



Digital Logic

Gateway to computer science

transistors, gates, circuits, Boolean algebra

Software

Program, Application

Programming Language

Compiler/Interpreter

Operating System

Instruction Set Architecture

Microarchitecture

Digital Logic

Devices (transistors, etc.)

Solid-State Physics

Hardware



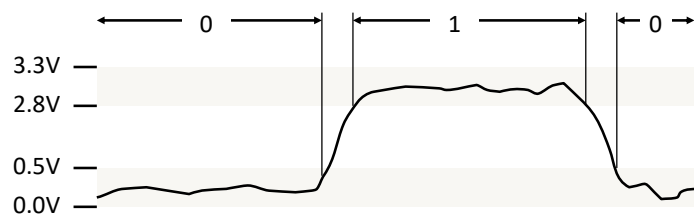
Digital data/computation = Boolean

Boolean value (*bit*): 0 or 1

Boolean functions (AND, OR, NOT, ...)

Electronically:

bit = high voltage vs. low voltage

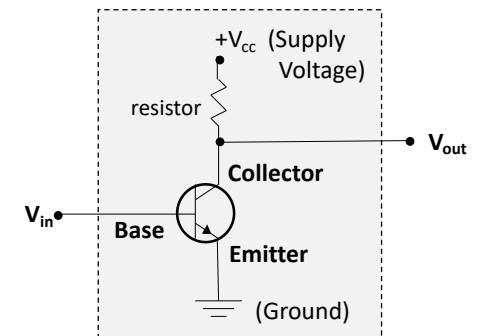


Boolean functions = logic gates, built from transistors

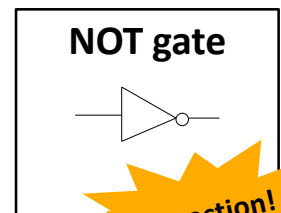
Transistors (more in lab)

If **Base voltage is high**:
Current may flow freely from *Collector* to *Emitter*.

If **Base voltage is low**:
Current may not flow from *Collector* to *Emitter*.



| Truth table | | | | | | | |
|-------------|-----------|---|----|-----|---|----|-----|
| V_{in} | V_{out} | = | in | out | = | in | out |
| low | high | = | 0 | 1 | = | F | T |
| high | low | | 1 | 0 | | T | F |



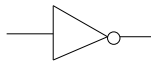
Digital Logic Gates



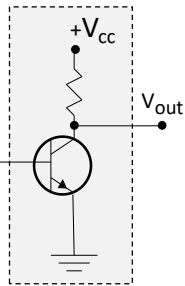
ex

Tiny electronic devices that compute basic Boolean functions.

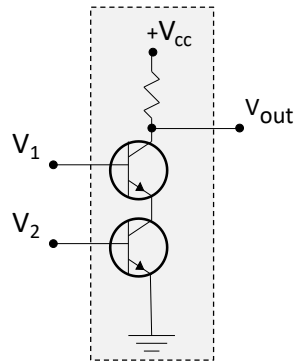
NOT



| V_{in} | V_{out} |
|----------|-----------|
| 0 | 1 |
| 1 | 0 |



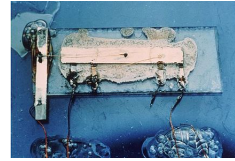
| V_1 | V_2 |
|-------|-------|
| 0 | 0 |
| 1 | 1 |



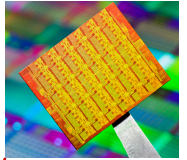
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Integrated Circuits (1950s -)

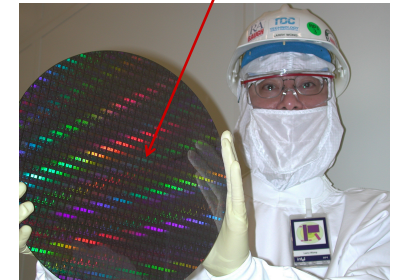
Early (first?) transistor



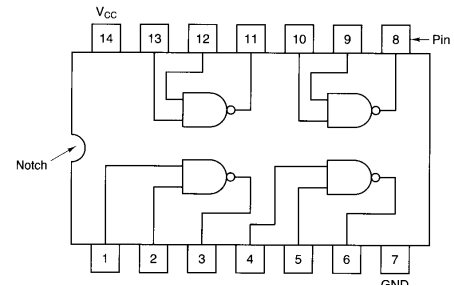
Chip



Wafer



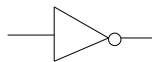
Small integrated circuit



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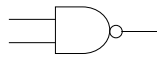
Five basic gates: define with truth tables

ex



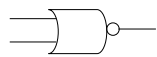
NOT

| | 1 |
|---|---|
| 0 | 1 |
| 1 | 0 |



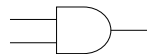
NAND

| | 0 | 1 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 0 |



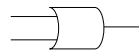
NOR

| | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 1 |



AND

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |



OR

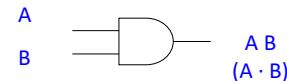
| | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

