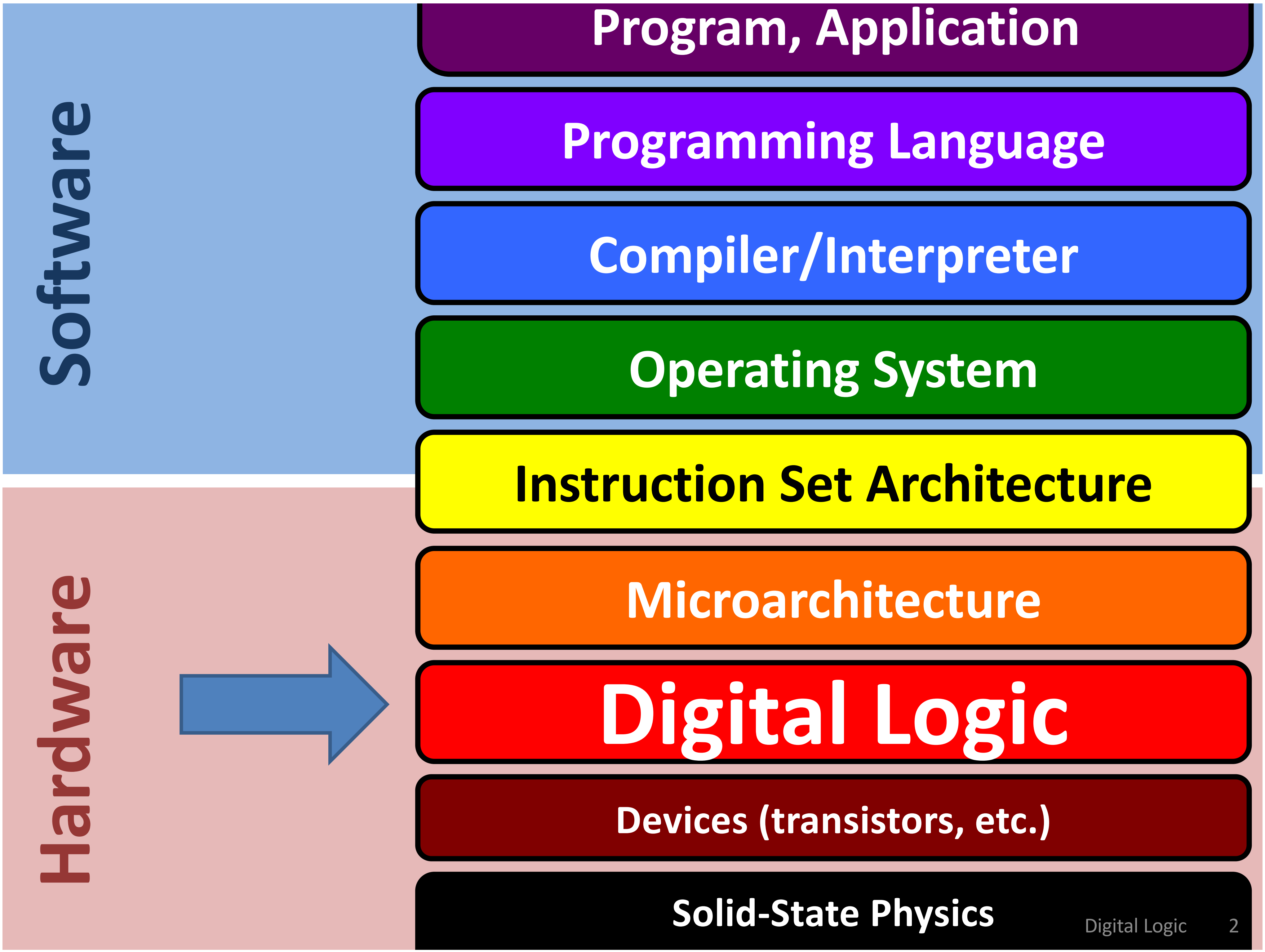
Digital Logic

*Gate*way to computer science

transistors, gates, circuits, Boolean algebra


Software
Hardware
Program, Application
Programming Language
Compiler/Interpreter
Operating System
Instruction Set Architecture
Microarchitecture
Digital Logic
Devices (transistors, etc.)
Solid-State Physics

