CS 240 Spring 2020
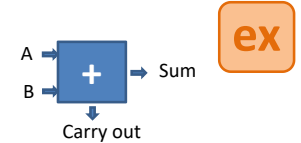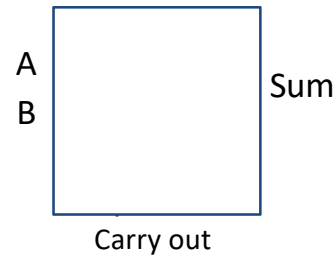Foundations of Computer Systems
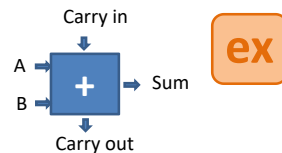Ben Wood

WELLESLEY

Logic for Arithmetic

adders
Arithmetic Logic Unit

Addition: 1-bit *half* adder

A → + → Sum
B →
↓
Carry out

A
B
Sum
Carry out

A	B	Carry out	Sum
0	0		
0	1		
1	0		
1	1		

Addition: 1-bit *full* adder

Carry in
↓
A → + → Sum
B →
↓
Carry out

Carry in

A
B
Sum

Carry out

Carry in	A	B	Carry out	Sum
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		
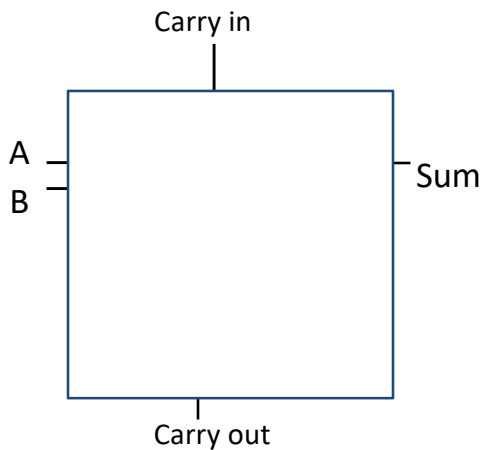
Addition: *n*-bit *ripple-carry* adder

Carry in

A
B
Sum

Carry out

Carry in$_0$
↓
A_0 → + → Sum$_0$
B_0 →
↓
A_1 → + → Sum$_1$
B_1 →
↓
A_2 → + → Sum$_2$
B_2 →
⋮
A_{n-1} → + → Sum$_{n-1}$
B_{n-1} →
↓
Carry out$_{n-1}$
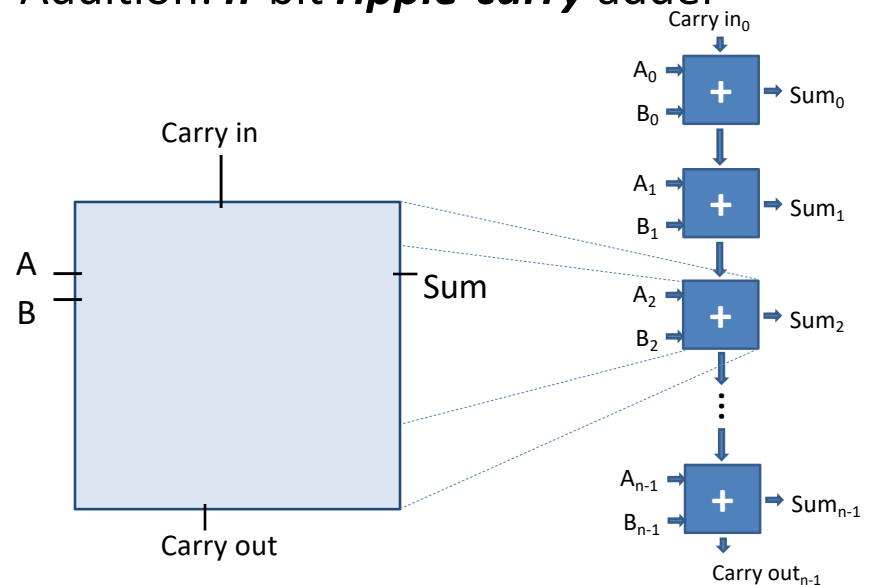
There are faster, more complicated ways too…

Processor Components

Abstraction!

4 — Instruction Fetch and Decode

2 — Registers

1 — ALU

3 — Memory

Arithmetic Logic Unit (ALU)

1

a few bits → **Condition Codes** (sign, overflow, carry-out, zero)

