Representing Data with Bits

bits, bytes, numbers, and notation

positional number representation

base = 10 (*decimal*)

2	4	0
100	10	1
10^2	10^1	10^0
2	1	0

$$= 2 \times 10^2 + 4 \times 10^1 + 0 \times 10^0$$

weight

position

Base determines:

Maximum digit (base – 1). Minimum digit is 0.

Weight of each position.

Each position holds a digit.

Represented value = sum of all position values

position value = digit value × base^{position}

binary = base 2

Binary digits are called ***bits***: 0, 1

base = 2 (*binary*)

1	0	1	1
8	4	2	1
2^3	2^2	2^1	2^0
3	2	1	0

$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

weight

position

When ambiguous, subscript with base:

101_{10} Dalmatians (movie)

101_2-Second Rule (folk wisdom for food safety)

irony

Powers of 2:
memorize up to ≥ 2^{10} (in base ten)

ex

Power: $2^?$	Decimal value
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

conversion from binary to decimal

ex

$$1\,0\,1\,1\,0\,1_{2} = ?_{10}$$

Interpret the positional representation according to the base:
sum the place weights where 1 appears (in either direction).

conversion from decimal to binary

ex

$$19_{10} = ?_2$$

Divide-by-2 Approach (Right to Left)

64	32	16	8	4	2	1

Quotient *Remainder?*

Powers-of-2 Approach (Left to Right)

64	32	16	8	4	2	1

Value *Power that fits?*

binary arithmetic

ex

$110_2 + 1011_2 = ?_2$

$1101_2 - 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

conversion and arithmetic

ex

$19_{10} = ?_2$

$1001_2 = ?_{10}$

$240_{10} = ?_2$

$11010011_2 = ?_{10}$

$101_2 + 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

What do you call 4 bits?

byte = 8 bits

a.k.a. octet

Smallest unit of data

used by a typical modern computer

Binary 00000000_2 -- 11111111_2

Decimal 000_{10} -- 255_{10}

Hexadecimal 00_{16} -- FF_{16}

Byte = 2 hex digits!

Programmer's hex notation (C, etc.):

`0xB4` = $B4_{16}$

Octal (base 8) also useful.

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

`char`: representing characters

A C-style string is represented by a series of bytes (`char`s).

- One-byte ASCII codes for each character.
- ASCII = American Standard Code for Information Interchange

32	space	48	0	64	@	80	P	96	`	112	p
33	!	49	1	65	A	81	Q	97	a	113	q
34	”	50	2	66	B	82	R	98	b	114	r
35	#	51	3	67	C	83	S	99	c	115	s
36	$	52	4	68	D	84	T	100	d	116	t
37	%	53	5	69	E	85	U	101	e	117	u
38	&	54	6	70	F	86	V	102	f	118	v
39	’	55	7	71	G	87	W	103	g	119	w
40	(	56	8	72	H	88	X	104	h	120	x
41	)	57	9	73	I	89	Y	105	I	121	y
42	*	58	:	74	J	90	Z	106	j	122	z
43	+	59	;	75	K	91	[	107	k	123	{
44	,	60	<	76	L	92	\	108	l	124	|
45	-	61	=	77	M	93	]	109	m	125	}
46	.	62	>	78	N	94	^	110	n	126	~
47	/	63	?	79	O	95	_	111	o	127	del

word *|wərd|,* **n.**

Natural unit of data used by processor.

Fixed size (e.g. 32 bits, 64 bits)

Defined by ISA: Instruction Set Architecture

machine instruction operands

word size = register size = address size

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	1	0	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Java/C int = 4 bytes: 11,501,584

MSB: most significant bit

LSB: least significant bit

fixed-size data representations

Java Data Type	C Data Type	(size in bytes) [word = 32 bits]	[word = 64 bits]
boolean		1	1
byte	char	1	1
char		2	2
short	short int	2	2
int	int	4	4
float	float	4	4
	long int	4	8
double	double	8	8
long	long long	8	8
	long double	8	16

Depends on word size!

bitwise operators

bit = Boolean
0 = false
1 = true

ex

Bitwise operators on fixed-width **bit vectors**.