Digital data/computation = Boolean

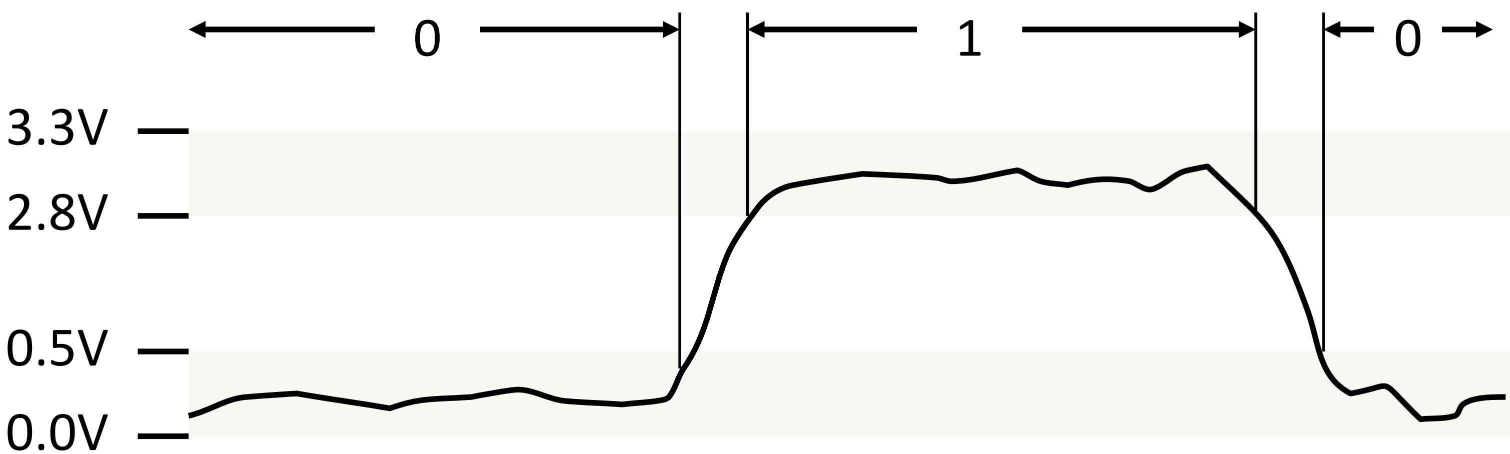
Abstraction!

Boolean value (***bit***): **0 or 1**

Boolean functions (AND, OR, NOT, …)

Electronically:

bit = high voltage vs. low voltage

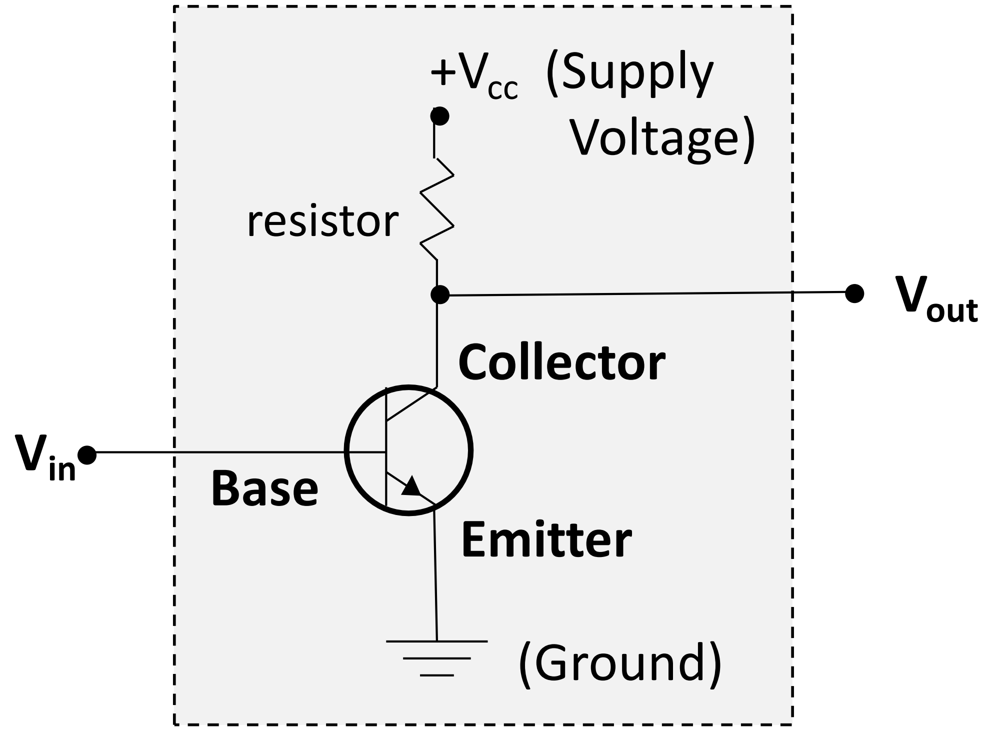


Boolean functions = logic gates, built from transistors

Transistors (more in lab)

If *Base* voltage is high:
Current may flow freely from *Collector* to *Emitter*.

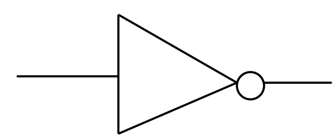
If *Base* voltage is low:
Current may not flow from *Collector* to *Emitter*.



Truth table

V_{in}	V_{out}		in	out		in	out
low	high	=	0	1	=	F	T
high	low		1	0		T	F

NOT gate



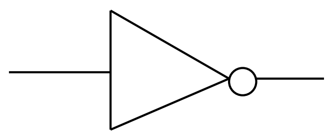
Digital Logic Gates

Abstraction!

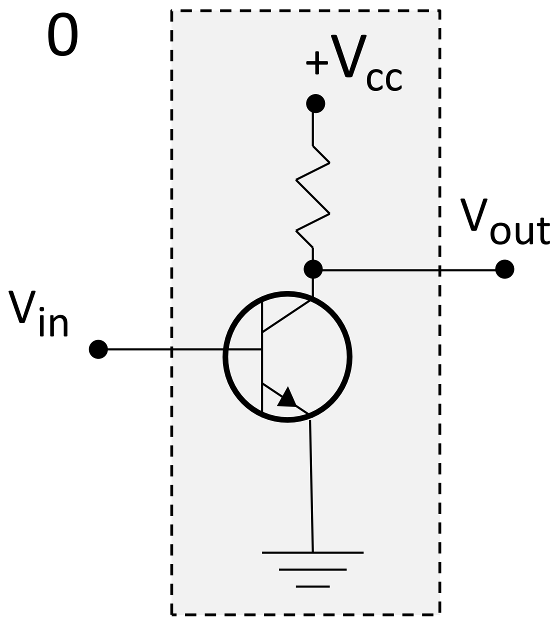
ex

Tiny electronic devices that compute basic Boolean functions.

