# Representing Data with Bits

bits, bytes, numbers, and notation

# positional number representation

base = 10 (*decimal*)

| | | |
|---|---|---|
| 2 | 4 | 0 |
| 100 | 10 | 1 |
| $10^2$ | $10^1$ | $10^0$ |
| 2 | 1 | 0 |

*weight*

*position*

$$= 2 \times 10^2 + 4 \times 10^1 + 0 \times 10^0$$

**Base** determines:

Maximum digit (base – 1). Minimum digit is 0.

Weight of each position.

Each position holds a digit.

Represented value = sum of all position values

*position value = digit value × base*$^{position}$

# binary = base 2

Binary digits are called ***bits***: 0, 1

base = 2 (*binary*)

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 1 |
| 8 | 4 | 2 | 1 |
| $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| 3 | 2 | 1 | 0 |

$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

*weight* (rows 8 4 2 1 and $2^3$ $2^2$ $2^1$ $2^0$)

*position* (row 3 2 1 0)

When ambiguous, subscript with base:

$101_{10}$ Dalmatians (movie)

$101_2$-Second Rule (folk wisdom for food safety)

irony

# Powers of 2:
# memorize up to ≥ $2^{12}$ (in base ten)

| Power: $2^?$ | Decimal value |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |

| Power: $2^?$ | Decimal value |
|---|---|
| 9 | 512 |
| 10 | 1024 |
| 11 | 2048 |
| 12 | 4096 |
| 13 | 8192 |
| 14 | 16384 |
| 15 | 32768 |
| 16 | 65536 |

# Shifting binary numbers

$11011_2 = 27_{10}$

- What is $110110_2$?

- What is $110111_2$?

- What is $1101_2$?

# Converting binary to decimal

$101011_2 = ?_{10}$

Start with $output_{10} = 0$

Right to left(traditional algorithm)

- Start with smallest power 2 = $2^{0.}$ = 1
- If corresponding bit is 1, add power of 2 to $output_{10}$
- Repeat until power of two for leftmost 1 in $input_2$ is found

Right to left (better algorithm)

- Start with leftmost 1 bit in $input_2$ and $output_{10} = 1$
- For every 0, double $output_{10}$
- For every 1, double $output_{10}$ and add 1.

# Converting binary to decimal

$110101_2$ => $??_{10}$

$10110111_2$ => $??_{10}$

# Converting decimal to binary

$19_{10} = ?_2$

Start with $output_2$ = the empty string of binary digits

Left to right (traditional algorithm)

- Find the largest **power of $2_{10}$** that is ≤ $input_{10}$.
- Subtract it from $input_{10}$.
- Add it to $output_2$.
- Repeat with until $input_{10}$ is *0*.

Right to left (better algorithm)

- **Divide** $input_{10}$ by $2_{10}$.
- Prepend the remainder as a bit on the left end of $output_2$.
- Repeat until $input_{10}$ is *0*.

# Converting decimal to binary

ex

$41_{10}$ => $??_2$

$123_{10}$ => $??_2$

# binary arithmetic

ex

$110_2 + 1011_2 = ?_2$

$1101_2 - 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

# conversion and arithmetic

ex

$19_{10} = ?_2$

$1001_2 = ?_{10}$

$240_{10} = ?_2$

$11010011_2 = ?_{10}$

$101_2 + 1011_2 = ?_2$

$1001011_2 \times 2_{10} = ?_2$

# *byte* = 8 bits

a.k.a. octet

*4 bits is a nibble (or nibble)*

**Smallest unit of data**

*used by a typical modern computer*

**Binary:** $00000000_2$ -- $11111111_2$

**Decimal:** $000_{10}$ -- $255_{10}$

**Hexadecimal (Hex):** $00_{16}$ -- $FF_{16}$

**Byte = 2 hex digits!**

Programmer's hex notation (C, etc.):

**`0xB4`** = $B4_{16}$

Stands for the following in binary:

0b10110100 = $10110100_2$

Octal (base 8) also useful.

| Hex | Decimal | Binary |
|---|---|---|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

# `char`: representing characters

A C-style string is represented by a series of bytes (`char`s).