words → **Operand A** →

ALU → **Result** ← *word*

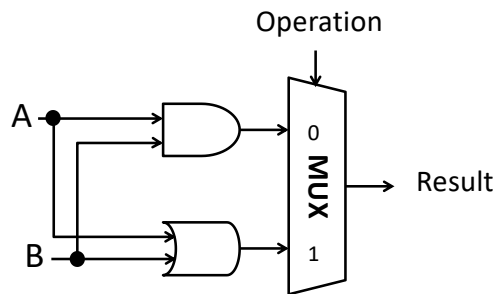**Operand B** →

Operation

a few bits
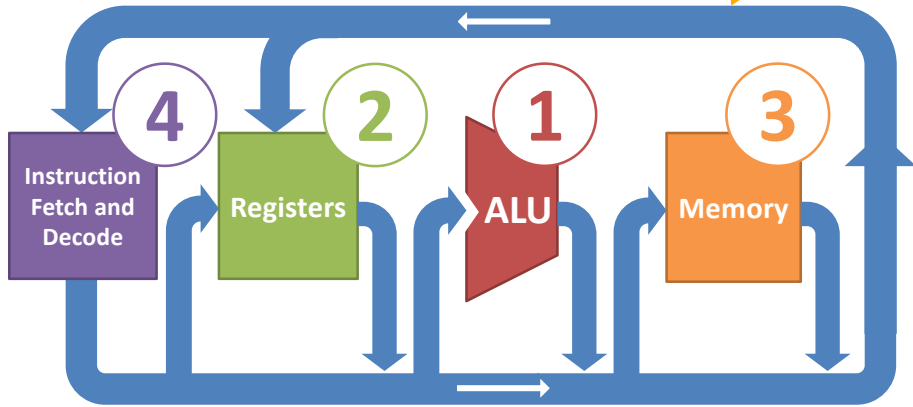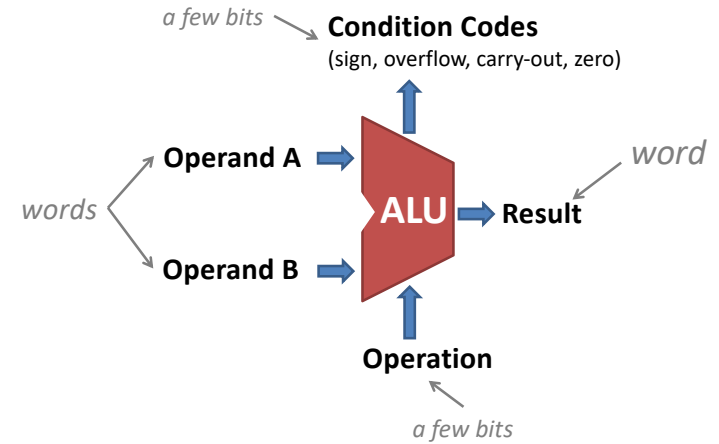
Hardware unit for arithmetic and bitwise operations.
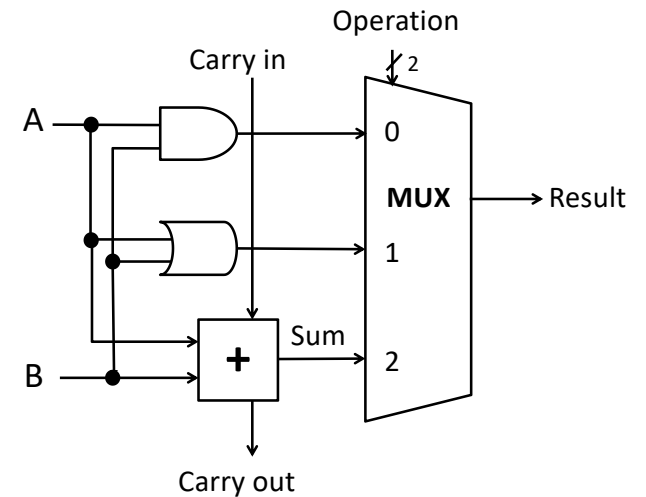

1-bit ALU for bitwise operations

ex

Build an n-bit ALU from n 1-bit ALUs.
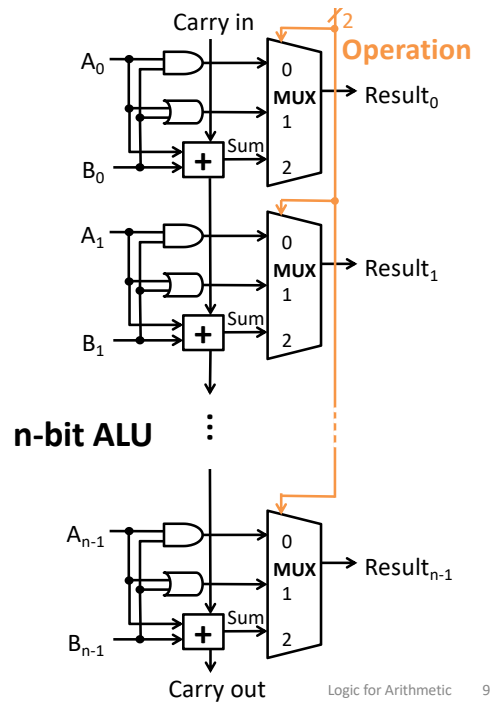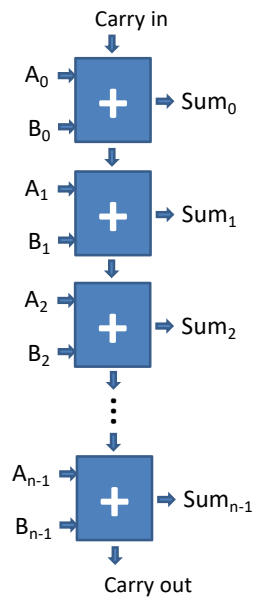Each bit i in the result is computed from the corresponding bit i in the two inputs.