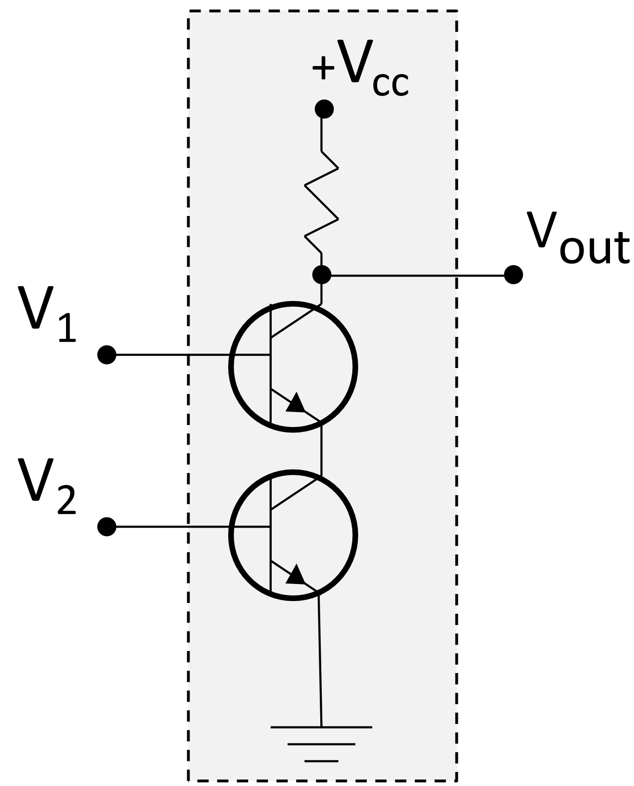
NOT



V_{in}	V_{out}
0	1
1	0

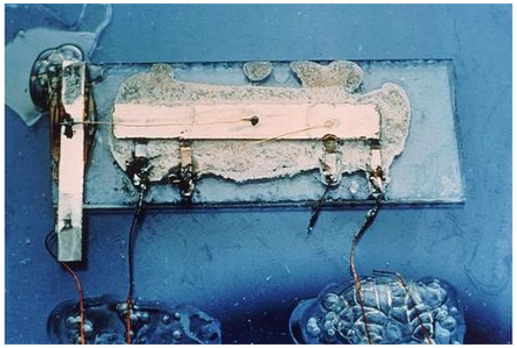


		V_2	
		0	1
V_1	0		
	1		

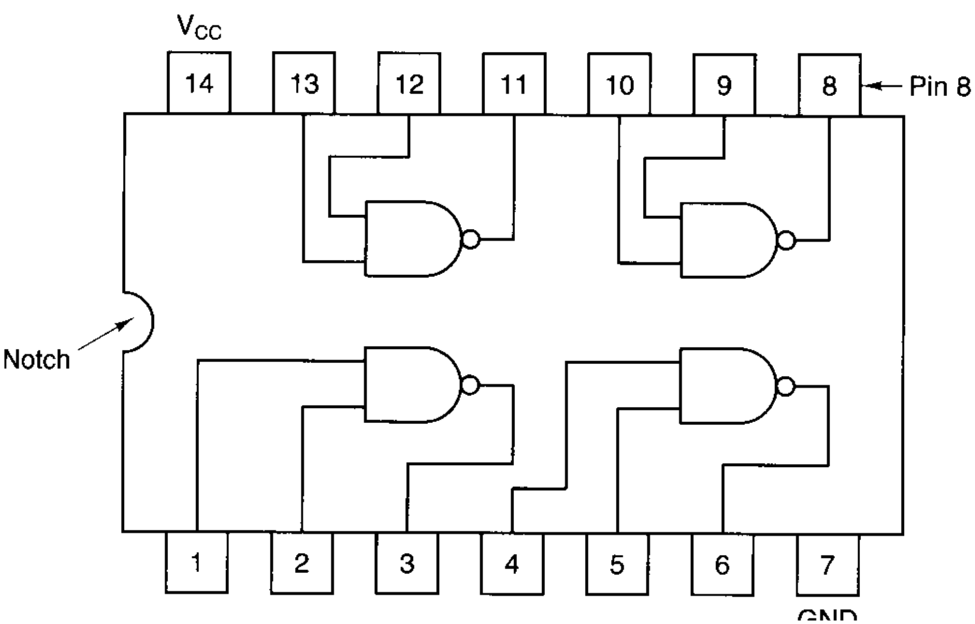


Integrated Circuits (1950s -)