AND &   OR | XOR ^   NOT ~

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001
```

```
  01010101
^ 01010101
```

Laws of Boolean algebra apply bitwise.

e.g., DeMorgan's Law: ~(A | B) = ~A & ~B

bitwise **operators in C**

ex

`& | ^ ~` apply to any *integral* data type

`long`, **`int`**, **`short`**, **`char`**, **`unsigned`**

Examples (**`char`**)

```
~0x41 =

~0x00 =

0x69 & 0x55 =

0x69 | 0x55 =
```

Many bit-twiddling puzzles in upcoming assignment

Representation Example 1: Sets as Bit Vectors

ex

Representation: *n*-bit vector gives subset of {0, …, *n*–1}.

$a_i = 1 \equiv i \in A$

```
a = 0b01101001        A = { 0, 3, 5, 6 }
      76543210

b = 0b01010101        B = { 0, 2, 4, 6 }
      76543210
```

Bitwise Operations		**Set Operations**
`a & b = 0b01000001`	{0, 6}	Intersection
`a | b = 0b01111101`	{0, 2, 3, 4, 5, 6}	Union
`a ^ b = 0b00111100`	{2, 3, 4, 5}	Symmetric difference
`~ b = 0b10101010`	{1, 3, 5, 7}	Complement

logical operations in C

ex

&& || !   apply to any "integral" data type
`long, int, short, char, unsigned`

0 is **false**   **nonzero** is **true**   **result** always **0 or 1**

early termination a.k.a. **short-circuit evaluation**

Examples (**`char`**)

```
 !0x41 =
 !0x00 =
!!0x41 =

0x69 && 0x55 =
0x69 || 0x55 =
```