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Boolean Algebra

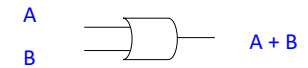
for combinational logic

- inputs* = *variables*
- wires* = *expressions*
- gates* = *operators/functions*
- circuits* = *functions*



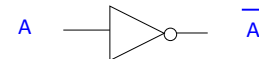
AND = Boolean product

| \cdot | 0 | 1 |
|---------|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |



OR = Boolean sum

| $+$ | 0 | 1 |
|-----|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |



NOT = inverse or complement

| | 1 |
|---|---|
| 0 | 1 |
| 1 | 0 |



wire = identity

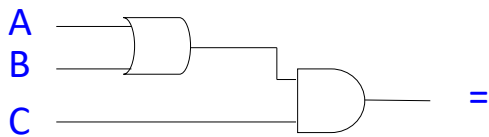
| | 0 |
|---|---|
| 0 | 0 |
| 1 | 1 |

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Circuits

ex

Connect inputs and outputs of gates with wires.
Crossed wires touch *only if* there is a dot.



What is the output if A=1, B=0, C=1?
What is the truth table of this circuit?
What is an equivalent Boolean expression?

Translation

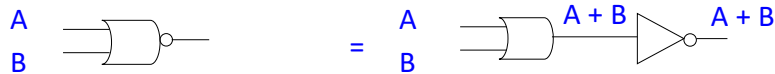
ex

Connect gates to implement these functions. Check with truth tables.
Use a direct translation -- it is straightforward and bidirectional.

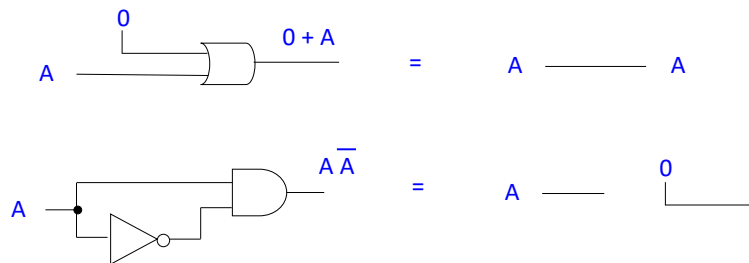
$$F = (A\bar{B} + C)D$$

$$Z = \bar{W} + (X + \bar{W}Y)$$

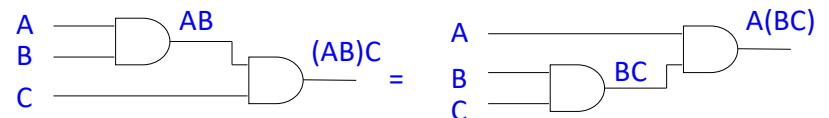
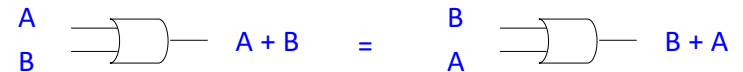
Note on notation: bubble = inverse/complement



Identity law, inverse law



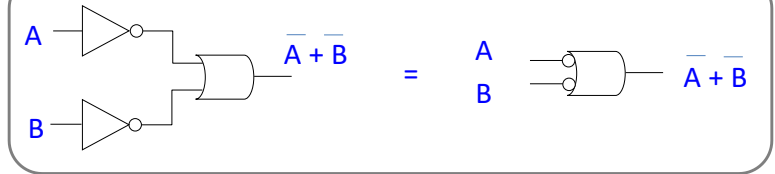
Commutativity, Associativity



Idempotent law, Null/Zero law

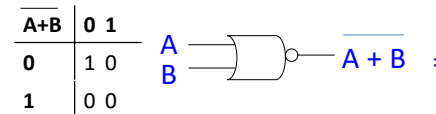
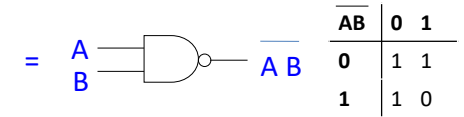


Note on notation: bubble = inverse/complement



DeMorgan's Law

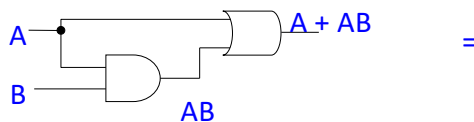
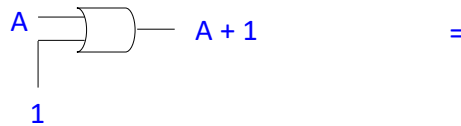
(double bubble, toil and trouble, in Randy's words...)