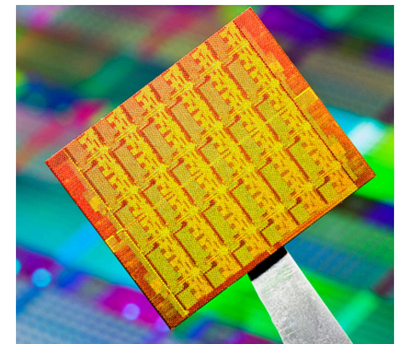
Early (first?) **transistor**



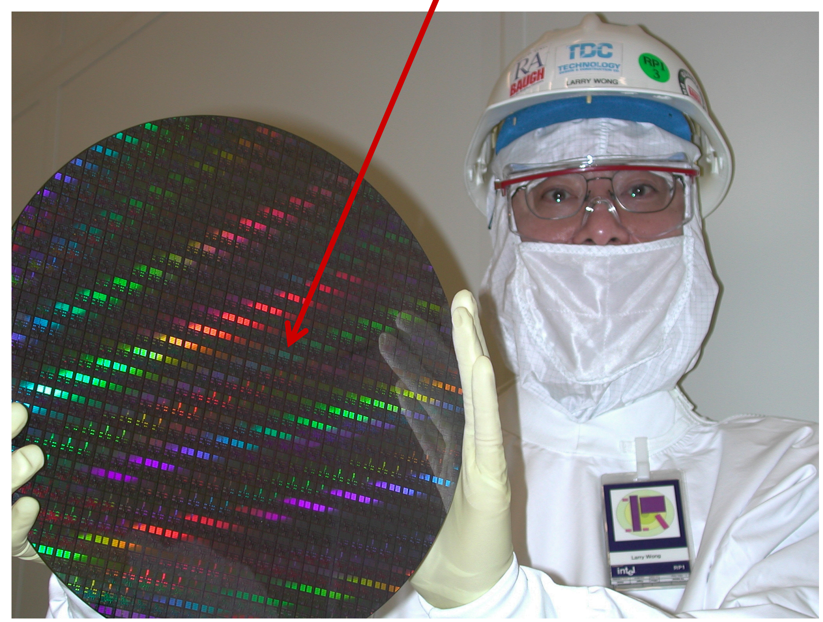
Small integrated circuit



Chip

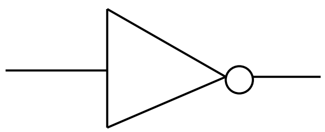


Wafer

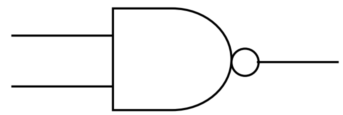


Five basic gates: define with truth tables

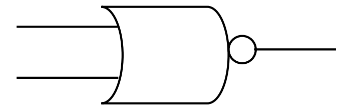
ex



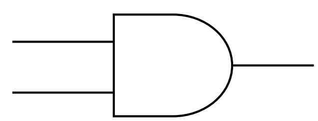
NOT	
0	1
1	0



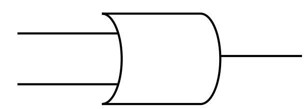
NAND	0	1
0	1	1
1	1	0



NOR	0	1
0		
1		



AND	0	1
0		
1		



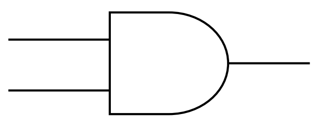
OR	0	1
0		
1		

Boolean Algebra

for combinational logic

inputs = *variables*
wires = *expressions*
gates = *operators/functions*
circuits = *functions*

A
B

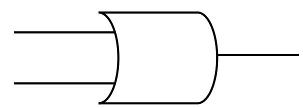


A B
(A · B)

AND = Boolean product

·	0	1
0	0	0
1	0	1

A
B

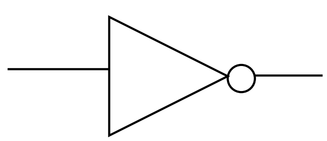


A + B

OR = Boolean sum

+	0	1
0	0	1
1	1	1

A



$\overline{A}$

NOT = inverse or complement

0	1
1	0

A A

wire = identity

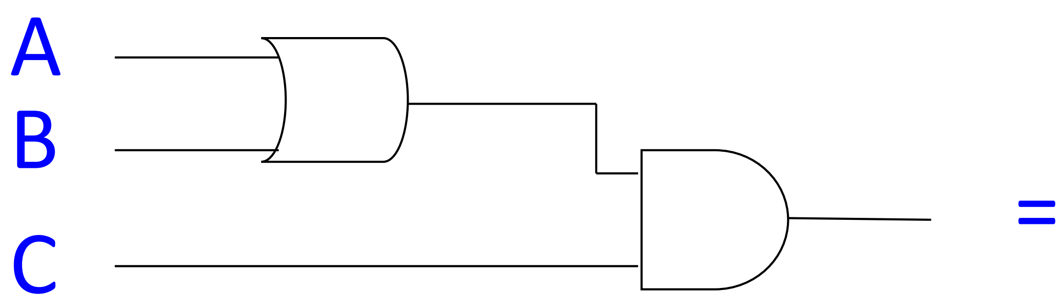
0	0
1	1

Circuits

ex

Connect inputs and outputs of gates with wires.

Crossed wires touch *only if* there is a dot.



What is the output if A=1, B=0, C=1?

What is the truth table of this circuit?

What is an equivalent Boolean expression?