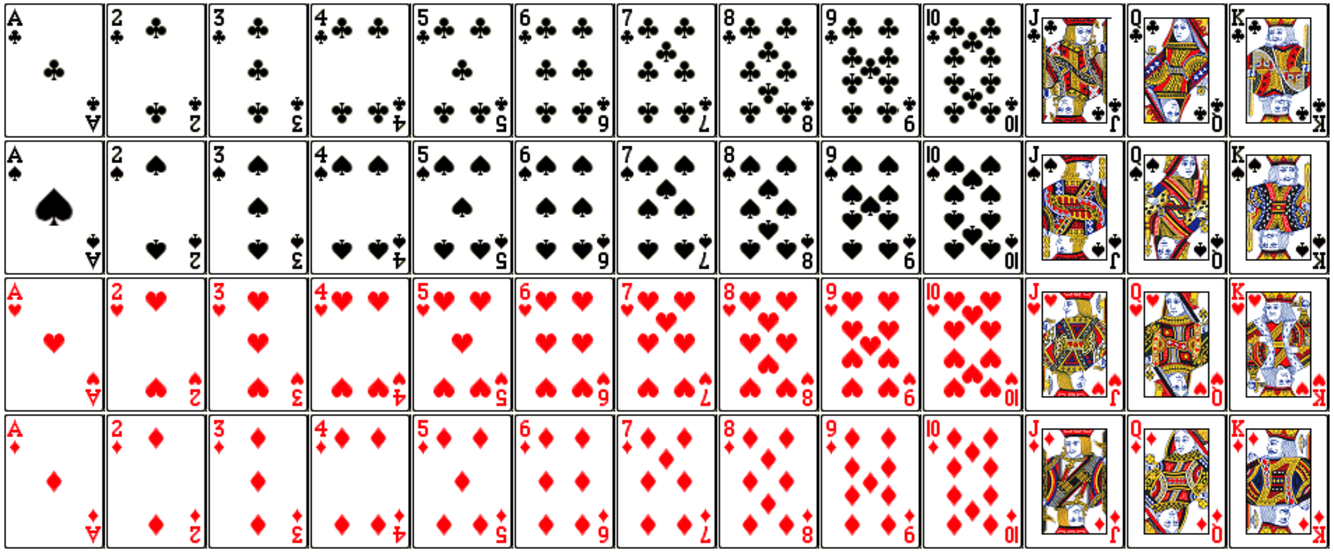
Representation Example 2: Playing Cards

52 cards in 4 suits

- How do we encode suits, face cards?

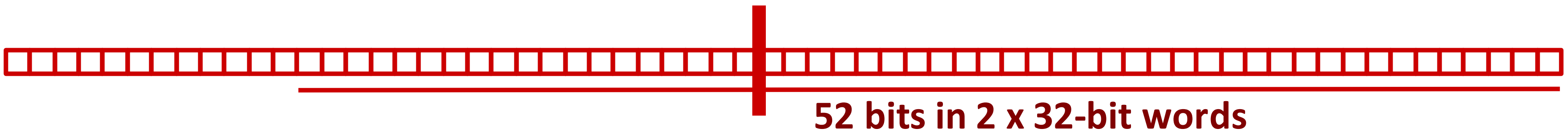
What operations should be easy to implement?

- Get and compare rank
- Get and compare suit



Two possible representations

52 cards – 52 bits with bit corresponding to card set to 1

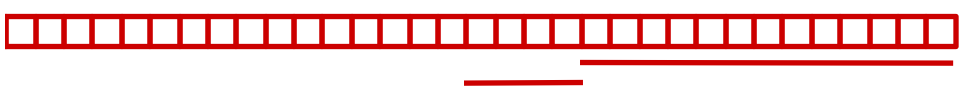


"One-hot" encoding

Hard to compare values and suits independently

Not space efficient

4 bits for suit, 13 bits for card value – 17 bits with two set to



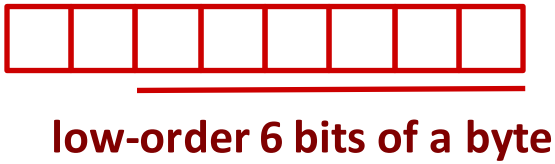
Pair of one-hot encoded values

Easier to compare suits and values independently

Smaller, but still not space efficient

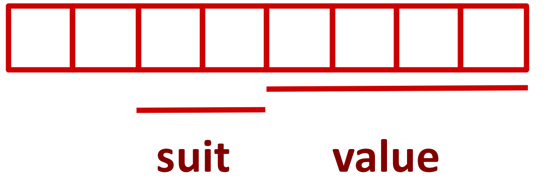
Two better representations

Binary encoding of all 52 cards – only 6 bits needed



- Number cards uniquely from 0
- Smaller than one-hot encodings.
- Hard to compare value and suit

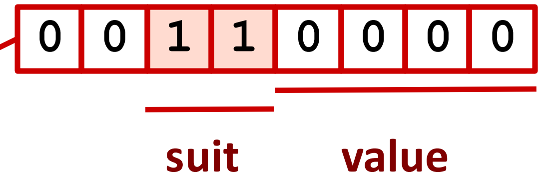
Binary encoding of suit (2 bits) and value (4 bits) separately



- Number each suit uniquely
- Number each value uniquely
- Still small
- Easy suit, value comparisons

Compare Card Suits

mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0



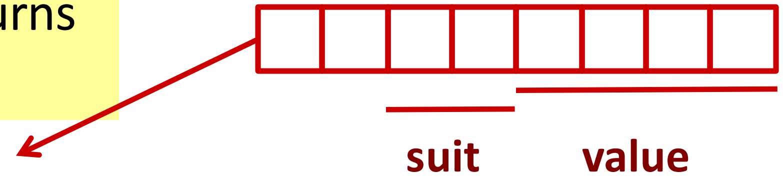

