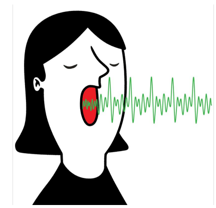
Hidden Markov Models

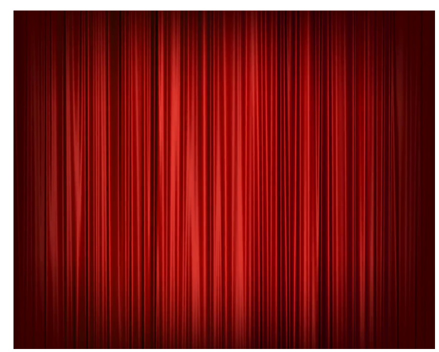
Hidden Markov Models (HMMs)



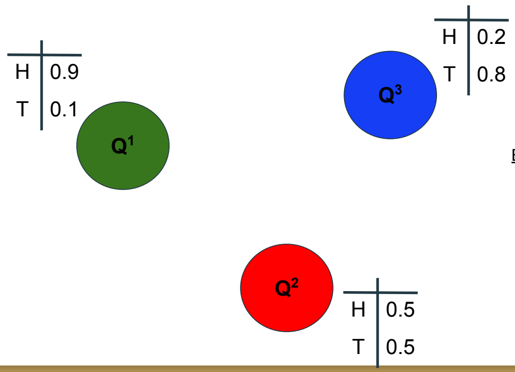
Coin Example

Coin Example

HTTHTTTHHT



HMM: Emission Probabilities, B



Emission Probabilities, *B*

$b_1(H) = 0.9$

$b_1(T) = 0.1$

$b_2(H) = 0.5$

$b_2(T) = 0.5$

$b_3(H) = 0.2$

$b_3(T) = 0.8$

Probability of Observation Sequence

If only state 1, i.e., the first coin, is used...

$O =$	H	T	T	H	T	T	T	H	H	T
	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1
$P(O) =$	$b_1(H)$	$b_1(T)$	$b_1(T)$	$b_1(H)$	$b_1(T)$	$b_1(T)$	$b_1(T)$	$b_1(H)$	$b_1(H)$	$b_1(T)$
	0.9	0.1	0.1	0.9	0.1	0.1	0.1	0.9	0.9	0.1

0.0000006561

Probability of Observation Sequence

If only state 2, i.e., the second coin, is used...

$O =$	H	T	T	H	T	T	T	H	H	T
	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2
$P(O) =$	$b_2(H)$	$b_2(T)$	$b_2(T)$	$b_2(H)$	$b_2(T)$	$b_2(T)$	$b_2(T)$	$b_2(H)$	$b_2(H)$	$b_2(T)$
	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

0.0009765625

Probability of Observation Sequence

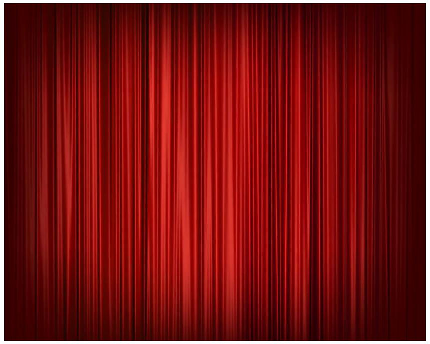
If only state 3, i.e., the third coin, is used...