- One-byte ASCII codes for each character.
- ASCII = American Standard Code for Information Interchange

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 32 | space | 48 | 0 | 64 | @ | 80 | P | 96 | ` | 112 | p |
| 33 | ! | 49 | 1 | 65 | A | 81 | Q | 97 | a | 113 | q |
| 34 | ” | 50 | 2 | 66 | B | 82 | R | 98 | b | 114 | r |
| 35 | # | 51 | 3 | 67 | C | 83 | S | 99 | c | 115 | s |
| 36 | $ | 52 | 4 | 68 | D | 84 | T | 100 | d | 116 | t |
| 37 | % | 53 | 5 | 69 | E | 85 | U | 101 | e | 117 | u |
| 38 | & | 54 | 6 | 70 | F | 86 | V | 102 | f | 118 | v |
| 39 | ’ | 55 | 7 | 71 | G | 87 | W | 103 | g | 119 | w |
| 40 | ( | 56 | 8 | 72 | H | 88 | X | 104 | h | 120 | x |
| 41 | ) | 57 | 9 | 73 | I | 89 | Y | 105 | I | 121 | y |
| 42 | * | 58 | : | 74 | J | 90 | Z | 106 | j | 122 | z |
| 43 | + | 59 | ; | 75 | K | 91 | [ | 107 | k | 123 | { |
| 44 | , | 60 | < | 76 | L | 92 | \ | 108 | l | 124 | \| |
| 45 | - | 61 | = | 77 | M | 93 | ] | 109 | m | 125 | } |
| 46 | . | 62 | > | 78 | N | 94 | ^ | 110 | n | 126 | ~ |
| 47 | / | 63 | ? | 79 | O | 95 | _ | 111 | o | 127 | del |

# ***word*** *|wərd|,* **n.**

## Natural unit of data used by processor.

**Fixed size** (e.g. 32 bits, 64 bits)

Defined by ISA: Instruction Set Architecture

machine instruction operands

word size = register size = address size

| 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

**Java/C int = 4 bytes:** 11,501,584

**MSB: most significant bit**

**LSB: least significant bit**

# fixed-size data representations

| Java Data Type | C Data Type | (size in bytes) [word = 32 bits] | [word = 64 bits] |
|---|---|---|---|
| boolean | | 1 | 1 |
| byte | char | 1 | 1 |
| char | | 2 | 2 |
| short | short int | 2 | 2 |
| int | int | 4 | 4 |
| float | float | 4 | 4 |
| | long int | 4 | 8 |
| double | double | 8 | 8 |
| long | long long | 8 | 8 |
| | long double | 8 | 16 |

**Depends on word size!**

# *bitwise* operators

bit = Boolean
0 = false
1 = true

ex

**Bitwise operators** on fixed-width **bit vectors**.

AND & OR | XOR ^ NOT ~

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
----------    ----------    ----------    ----------
  01000001
```

```
  01010101
^ 01010101
----------
```

Laws of Boolean algebra apply bitwise.

*e.g.,* DeMorgan's Law: ~(A | B) = ~A & ~B

# ***bitwise*** **operators in C**

ex

**& | ^ ~** apply to any *integral* data type

**`long, int, short, char, unsigned`**

Examples (**`char`**)

```
~0x41 =

~0x00 =

0x69 & 0x55 =

0x69 | 0x55 =
```

Many bit-twiddling puzzles in upcoming assignment

# Representation Example 1: Sets as Bit Vectors

ex

**Representation:** *n*-bit vector gives subset of {0, ..., *n*–1}.

$a_i = 1 \equiv i \in A$

```
a = 0b01101001      A = { 0, 3, 5, 6 }
      76543210

b = 0b01010101      B = { 0, 2, 4, 6 }
      76543210
```

**Bitwise Operations** / **Set Operations**