Translation

ex

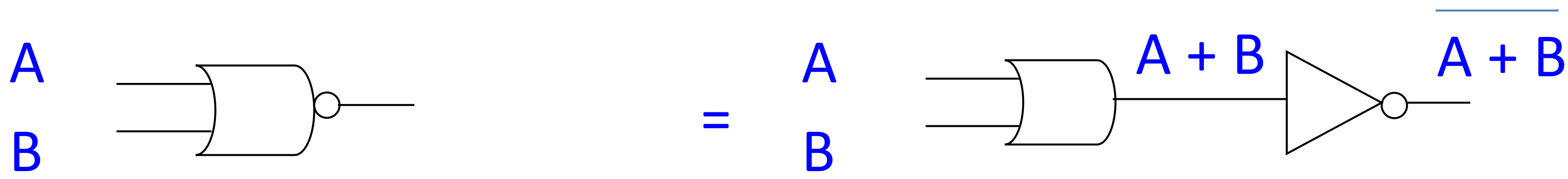
Connect gates to implement these functions. Check with truth tables.

Use a direct translation -- it is straightforward and bidirectional.

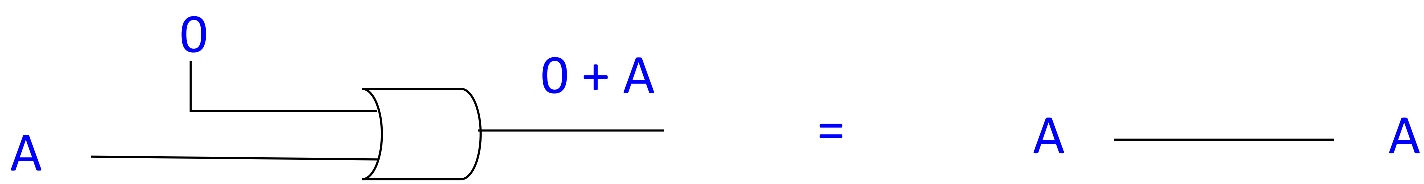
$$F = (A\overline{B} + C)D$$

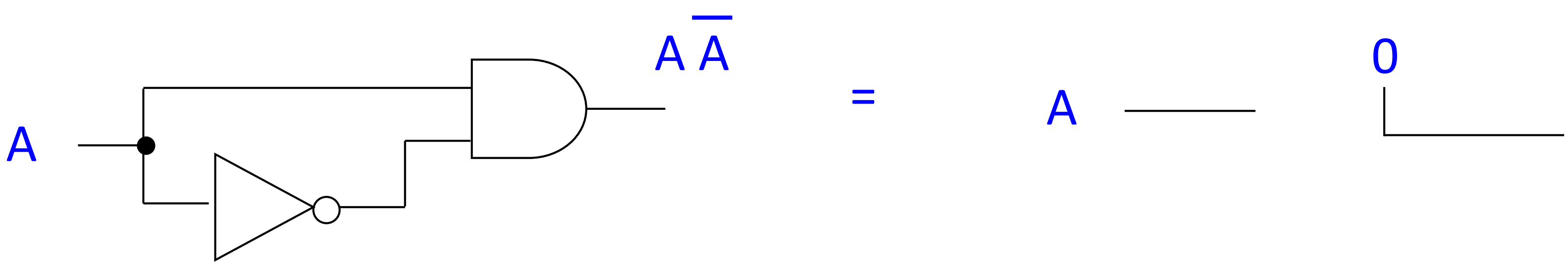
$$Z = \overline{W} + (X + \overline{WY})$$