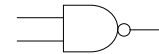
One law, Absorption law

Write truth tables. Do they correspond to simpler circuits?

ex



NAND is universal.

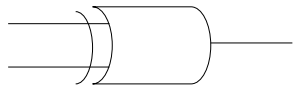


ex

All Boolean functions can be implemented using only NANDs.
Build NOT, AND, OR, NOR, using only NAND gates.

XOR: Exclusive OR

ex



Output = 1 if exactly one input = 1.

Truth table:

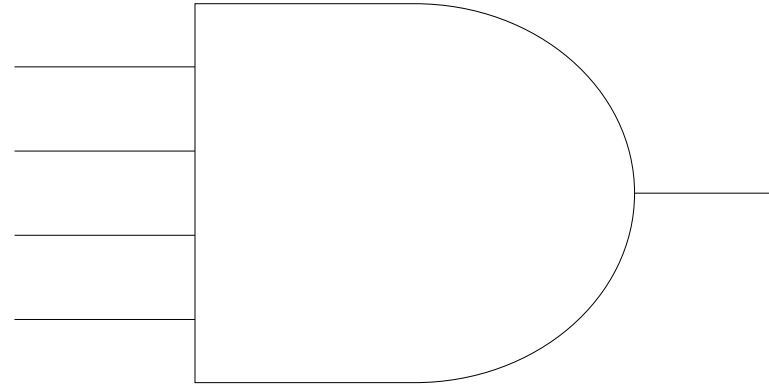
Build from earlier gates:

Often used as a one-bit comparator.

Larger gates

ex

Build a 4-input AND gate using any number of 2-input gates.

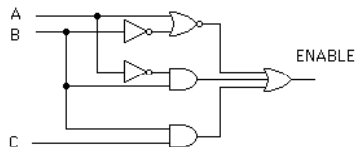


Circuit simplification

Why simplify?

ex

Is there a simpler circuit that performs the same function?



Start with an equivalent Boolean expression, then simplify with algebra.

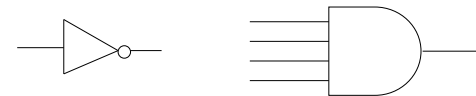
$F(A, B, C) =$

Check the answer with a truth table.

Circuit derivation: *code detectors*

ex

AND gate + NOT gates = code detector, recognizes exactly one input code.



Design a 4-input code detector to output 1 if ABCD = 1001, and 0 otherwise.

A _____
B _____
C _____
D _____

Design a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others. (accept = 1, reject = 0)

Circuit derivation: *sum-of-products* form



logical sum (OR)
of products (AND)
of inputs or their complements (NOT)

Draw the truth table and design a sum-of-products circuit for a 4-input code detector to accept two codes (ABCD=1001, ABCD=1111) and reject all others. How are the truth table and the sum-of-products circuit related?

Voting machines



A majority circuit outputs 1 if and only if a majority of its inputs equal 1.
Design a majority circuit for three inputs. Use a sum of products.

| A | B | C | Majority |
|---|---|---|----------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Triply redundant computers in spacecraft

- Space program also hastened Integrated Circuits.



Mary Jackson



Katherine Johnson

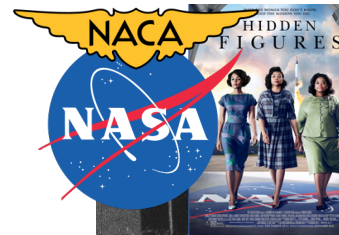
- Supported Mercury, Apollo, Space Shuttle, ...

Dorothy Vaughn

- First black supervisor within NACA
- Early self-taught FORTRAN programmer for NASA move to digital computers.

Computers

- Manual calculations
- powered all early US space missions.
- Facilitated transition to digital computers.



Early pioneers in reliable computing



Katherine Johnson

- Calculated first US human space flight trajectories
- Mercury, Apollo 11, Space Shuttle, ...
- Reputation for accuracy in manual calculations, verified early code
- Called to verify results of code for launch calculations for first US human in orbit
- Backup calculations helped save Apollo 13
- Presidential Medal of Freedom 2015

Margaret Hamilton

- Led software team for Apollo 11 Guidance Computer, averted mission abort on first moon landing.
- Coined "software engineering", developed techniques for correctness and reliability.
- Presidential Medal of Freedom 2016

Apollo 11 code print-out

