CS 240
Foundations of Computer Systems

WELLESLEY

Representing Data with Bits

bits, bytes, numbers, and notation

positional number representation

base = 10 (*decimal*)

2	4	0	$= 2 \times 10^2 + 4 \times 10^1 + 0 \times 10^0$
100	10	1	*weight*
10^2	10^1	10^0	*weight*
2	1	0	*position*

Base determines:

Maximum digit (base – 1). Minimum digit is 0.

Weight of each position.

Each position holds a digit.

Represented value = sum of all position values

position value = digit value × base^{position}

binary = base 2

Binary digits are called ***bits***: 0, 1

base = 2 (*binary*)

1	0	1	1	$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
8	4	2	1	*weight*
2^3	2^2	2^1	2^0	*weight*
3	2	1	0	*position*

When ambiguous, subscript with base:

101_{10} Dalmatians (movie)

101_2-Second Rule (folk wisdom for food safety)

irony

Powers of 2: memorize up to ≥ 2^{12} (in base ten)

Power: $2^?$	Decimal value
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256

Power: $2^?$	Decimal value
9	512
10	1024
11	2048
12	4096
13	8192
14	16384
15	32768
16	65536

Shifting binary numbers

$11011_2 = 27_{10}$

- What is 110110_2?
- What is 110111_2?
- What is 1101_2?

Converting binary to decimal

$101011_2 = ?_{10}$

Start with $output_{10} = 0$

Right to left(traditional algorithm)

- Start with smallest power 2 = 2^0 = 1
- If corresponding bit is 1, add power of 2 to $output_{10}$
- Repeat until power of two for leftmost 1 in $input_2$ is found

Right to left (better algorithm)

- Start with leftmost 1 bit in $input_2$ and $output_{10} = 1$
- For every 0, double $output_{10}$
- For every 1, double $output_{10}$ and add 1.

Converting binary to decimal

$110101_2 => ??_{10}$

$10110111_2 => ??_{10}$

Converting decimal to binary

$19_{10} = ?_2$

Start with $output_2$ = the empty string of binary digits

Left to right (traditional algorithm)

- Find the largest **power of** $\boldsymbol{2}_{10}$ that is ≤ $input_{10}$.
- Subtract it from $input_{10}$.
- Add it to $output_2$.
- Repeat with until $input_{10}$ is *0*.

Right to left (better algorithm)

- **Divide** $input_{10}$ by 2_{10}.
- Prepend the remainder as a bit on the left end of $output_2$.
- Repeat until $input_{10}$ is *0*.

Converting decimal to binary

ex

41_{10} => $??_2$

123_{10} => $??_2$

binary arithmetic

ex

$110_2 + 1011_2 = ?_2$

$1101_2 - 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

conversion and arithmetic

ex

$19_{10} = ?_2$

$1001_2 = ?_{10}$

$240_{10} = ?_2$

$11010011_2 = ?_{10}$

$101_2 + 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

byte = 8 bits

a.k.a. octet

4 bits is a nibble (or nibble)

Smallest unit of data

used by a typical modern computer

Binary: 00000000_2 -- 11111111_2

Decimal: 000_{10} -- 255_{10}

Hexadecimal (Hex): 00_{16} -- FF_{16}

Byte = 2 hex digits!

Programmer's hex notation (C, etc.):

`0xB4` = $B4_{16}$

Stands for the following in binary:

0b10110100 = 10110100_2

Octal (base 8) also useful.

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

`char`: representing characters

A C-style string is represented by a series of bytes (`chars`).

- One-byte ASCII codes for each character.
- ASCII = American Standard Code for Information Interchange

32	space	48	0	64	@	80	P	96	`	112	p
33	!	49	1	65	A	81	Q	97	a	113	q
34	”	50	2	66	B	82	R	98	b	114	r
35	#	51	3	67	C	83	S	99	c	115	s
36	$	52	4	68	D	84	T	100	d	116	t
37	%	53	5	69	E	85	U	101	e	117	u
38	&	54	6	70	F	86	V	102	f	118	v
39	’	55	7	71	G	87	W	103	g	119	w
40	(	56	8	72	H	88	X	104	h	120	x
41	)	57	9	73	I	89	Y	105	I	121	y
42	*	58	:	74	J	90	Z	106	j	122	z
43	+	59	;	75	K	91	[	107	k	123	{
44	,	60	<	76	L	92	\	108	l	124	|
45	-	61	=	77	M	93	]	109	m	125	}
46	.	62	>	78	N	94	^	110	n	126	~
47	/	63	?	79	O	95	_	111	o	127	del

word |wərd|, n.