Note on notation: bubble = inverse/complement

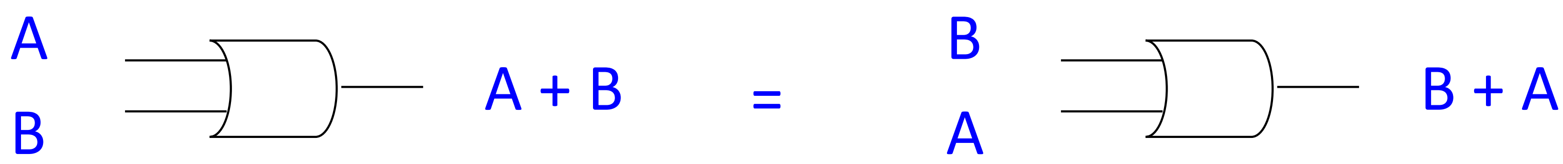


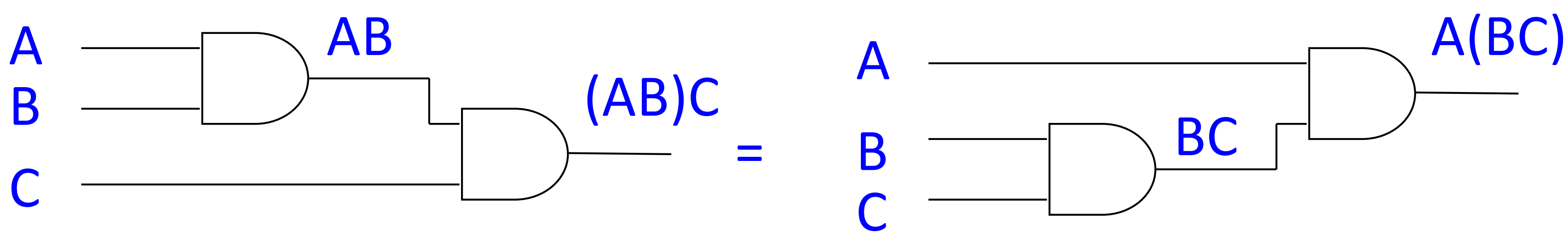
Identity law, inverse law



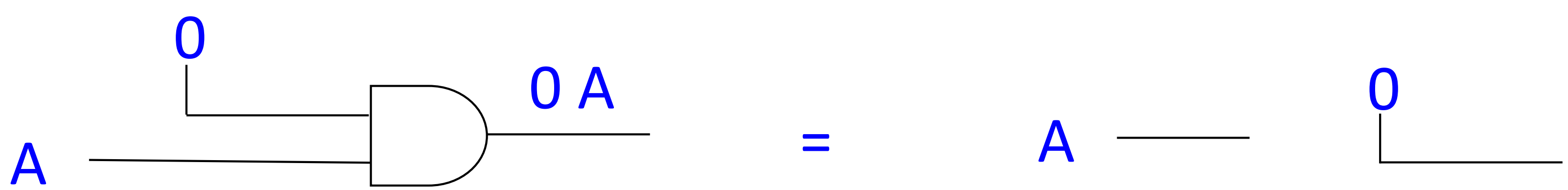


Commutativity, Associativity

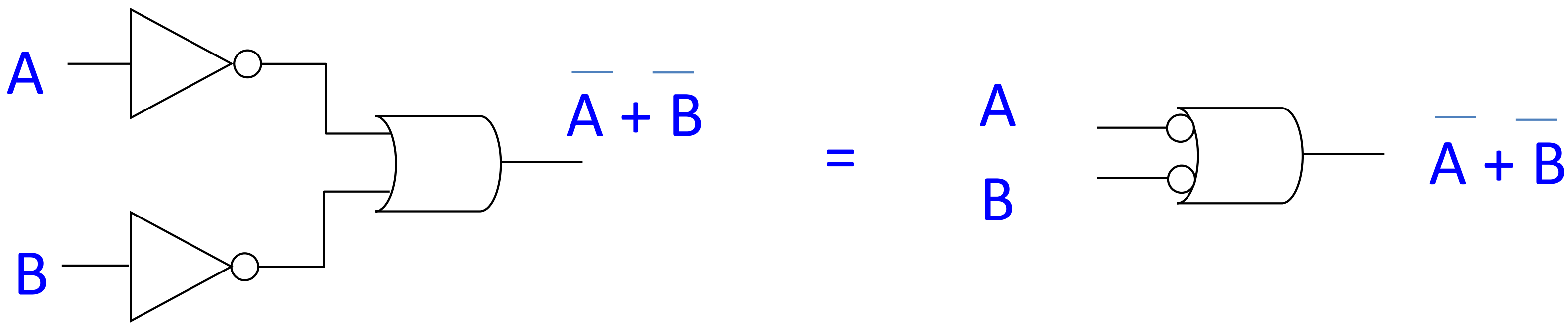




Idempotent law, Null/Zero law



Note on notation: bubble = inverse/complement



DeMorgan's Law

(**double bubble**, toil and trouble, in Randy's words...)

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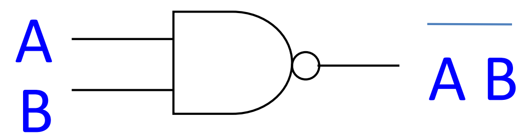
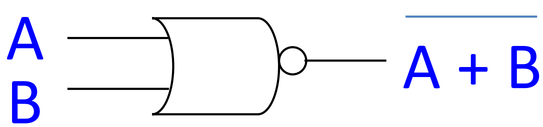


$\overline{AB}$	0	1
0	1	1
1	1	0

$\overline{A+B}$	0	1
0	1	0
1	0	0



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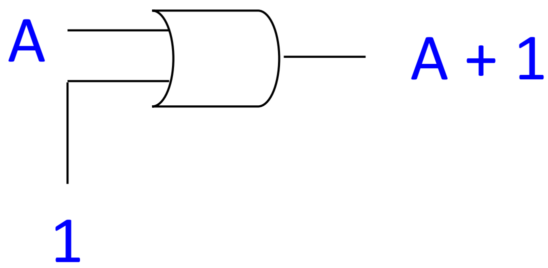
One law, Absorption law

ex

Write truth tables. Do they correspond to simpler circuits?



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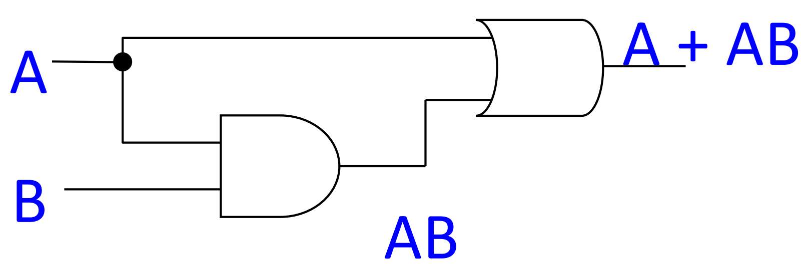

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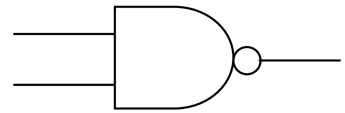
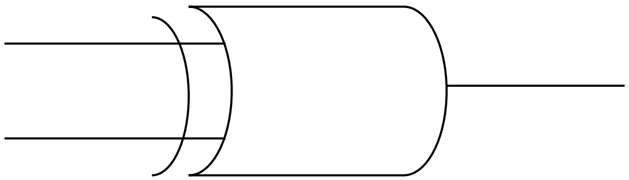
NAND is *universal*.