```
a & b = 0b01000001  {0, 6}              Intersection
a | b = 0b01111101  {0, 2, 3, 4, 5, 6}  Union
a ^ b = 0b00111100  {2, 3, 4, 5}        Symmetric difference
  ~ b = 0b10101010  {1, 3, 5, 7}        Complement
```

# *logical* operations in C

ex

**`&&`    `||`    `!`**    apply to any "integral" data type
**`long, int, short, char, unsigned`**

**0** is **false**    **nonzero** is **true**    **result** always **0 or 1**

**early termination**  a.k.a.  **short-circuit evaluation**

Examples (**`char`**)

```
  !0x41 =
  !0x00 =
 !!0x41 =

0x69 && 0x55 =
0x69 || 0x55 =
```

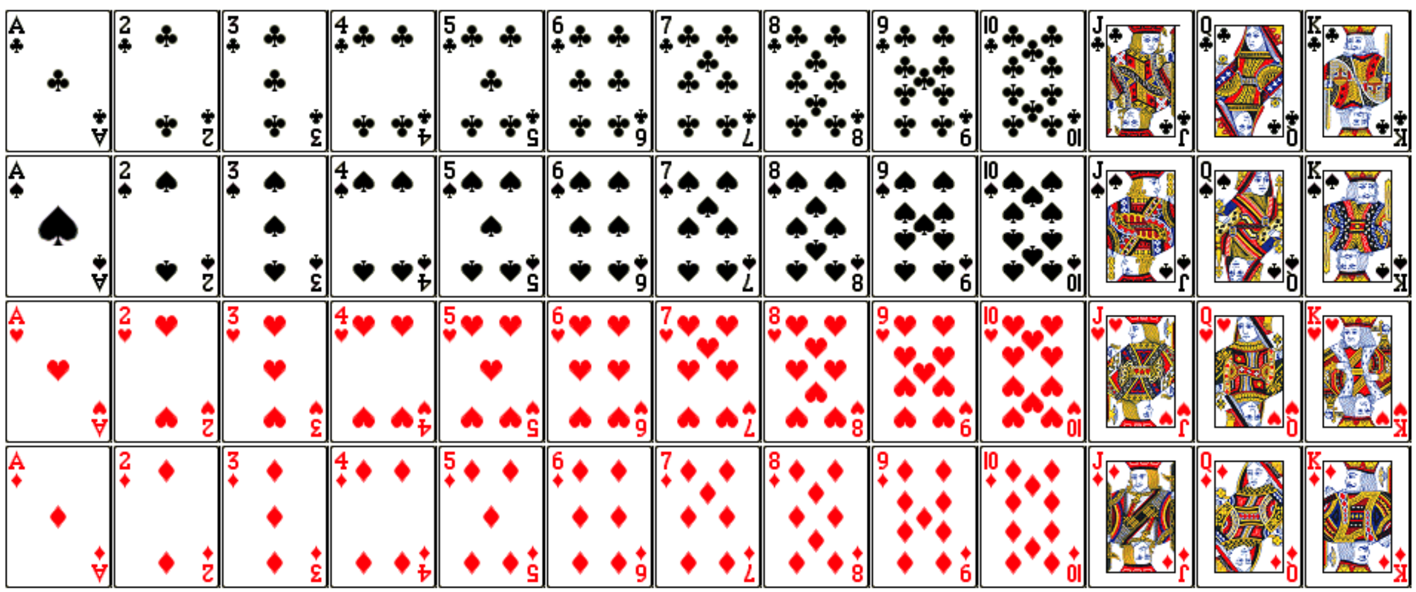
# Representation Example 2: Playing Cards

52 cards in 4 suits

- How do we encode suits, face cards?

What operations should be easy to implement?

- Get and compare rank
- Get and compare suit



# Two possible representations

52 cards – 52 bits with bit corresponding to card set to 1

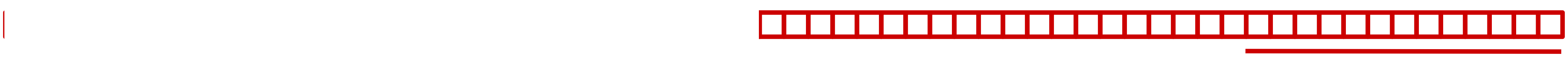