$O =$	H	T	T	H	T	T	T	H	H	T
	Q^3	Q^3	Q^3	Q^3	Q^3	Q^3	Q^3	Q^3	Q^3	Q^3
$P(O) =$	$b_3(H)$	$b_3(T)$	$b_3(T)$	$b_3(H)$	$b_3(T)$	$b_3(T)$	$b_3(T)$	$b_3(H)$	$b_3(H)$	$b_3(T)$
	0.2	0.8	0.8	0.2	0.8	0.8	0.8	0.2	0.2	0.8

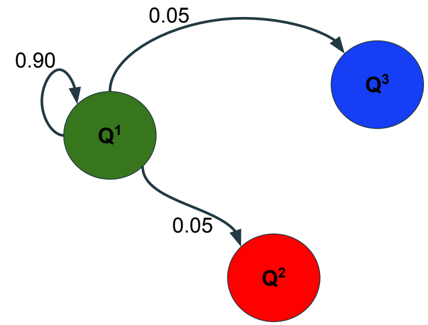
0.0004194304

Coin Example

HTTHTTTHHT



HMM: Transition Probabilities, A

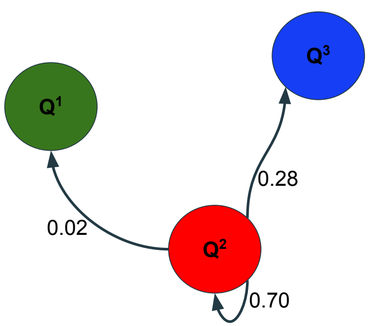


<u>Transition Probabilities, *A*</u>

$a_{11} = 0.90$
$a_{12} = 0.05$
$a_{13} = 0.05$

HMM: Transition Probabilities, A

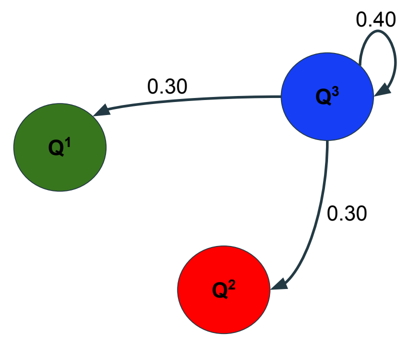
<u>Transition Probabilities, *A*</u>



$a_{21} = 0.02$
$a_{22} = 0.70$
$a_{23} = 0.28$

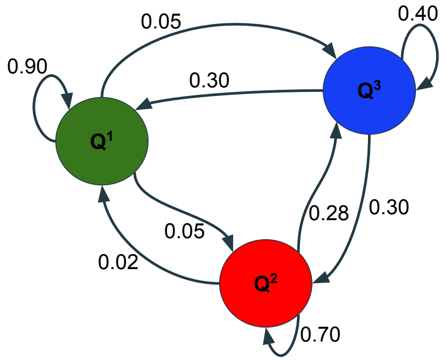
HMM: Transition Probabilities, A

<u>Transition Probabilities, *A*</u>



$a_{31} = 0.30$
$a_{32} = 0.30$
$a_{33} = 0.40$

HMM: Transition Probabilities, A



Transition Probabilities, *A*

a_{11} = 0.90
a_{12} = 0.05
a_{13} = 0.05

a_{21} = 0.02
a_{22} = 0.70
a_{23} = 0.28

a_{31} = 0.30
a_{32} = 0.30
a_{33} = 0.40

Probability of Observation Sequence

If we can transition between states (coins) ...

O =	H	T	T	H	T	T	T	H	H	T
	Q^1	Q^1	Q^1	Q^1	Q^3	Q^3	Q^3	Q^3	Q^3	Q^3

$P(O) = b_1(H)a_{11}\, b_1(T)a_{11}\, b_1(T)a_{11}\, b_1(H)a_{13}\, b_3(T)a_{33}\, b_3(T)a_{33}\, b_3(T)a_{33}\, b_3(H)a_{33}\, b_3(H)a_{33}\, b_3(T)$

0.90 0.90 0.10 0.90 0.10 0.90 0.90 0.05 0.80 0.40 0.80 0.40 0.80 0.40 0.20 0.40 0.20 0.40 0.80

0.0000000220150628352

* Assuming we start in state 1, i.e., the first coin

Probability of Observation Sequence

If we can transition between states (coins) ...