All Boolean functions can be implemented using only NANDs.
Build NOT, AND, OR, NOR, using only NAND gates.

XOR: Exclusive OR

ex



Output = 1 if exactly one input = 1.

Truth table:

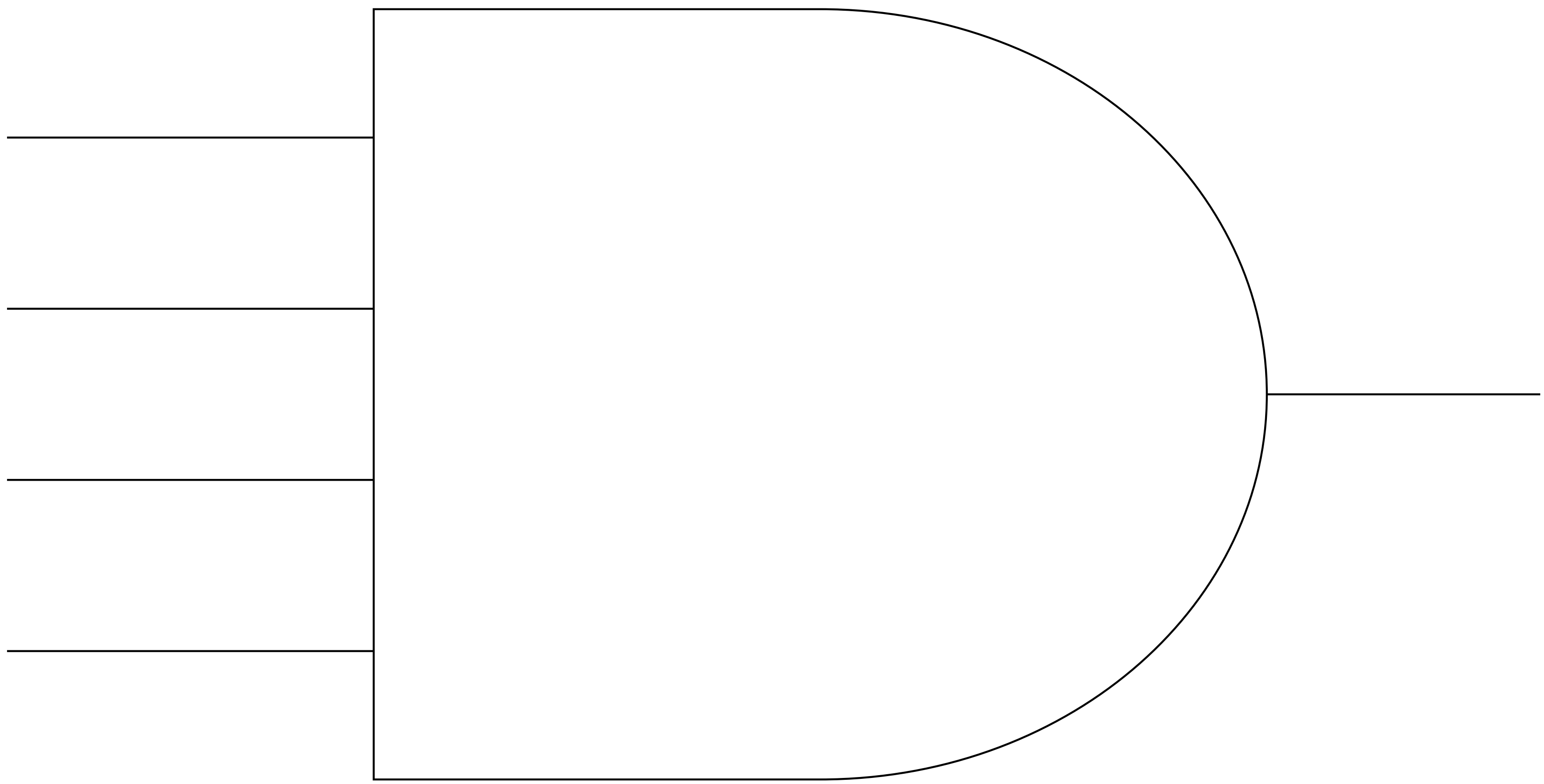
Build from earlier gates:

Often used as a one-bit comparator.

Larger gates

ex

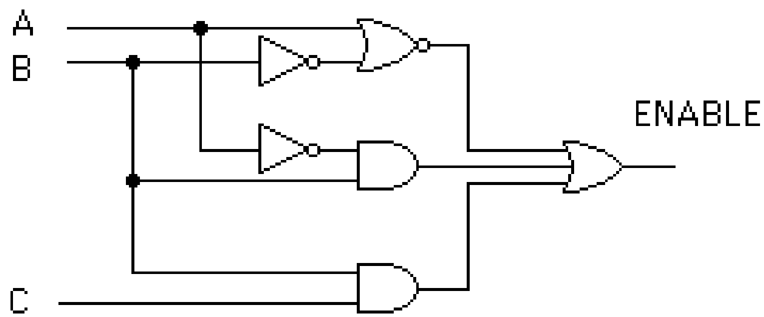
Build a 4-input AND gate using any number of 2-input gates.



ex

Circuit simplification

Is there a simpler circuit that performs the same function?



