to remove terms, and repeat step 1.

EXAMPLE: Is $(A'B)'(AB)' + A'B'$ equivalent to $(AB)'$?

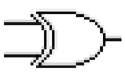
$F = (A'B)'(AB)' + A'B'$	-- can't distribute
$= (A + B')(A' + B') + A'B'$	DeMorgan's
$= AA' + AB' + A'B + B'B' + A'B'$	distributive
$= 0 + AB' + A'B + B' + A'B'$	inverse and idempotent
$= AB' + A'B + A'B'$	identity
$= B'(A+ A') + A'B$	distributive
$= B'(1) + A'B$	inverse
$= B' + A'B$	identity
$= B' + (A + B')'$	DeMorgan's
$= (B(A + B'))'$	DeMorgan's
$= (AB + BB')'$	distributive
$= (AB + 1)'$	inverse
$= (AB)'$	identity

Exclusive OR (XOR)

$F = AB' + A'B = A \oplus B$

A	**B**	**F**
0	0	0
0	1	1
1	0	1
1	1	0

Available on IC as a gate, useful for comparison problems



Example: Even parity $F = A \oplus B \oplus C$

A	**B**	**C**	**F**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Binary Numbers

Hex	Binary			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
A	1	0	1	0
B	1	0	1	1
C	1	1	0	0
D	1	1	0	1
E	1	1	1	0
F	1	1	1	1

Binary can be converted to decimal using positional representation of powers of 2:

$$0111_2 = 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0, \quad \text{result} = 7_{10}$$

Decimal can be also be converted to binary by finding the largest power of 2 which fits, subtract, and repeat with the remainders until remainder is 0 (assigning 1 to the positions where a power of 2 is used):

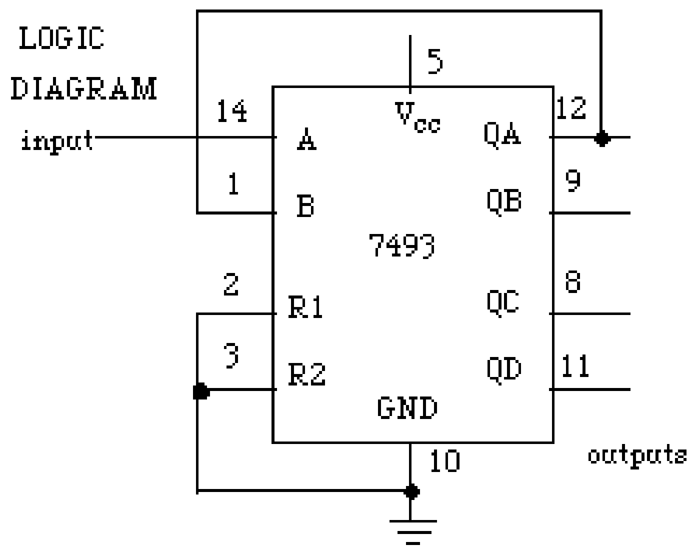
$$6_{10} = 6 - 2^2 = 2 - 2^1 = 0, \quad \text{result} = 0110_2$$

Hex can be converted to binary and vice versa by grouping into 4 bits.

$11110101_2 = F5_{16}$ **$37_{16} = 00110111$**

Logic diagrams vs. pin-outs

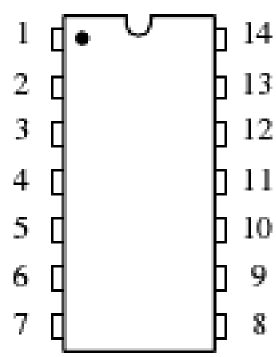
Logic diagrams are not the same as pin-outs! Logic diagrams show information about the logical operation of the device.



Pin-outs (found in **TTL Data Book** or online) show the physical layout of the pins:

Top left pin is pin 1, always to left of notch in chip, and often marked with a dot

Pins are numbered, starting with "1" at the top left corner and incremented counter-clockwise around the device



Bottom left pin is almost always connected to ground (0V)

Top right pin is almost always connected to Vcc (+5V)

The chip will not work if it is not connected to power and ground!

Circuit Simulation/LogicWorks (demo)