O =	H	T	T	H	T	T	T	H	H	T
	Q^1	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2	Q^2

$P(O) = b_1(H)a_{12}\, b_2(T)a_{22}\, b_2(T)a_{22}\, b_2(H)a_{22}\, b_2(T)a_{22}\, b_2(T)a_{22}\, b_2(T)a_{22}\, b_2(H)a_{22}\, b_2(H)a_{22}\, b_2(T)$

0.90 0.05 0.50 0.70 0.50 0.70 0.50 0.70 0.50 0.70 0.50 0.70 0.50 0.70 0.50 0.70 0.50 0.70 0.50

0.00000506671962890525

* Assuming we start in state 1, i.e., the first coin

Probability of Observation Sequence

If we can transition between states (coins) ...

O =	H	T	T	H	T	T	T	H	H	T
	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1	Q^1

$P(O) = b_1(H)a_{11}\, b_1(T)a_{11}\, b_1(T)a_{11}\, b_1(H)a_{11}\, b_1(T)a_{11}\, b_1(T)a_{11}\, b_1(T)a_{11}\, b_1(H)a_{11}\, b_1(H)a_{11}\, b_1(T)$

0.90 0.90 0.10 0.90 0.10 0.90 0.90 0.90 0.10 0.90 0.10 0.90 0.10 0.90 0.90 0.90 0.90 0.90 0.10

0.0000002541865828329

* Assuming we start in state 1, i.e., the first coin

Probability of Observation Sequence

If we can transition between states (coins) ...

O =	H	T	T	H	T	T	T	H	H	T
	Q^1	Q^3	Q^2	Q^2	Q^3	Q^3	Q^3	Q^1	Q^2	Q^2

$$P(O) = b_1(H)a_{13}\,b_3(T)a_{32}\,b_2(T)a_{22}\,b_2(H)a_{23}\,b_3(T)a_{33}\,b_3(T)a_{33}\,b_3(T)a_{31}\,b_1(H)a_{12}\,b_2(H)a_{22}\,b_2(T)$$

0.90 0.05 0.80 0.30 0.50 0.70 0.50 0.28 0.80 0.40 0.80 0.40 0.80 0.30 0.90 0.05 0.50 0.70 0.50

0.0000001024192512

* Assuming we start in state 1, i.e., the first coin

HMM: Model, λ

$$\lambda = (A, B)$$

Transition Probabilities, A

$a_{11} = 0.90$
$a_{12} = 0.05$
$a_{13} = 0.05$

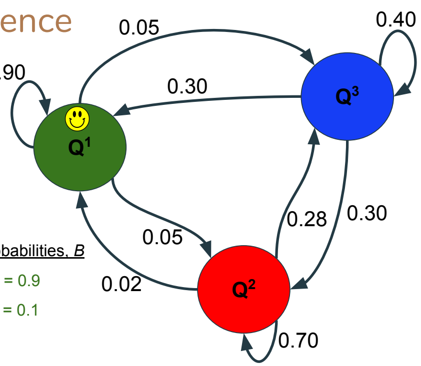
$a_{21} = 0.02$
$a_{22} = 0.70$
$a_{23} = 0.28$

$a_{31} = 0.30$
$a_{32} = 0.30$
$a_{33} = 0.40$

Emission Probabilities, B

$b_1(H) = 0.9$
$b_1(T) = 0.1$
$b_2(H) = 0.5$
$b_2(T) = 0.5$
$b_3(H) = 0.2$
$b_3(T) = 0.8$

Generating an Observation Sequence



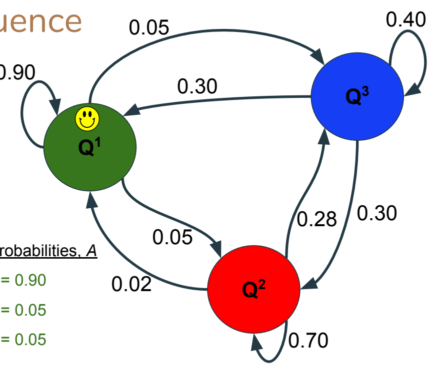
Emission Probabilities, B

$b_1(H) = 0.9$
$b_1(T) = 0.1$

O = H

- Begin in the first state, i.e., the first coin
- Emit an output character from the current state, i.e., flip the coin