Natural unit of data used by processor.

Fixed size (e.g. 32 bits, 64 bits)

Defined by ISA: Instruction Set Architecture

machine instruction operands

word size = register size = address size

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	1	0	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Java/C int = 4 bytes: 11,501,584

MSB: most significant bit

LSB: least significant bit

fixed-size data representations

Java Data Type	C Data Type	(size in bytes) [word = 32 bits]	[word = 64 bits]
boolean		1	1
byte	char	1	1
char		2	2
short	short int	2	2
int	int	4	4
float	float	4	4
	long int	4	8
double	double	8	8
long	long long	8	8
	long double	8	16

Depends on word size!

bitwise operators

bit = Boolean
0 = false
1 = true

ex

Bitwise operators on fixed-width **bit vectors**.

AND &   OR | XOR ^   NOT ~

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001
```

Laws of Boolean algebra apply bitwise.

e.g., DeMorgan’s Law: ~(A | B) = ~A & ~B

```
  01010101
^ 01010101
```

bitwise operators in C

`& | ^ ~` apply to any *integral* data type
`long, int, short, char, unsigned`

Examples (**`char`**)

```
~0x41 =

~0x00 =

0x69 & 0x55 =

0x69 | 0x55 =
```

Many bit-twiddling puzzles in upcoming assignment

Representation Example 1: Sets as Bit Vectors

Representation: *n*-bit vector gives subset of {0, …, *n*–1}.

$a_i = 1 \equiv i \in A$

a = 0b**01101001** A = { 0, 3, 5, 6 }
      76543210

b = 0b**01010101** B = { 0, 2, 4, 6 }
      76543210

Bitwise Operations		**Set Operations**
a & b = 0b**01000001**	{0, 6}	Intersection
a | b = 0b**01111101**	{0, 2, 3, 4, 5, 6}	Union
a ^ b = 0b**00111100**	{2, 3, 4, 5}	Symmetric difference
~ b = 0b**10101010**	{1, 3, 5, 7}	Complement

logical operations in C

`&& || !` apply to any "integral" data type
`long, int, short, char, unsigned`

0 is **false** **nonzero** is **true** **result** always **0 or 1**

early termination a.k.a. **short-circuit evaluation**

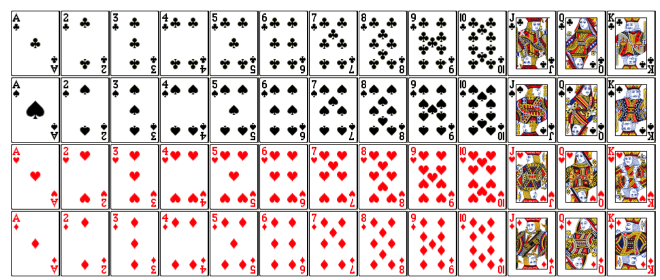
Examples (**`char`**)