52 bits in 2 x 32-bit words

**“One-hot” encoding**

Hard to compare values and suits independently

Not space efficient

4 bits for suit, 13 bits for card value – 17 bits with two set to 1



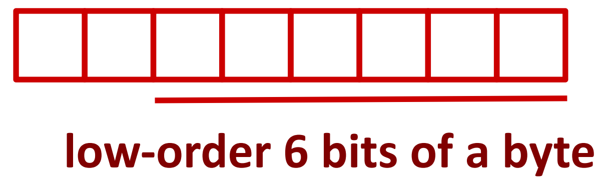
**Pair of one-hot encoded values**

Easier to compare suits and values independently

Smaller, but still not space efficient

# Two better representations

Binary encoding of all 52 cards – only 6 bits needed

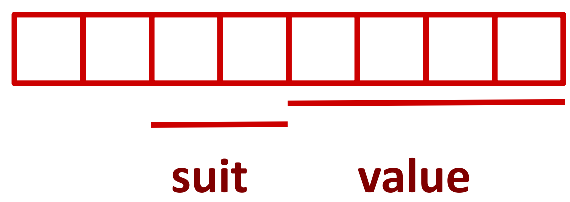



Number cards uniquely from 0

Smaller than one-hot encodings.

Hard to compare value and suit

Binary encoding of suit (2 bits) and value (4 bits) separately




Number each suit uniquely

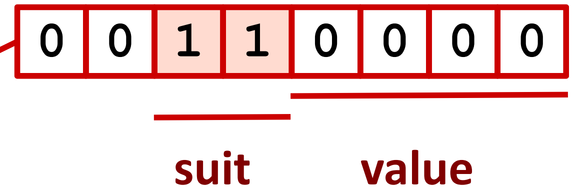
Number each value uniquely

Still small

Easy suit, value comparisons

# Compare Card **Suits**

**mask:** a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0



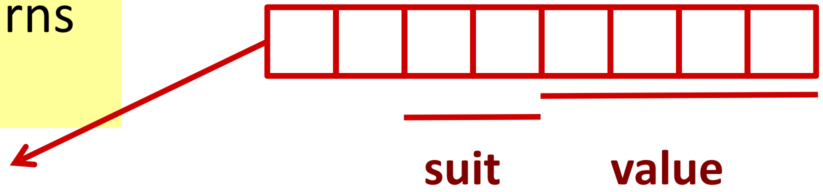