Generating an Observation Sequence



Transition Probabilities, A

$a_{11} = 0.90$
$a_{12} = 0.05$
$a_{13} = 0.05$

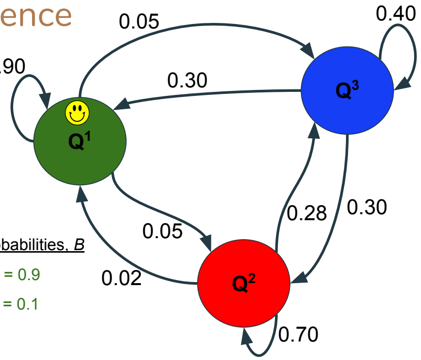
O = H

- Begin in the first state, i.e., the first coin
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next

Generating an Observation Sequence

O = HT

- Begin in the first state, i.e., the first coin
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next
- Emit an output character from the current state, i.e., flip the coin



Emission Probabilities, *B*

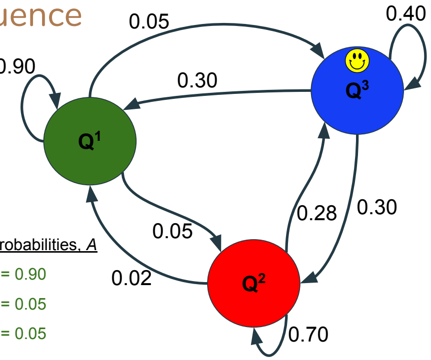
$b_1(H) = 0.9$

$b_1(T) = 0.1$

Generating an Observation Sequence

O = HT

- Begin in the first state, i.e., the first coin
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next



Transition Probabilities, *A*

$a_{11} = 0.90$

$a_{12} = 0.05$

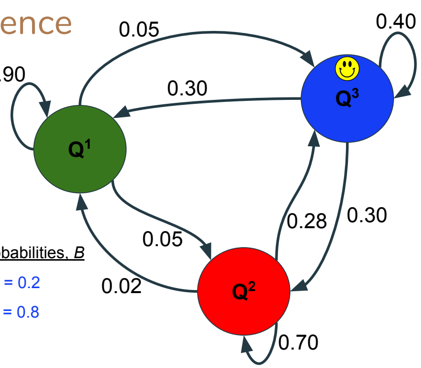
$a_{13} = 0.05$

Generating an Observation Sequence

O = HTT

- Begin in the first state, i.e., the first coin
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next
- Emit an output character from the current state, i.e., flip the coin

...



Emission Probabilities, *B*

$b_3(H) = 0.2$

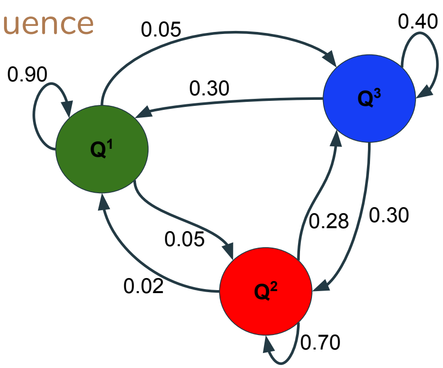
$b_3(T) = 0.8$

Generating an Observation Sequence

O = HTTHTTTHHT

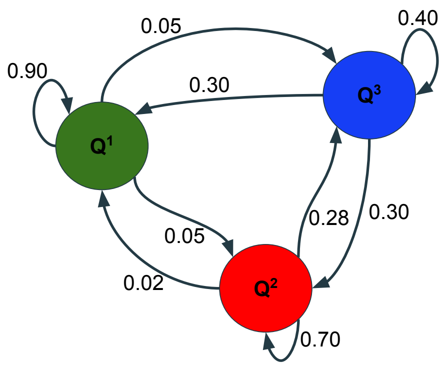
- Begin in the first state, i.e., the first coin
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next
- Emit an output character from the current state, i.e., flip the coin
- Transition to the next state, i.e., choose a coin to flip next
- Emit an output character from the current state, i.e., flip the coin

...