Op	A	B	Result
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

1-bit ALU

Operation

Carry in

A

B

MUX → Result

Sum

Carry out

n-bit ripple carry adder

n-bit ALU

ALU conditions

Extra ALU outputs
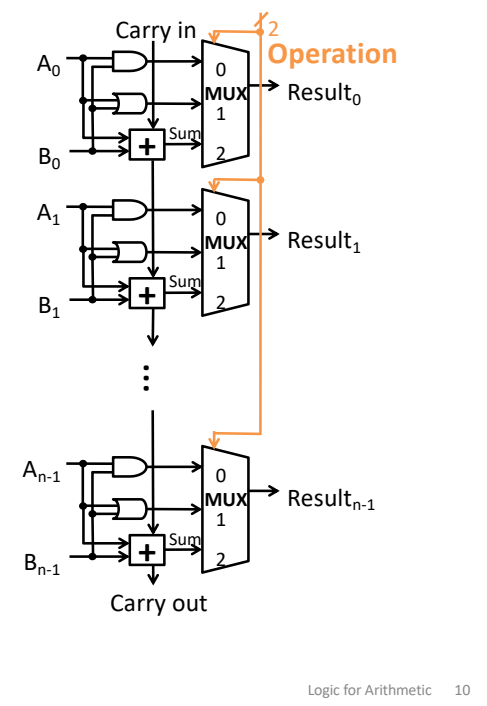describing properties of result.

Zero Flag: `ex`
1 if result is 00...0 else 0

Sign Flag: `ex`
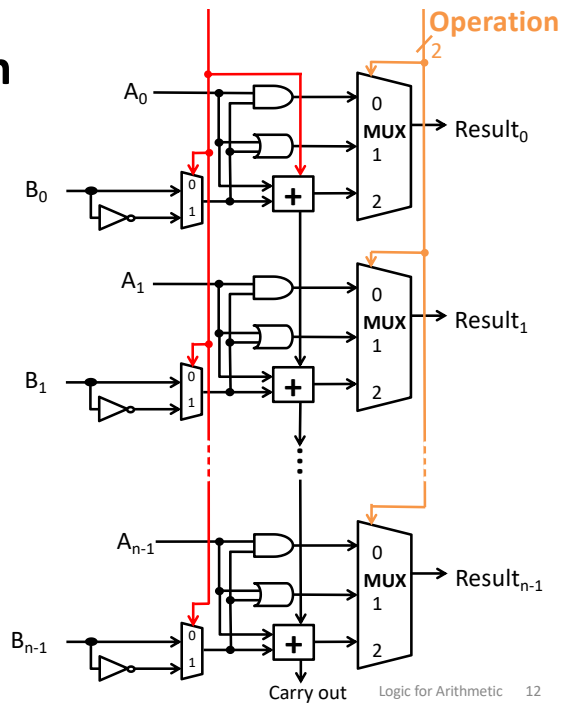1 if result is negative else 0

Carry Flag:
1 if carry out else 0

(Signed) **Overflow Flag:**
1 if signed overflow else 0

Implement these.

Add subtraction

How can we control ALU inputs
or add minimal new logic
to **compute A-B**?

ex

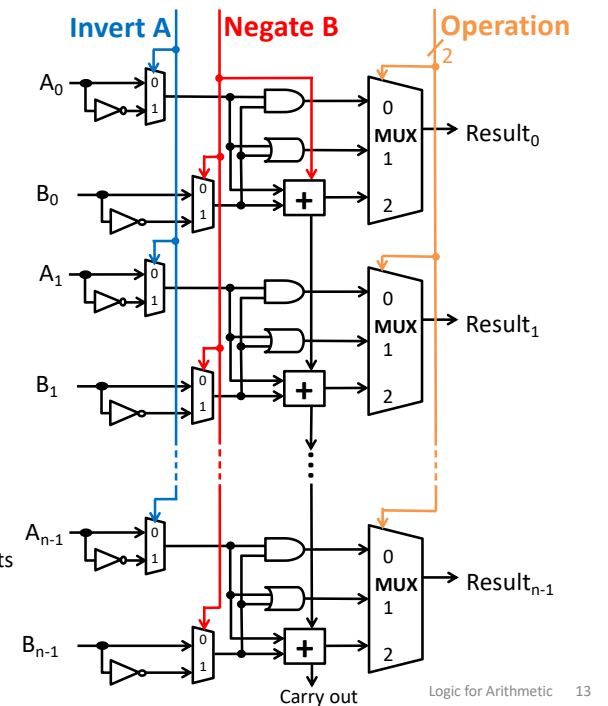**A NAND B**

A NOR B

A<B

A==B

How can we control ALU inputs
or add minimal new logic
to compute each?

Controlling the ALU

Abstraction!

ALU control lines	Function
0000	AND
0001	OR
0010	add
0110	subtract
1100	NOR

Condition Codes

Operand A →

ALU → Result

Operand B →

Control Lines