Start with an equivalent Boolean expression, then simplify with algebra.

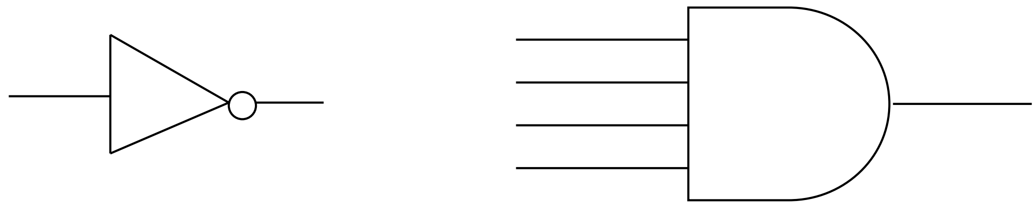
F(A, B, C) =

Check the answer with a truth table.

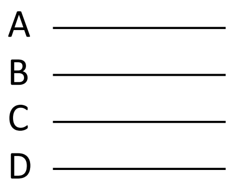
Circuit derivation: ***code detectors***

ex

AND gate + NOT gates = code detector, recognizes exactly one input code.



Design a 4-input code detector to output 1 if ABCD = 1001, and 0 otherwise.



Design a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others. (accept = 1, reject = 0)

Circuit derivation: ***sum-of-products*** form

ex

logical sum (OR)

of products (AND)

of inputs or their complements (NOT)

Draw the truth table and design a sum-of-products circuit for a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others.

How are the truth table and the sum-of-products circuit related?

Voting machines

ex

A majority circuit outputs 1 if and only if a majority of its inputs equal 1.

Design a majority circuit for three inputs. Use a sum of products.

A	B	C	Majority
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Triply redundant computers in spacecraft

- Space program also hastened Integrated Circuits.

Computers

- Manual calculations
- powered all early US **space** missions.
- Facilitated transition to digital computers.

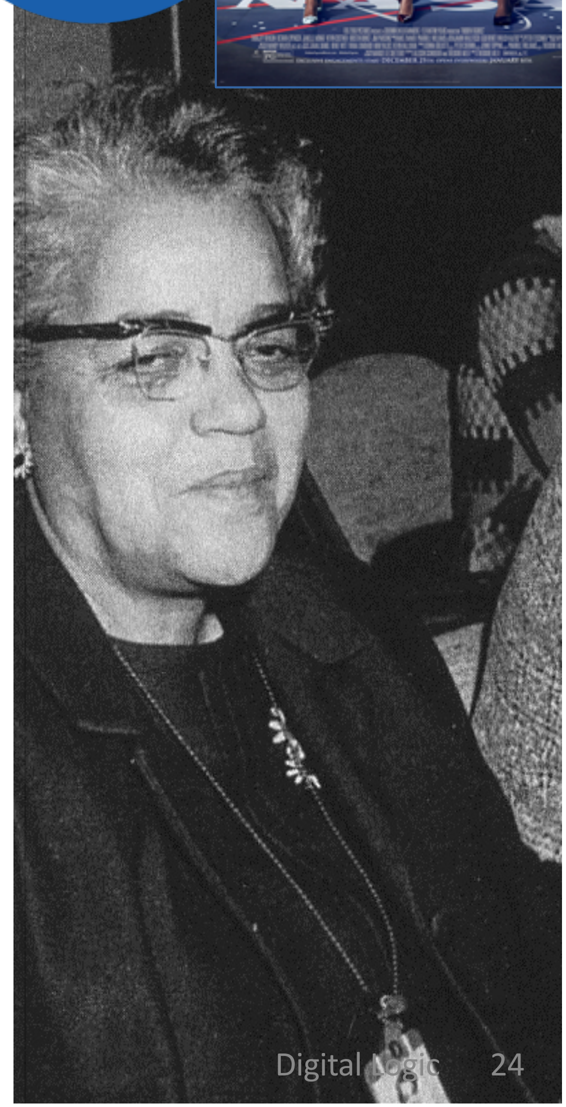
Mary Jackson

Katherine Johnson

- Supported Mercury, Apollo, Space Shuttle, ...

Dorothy Vaughn

- First black supervisor within NACA
- Early self-taught FORTRAN programmer for NASA move to digital computers.



Early pioneers in reliable computing

Katherine Johnson

- Calculated first US human space flight trajectories
- Mercury, Apollo 11, Space Shuttle, …
- Reputation for accuracy in manual calculations, verified early code
- Called to verify results of code for launch calculations for first US human in orbit
- Backup calculations helped save Apollo 13
- Presidential Medal of Freedom 2015

Margaret Hamilton

- Led software team for Apollo 11 Guidance Computer, averted mission abort on first moon landing.
- Coined “software engineering”, developed techniques for correctness and reliability.
- Presidential Medal of Freedom 2016

Apollo 11 code print-out