Hidden Information

O = HTTHTTTHHT



$Q = Q^1Q^1Q^3Q^3Q^3Q^3Q^3Q^1Q^1Q^1$

HMMs are Memoryless

The likelihood of a given future state depends only on the present state and not on past states

A Common Application of HMMs: Induction

Given an observation sequence $O = O_1O_2O_3 \ldots O_T$ and a model $\lambda = (A, B)$, what is the optimal state sequence?

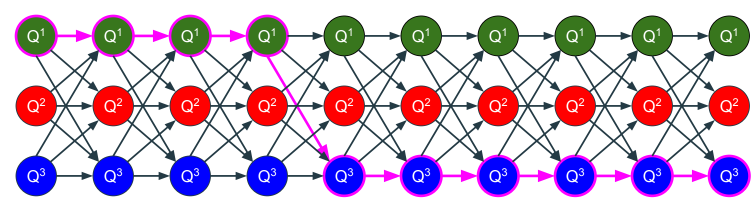
- We want to uncover the hidden information.
  We want to maximize $P(Q \mid O, \lambda)$

$$\underset{Q_1,Q_2,\ldots,Q_T}{\operatorname{argmax}} \left(b_{Q_1}(O_1)a_{Q_1Q_2}b_{Q_2}(O_2)a_{Q_2Q_3}b_{Q_3}(O_3)\cdots a_{Q_{T-1}Q_T}b_{Q_T}(O_T)\right)$$

Path (State Sequence) Through HMM

Q^1 Q^1 Q^1 Q^1 Q^3 Q^3 Q^3 Q^3 Q^3 Q^3

$b_1(H)a_{11}b_1(T)a_{11}b_1(T)a_{11}b_1(H)a_{13}b_3(T)a_{33}b_3(T)a_{33}b_3(T)a_{33}b_3(H)a_{33}b_3(H)a_{33}\ b_3(T)$

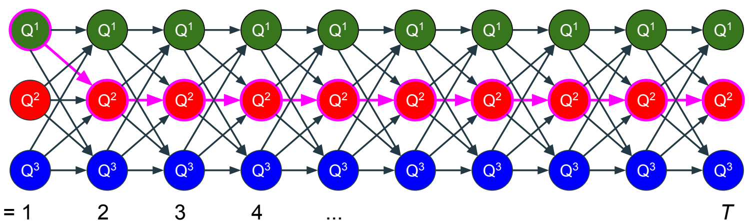


t = 1 2 3 4 ... T

Path (State Sequence) Through HMM

Q^1 Q^2 Q^2 Q^2 Q^2 Q^2 Q^2 Q^2 Q^2 Q^2

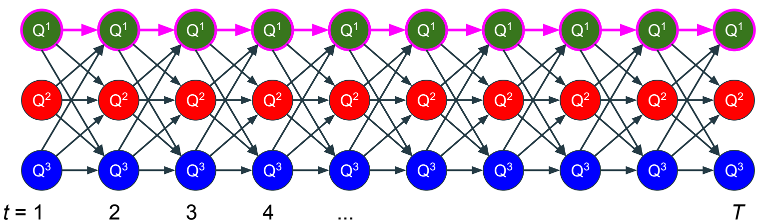
$b_1(H)a_{12}b_2(T)a_{22}b_2(T)a_{22}b_2(H)a_{22}b_2(T)a_{22}b_2(T)a_{22}b_2(T)a_{22}b_2(H)a_{22}b_2(H)a_{22}\ b_2(T)$