```
#define SUIT_MASK 0x30

int sameSuit(char card1, char card2) {
  return !((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK));

  //same as (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

```
char hand[5];          // represents a 5-card hand
...
if ( sameSuit(hand[0], hand[1]) ) { ... }
```

Compare Card Values

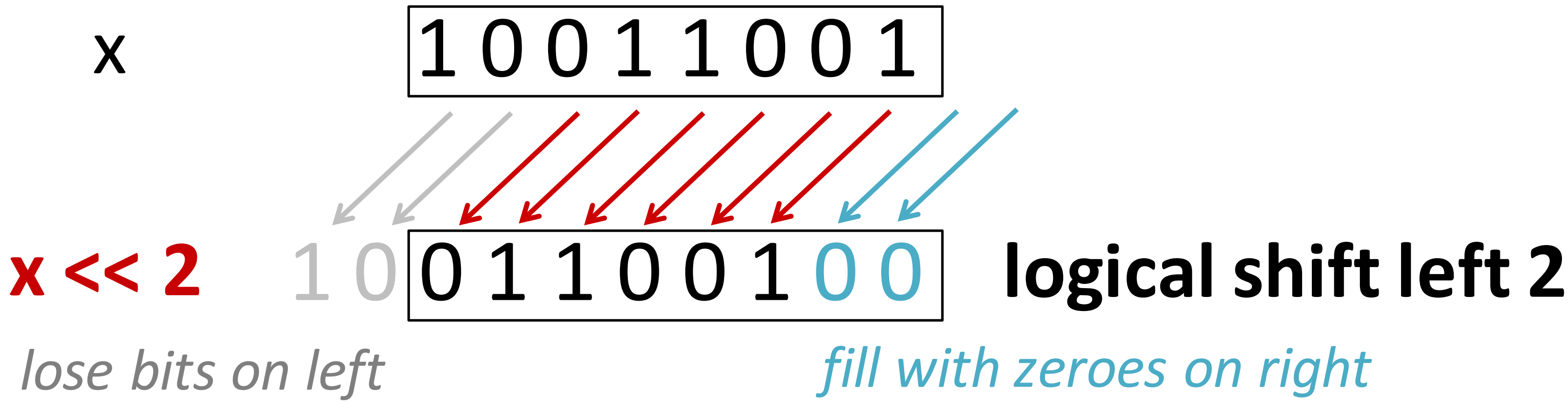
ex

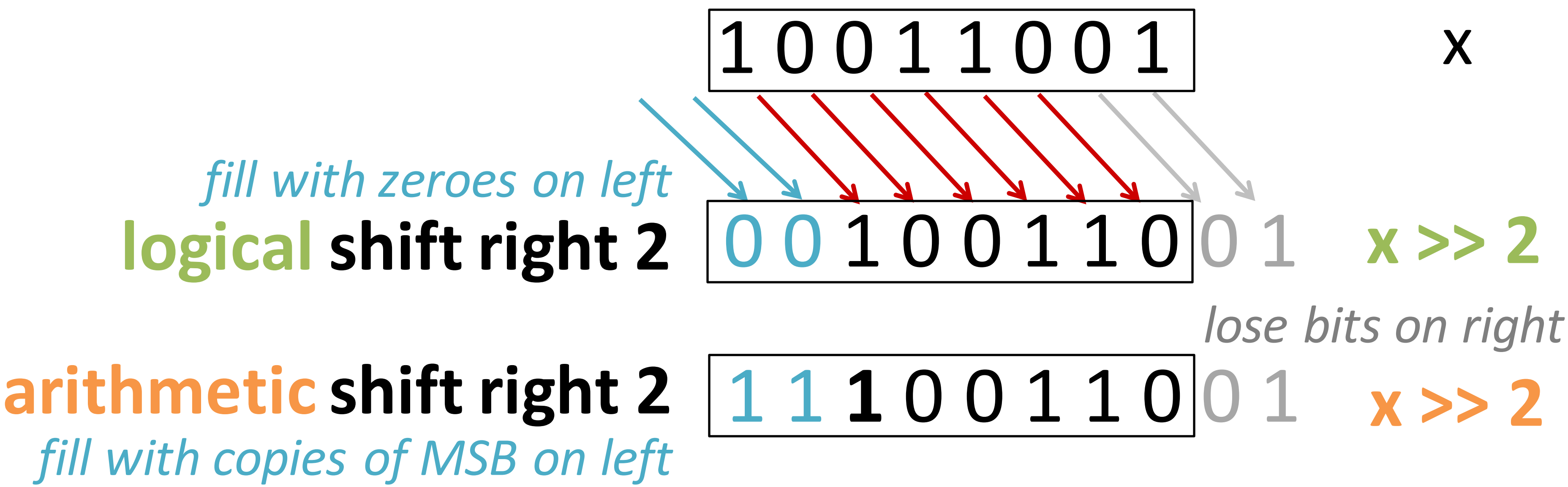
mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0




```
#define VALUE_MASK

int greaterValue(char card1, char card2) {






}


char hand[5];          // represents a 5-card hand
...
if ( greaterValue(hand[0], hand[1]) ) { ... }
```

Bit shifting





Shift gotchas

!!!

Logical or arithmetic shift right: how do we tell?

C: compiler chooses

- Usually based on type: rain check!

Java: >> is arithmetic, >>> is logical

Shift an *n*-bit type by at least 0 and no more than n-1.

C: other shift distances are undefined.

- *anything* could happen

Java: shift distance is used modulo number of bits in shifted type

- Given int x: x << 34 == x << 2

Shift and mask: extract a bit field

ex

Write a C function that
extracts the 2nd *most significant byte*
from its 32-bit integer argument.

Example behavior:

argument: 0b 01100001 01100010 01100011 01100100

expected result: 0b 00000000 00000000 00000000 01100010

All other bits are zero.

Desired bits in least significant byte.

```
int get2ndMSB(int x) {
```