CS 240 Spring 2020
Foundations of Computer Systems
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WELLESLEY

Representing Data with Bits

bits, bytes, numbers, and notation

https://cs.wellesley.edu/~cs240/s20/

positional number representation

2	4	0
100	10	1
10^2	10^1	10^0
2	1	0

$= 2 \times 10^2 + 4 \times 10^1 + 0 \times 10^0$

weight (the row 100, 10, 1 and the row 10^2, 10^1, 10^0)

position (the row 2, 1, 0)

Base determines:

- Maximum digit (base – 1). Minimum digit is 0.
- Weight of each position.

Each position holds a digit.

Represented value = sum of all position values

- Position value = digit value x $\text{base}^{\text{position}}$

binary = base 2

1	0	1	1
8	4	2	1
2^3	2^2	2^1	2^0
3	2	1	0

$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

weight (the row 8, 4, 2, 1 and the row 2^3, 2^2, 2^1, 2^0)

position (the row 3, 2, 1, 0)

When ambiguous, subscript with base:

101_{10} Dalmatians (movie)

101_2-Second Rule (folk wisdom for food safety)

irony

Powers of 2: memorize up to ≥ 2^{10} (in base ten)

ex

Show powers, strategies.

conversion and arithmetic

ex

$19_{10} = ?_2$

$1001_2 = ?_{10}$

$240_{10} = ?_2$

$11010011_2 = ?_{10}$

$101_2 + 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

numbers and wires

One wire carries one bit.

How many wires to represent a given number?

1 0 0 1

1 0 0 0 1 0 0 1

What if I want to build a computer (and not change the hardware later)?

byte = 8 bits

a.k.a. octet

Smallest unit of data

used by a typical modern computer

Binary 00000000_2 -- 11111111_2

Decimal 000_{10} -- 255_{10}

Hexadecimal 00_{16} -- FF_{16}

Byte = 2 hex digits!

Programmer's hex notation (C, etc.):

`0xB4` = $B4_{16}$

Octal (base 8) also useful.

What do you call 4 bits?

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hex encoding practice

ex

`char`: representing characters

A C-style string is represented by a series of bytes (`char`s).

- One-byte ASCII codes for each character.
- ASCII = American Standard Code for Information Interchange

32	space	48	0	64	@	80	P	96	`	112	p
33	!	49	1	65	A	81	Q	97	a	113	q
34	”	50	2	66	B	82	R	98	b	114	r
35	#	51	3	67	C	83	S	99	c	115	s
36	$	52	4	68	D	84	T	100	d	116	t
37	%	53	5	69	E	85	U	101	e	117	u
38	&	54	6	70	F	86	V	102	f	118	v
39	’	55	7	71	G	87	W	103	g	119	w
40	(	56	8	72	H	88	X	104	h	120	x
41	)	57	9	73	I	89	Y	105	I	121	y
42	*	58	:	74	J	90	Z	106	j	122	z
43	+	59	;	75	K	91	[	107	k	123	{
44	,	60	<	76	L	92	\	108	l	124	|
45	-	61	=	77	M	93	]	109	m	125	}
46	.	62	>	78	N	94	^	110	n	126	~
47	/	63	?	79	O	95	_	111	o	127	del

word |wərd|, n.

Natural unit of data used by processor.

Fixed size (e.g. 32 bits, 64 bits)

Defined by ISA: Instruction Set Architecture

machine instruction operands

word size = register size = address size

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	1	0	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Java/C int = 4 bytes: 11,501,584

MSB: most significant bit

LSB: least significant bit

fixed-size data representations

Java Data Type	C Data Type	32-bit	64-bit
		(size in bytes)	
boolean		1	1
byte	char	1	1
char		2	2
short	short int	2	2
int	int	4	4
float	float	4	4
	long int	4	8
double	double	8	8
long	long long	8	8
	long double	8	16

Depends on word size!

bitwise operators

ex

Bitwise operators on fixed-width **bit vectors**.

AND & OR | XOR ^ NOT ~

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001
```

Laws of Boolean algebra apply bitwise.

e.g., DeMorgan's Law: ~(A | B) = ~A & ~B

```
  01010101
^ 01010101
```

Aside: sets as bit vectors

Representation: *n*-bit vector gives subset of {0, …, *n*–1}.

$a_i = 1 \equiv i \in A$

```
01101001    { 0, 3, 5, 6 }
76543210

01010101    { 0, 2, 4, 6 }
76543210
```

Bitwise Operations

&	`01000001`	{ 0, 6 }
|	`01111101`	{ 0, 2, 3, 4, 5, 6 }
^	`00111100`	{ 2, 3, 4, 5 }
~	`10101010`	{ 1, 3, 5, 7 }

Set Operations?

bitwise operators in C

`& | ^ ~` apply to any *integral* data type
`long, int, short, char, unsigned`

Examples (**`char`**)

```
~0x41 =

~0x00 =

0x69 & 0x55 =

0x69 | 0x55 =
```

Many bit-twiddling puzzles in upcoming assignment

logical operations in C

`&& || !` apply to any "integral" data type
`long, int, short, char, unsigned`

0 is **false** **nonzero** is **true** **result** always **0 or 1**

early termination a.k.a. **short-circuit evaluation**

Examples (**`char`**)