```
 !0x41 =
 !0x00 =
!!0x41 =

0x69 && 0x55 =
0x69 || 0x55 =
```


Representation Example 2: Playing Cards

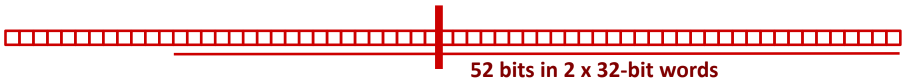
52 cards in 4 suits
- How do we encode suits, face cards?

What operations should be easy to implement?
- Get and compare rank
- Get and compare suit



Two possible representations

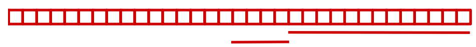
52 cards – 52 bits with bit corresponding to card set to 1



"One-hot" encoding

- Hard to compare values and suits independently
- Not space efficient

4 bits for suit, 13 bits for card value – 17 bits with two set to 1



Pair of one-hot encoded values

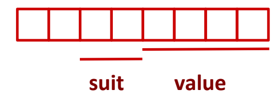
- Easier to compare suits and values independently
- Smaller, but still not space efficient

Two better representations

Binary encoding of all 52 cards – only 6 bits needed

- Number cards uniquely from 0
- Smaller than one-hot encodings.
- Hard to compare value and suit

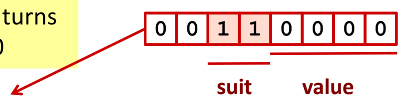
Binary encoding of suit (2 bits) and value (4 bits) separately



- Number each suit uniquely
- Number each value uniquely
- Still small
- Easy suit, value comparisons

Compare Card **Suits**

mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0

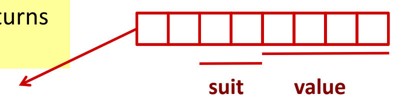



```
#define SUIT_MASK 0x30

int sameSuit(char card1, char card2) {
  return !((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK));

  //same as (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}



char hand[5];       // represents a 5-card hand
...
if ( sameSuit(hand[0], hand[1]) ) { ... }
```


Compare Card **Values**

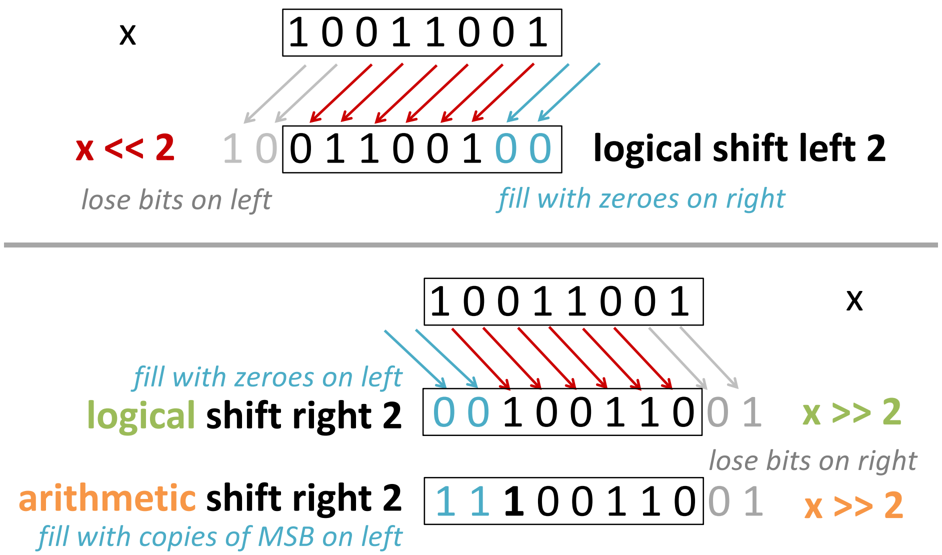
ex

mask: a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0




```
#define VALUE_MASK

int greaterValue(char card1, char card2) {




}

char hand[5];       // represents a 5-card hand
...
if ( greaterValue(hand[0], hand[1]) ) { ... }
```

Bit shifting



Shift gotchas

!!!

Logical or arithmetic shift right: how do we tell?

C: compiler chooses

  Usually based on type: rain check!

Java: >> is arithmetic, >>> is logical

Shift an *n*-bit type by at least 0 and no more than n-1.

C: other shift distances are undefined.

  anything could happen

Java: shift distance is used modulo number of bits in shifted type

  Given int x: x << 34 == x << 2

Shift and mask: extract a bit field

ex

Write a C function that
extracts the *2nd most significant byte*
from its 32-bit integer argument.

Example behavior:

argument: 0b 01100001 01100010 01100011 01100100

expected result: 0b 00000000 00000000 00000000 01100010

All other bits are zero. Desired bits in least significant byte.

```
int get2ndMSB(int x) {
```