```
#define SUIT_MASK 0x30

int sameSuit(char card1, char card2) {
  return !((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK));

  //same as (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

```
char hand[5];        // represents a 5-card hand
...
if ( sameSuit(hand[0], hand[1]) ) { ... }
```

# Compare Card Values

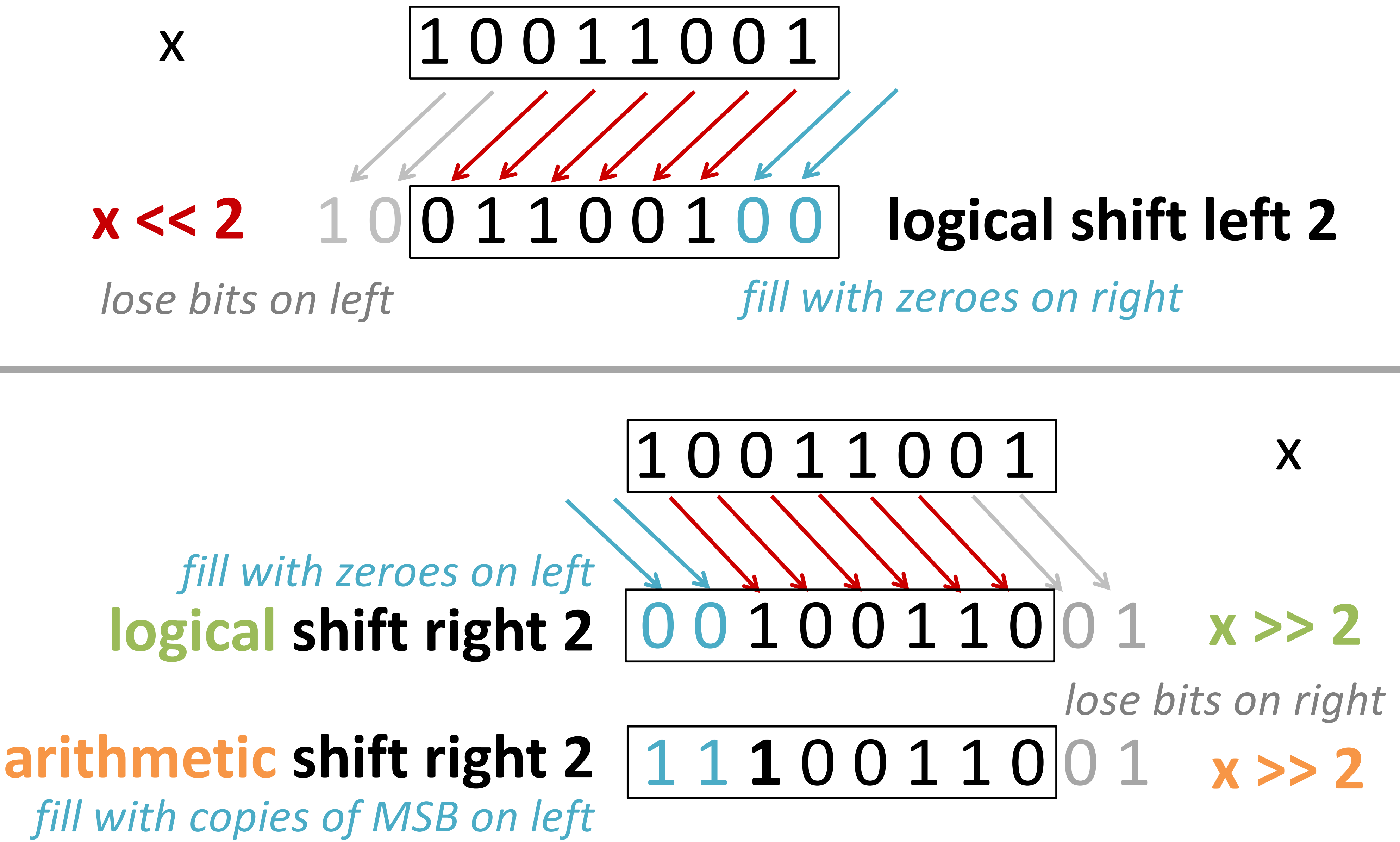
ex

**mask:** a bit vector that, when bitwise ANDed with another bit vector *v*, turns all *but* the bits of interest in *v* to 0




```
#define VALUE_MASK

int greaterValue(char card1, char card2) {




}


char hand[5];        // represents a 5-card hand
...
if ( greaterValue(hand[0], hand[1]) ) { ... }
```

# Bit shifting



# Shift gotchas

!!!

Logical or arithmetic shift right: how do we tell?

C: compiler chooses

- Usually based on type: rain check!

Java: >> is arithmetic, >>> is logical

Shift an *n*-bit type by at least 0 and no more than n-1.

C: other shift distances are undefined.

- *anything* could happen

Java: shift distance is used modulo number of bits in shifted type

- Given int x: x << 34 == x << 2

# **Shift and mask:** extract a bit field

ex

**Write a C function** that
extracts the $2^{nd}$ *most significant byte*
from its 32-bit integer argument.

**Example behavior:**

argument: `0b 01100001 01100010 01100011 01100100`

expected result: `0b 00000000 00000000 00000000 01100010`

All other bits are zero. Desired bits in least significant byte.

```
int get2ndMSB(int x) {
```