```
 !0x41 =
 !0x00 =
!!0x41 =

0x69 && 0x55 =
0x69 || 0x55 =
```

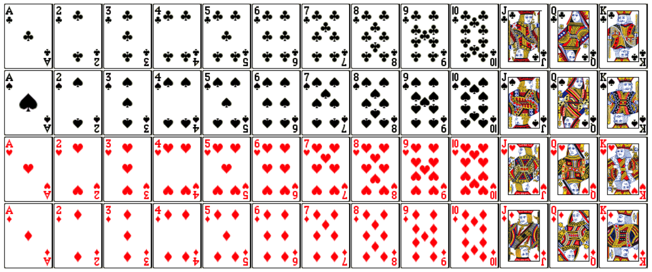

Encode playing cards.

52 cards in 4 suits

- How do we encode suits, face cards?

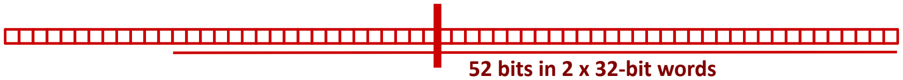
What operations should be easy to implement?

- Get and compare rank
- Get and compare suit



Two possible representations

52 cards – 52 bits with bit corresponding to card set to 1



"One-hot" encoding

Hard to compare values and suits independently

Not space efficient

4 bits for suit, 13 bits for card value – 17 bits with two set to

Pair of one-hot encoded values

Easier to compare suits and values independently

Smaller, but still not space efficient

Two better representations

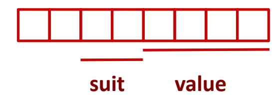
Binary encoding of all 52 cards – only 6 bits needed

Number cards uniquely from 0

Smaller than one-hot encodings.

Hard to compare value and suit

Binary encoding of suit (2 bits) and value (4 bits) separately



Number each suit uniquely

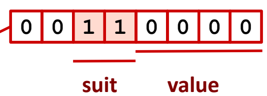
Number each value uniquely

Still small

Easy suit, value comparisons

Compare Card Suits

mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0

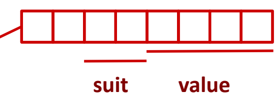



```
#define SUIT_MASK 0x30

int sameSuit(char card1, char card2) {
  return !((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK));

  //same as (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}


char hand[5];        // represents a 5-card hand
char card1, card2;   // two cards to compare
...
if ( sameSuit(hand[0], hand[1]) ) { ... }
```


Compare Card Values

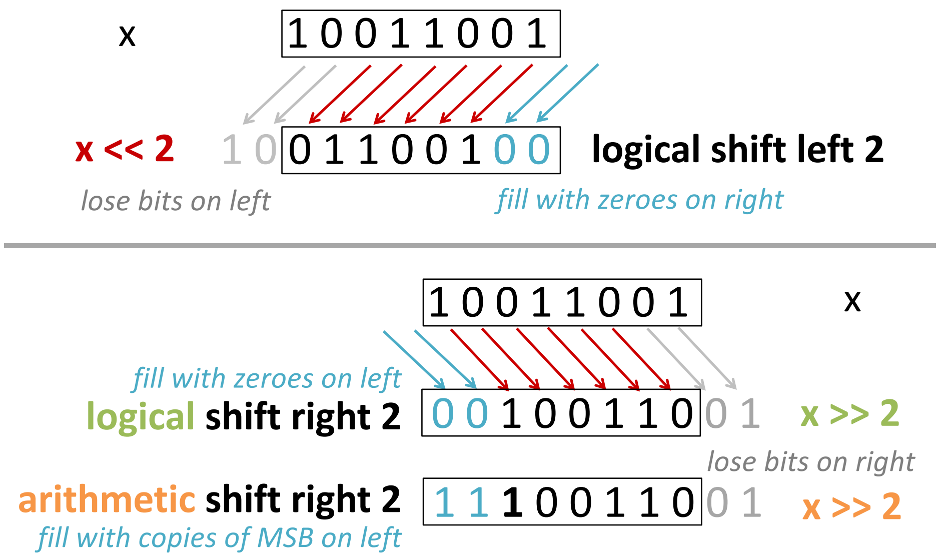
ex

mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0




```
#define VALUE_MASK

int greaterValue(char card1, char card2) {



}

char hand[5];        // represents a 5-card hand
char card1, card2;   // two cards to compare
...
if ( greaterValue(hand[0], hand[1]) ) { ... }
```

Bit shifting



Shift gotchas

!!!

Logical or arithmetic shift right: how do we tell?

C: compiler chooses

Usually based on type: rain check!

Java: >> is arithmetic, >>> is logical

Shift an *n*-bit type by at least 0 and no more than n-1.

C: other shift distances are undefined.

anything could happen

Java: shift distance is used modulo number of bits in shifted type

Given int x: x << 34 == x << 2

Shift and mask: extract a bit field

ex

Write a C function that
extracts the *2nd most significant byte*
from its 32-bit integer argument.

Example behavior:

argument: 0b 01100001 01100010 01100011 01100100

expected result: 0b 00000000 00000000 00000000 01100010

All other bits are zero. Desired bits in least significant byte.

```
int get2ndMSB(int x) {
```