Path (State Sequence) Through HMM

Q^1 Q^1 Q^1 Q^1 Q^1 Q^1 Q^1 Q^1 Q^1 Q^1

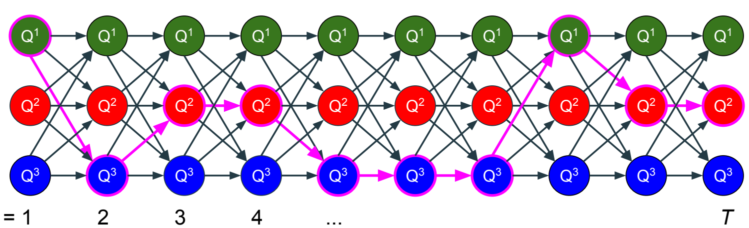
$b_1(H)a_{11}b_1(T)a_{11}b_1(T)a_{11}b_1(H)a_{11}b_1(T)a_{11}b_1(T)a_{11}b_1(T)a_{11}b_1(H)a_{11}b_1(H)a_{11}\ b_1(T)$



Path (State Sequence) Through HMM

Q^1 Q^3 Q^2 Q^2 Q^3 Q^3 Q^3 Q^1 Q^2 Q^2

$b_1(H)a_{13}b_3(T)a_{32}b_2(T)a_{22}b_2(H)a_{23}b_3(T)a_{33}b_3(T)a_{33}b_3(T)a_{31}b_1(H)a_{12}b_2(H)a_{22}\ b_2(T)$



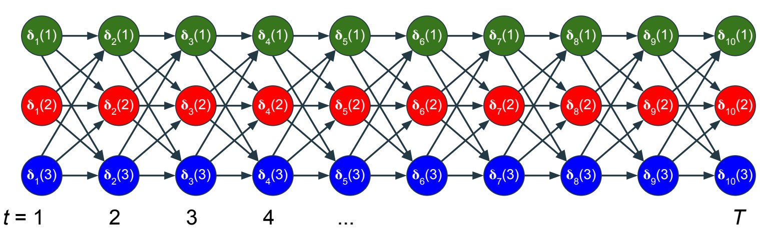
Viterbi Algorithm

$$\delta_t(j) = \begin{cases} b_1(O_1) & \text{if } t=1,\ j=1 \\ 0.0 & \text{if } t=1,\ j\neq 1 \\ \max\limits_{1\leq i\leq N}(\delta_{t-1}(i)*a_{ij})*b_j(O_t) & \text{if } 2\leq t\leq T \end{cases}$$

base case, start in state 1

base case, do not start in
state other than state 1

recursive case



Dynamic Programming Table

$$\delta_t(j) = \begin{cases} b_1(O_1) & \text{if } t=1,\ j=1 \\ 0.0 & \text{if } t=1,\ j\neq 1 \\ \max\limits_{1\leq i\leq N}(\delta_{t-1}(i)*a_{ij})*b_j(O_t) & \text{if } 2\leq t\leq T \end{cases}$$

base case, start in state 1
base case, do not start in
state other than state 1
recursive case

$\delta_1(1)$	$\delta_2(1)$	$\delta_3(1)$	$\delta_4(1)$	$\delta_5(1)$	$\delta_6(1)$	$\delta_7(1)$	$\delta_8(1)$	$\delta_9(1)$	$\delta_{10}(1)$
$\delta_1(2)$	$\delta_2(2)$	$\delta_3(2)$	$\delta_4(2)$	$\delta_5(2)$	$\delta_6(2)$	$\delta_7(2)$	$\delta_8(2)$	$\delta_9(2)$	$\delta_{10}(2)$
$\delta_1(3)$	$\delta_2(3)$	$\delta_3(3)$	$\delta_4(3)$	$\delta_5(3)$	$\delta_6(3)$	$\delta_7(3)$	$\delta_8(3)$	$\delta_9(3)$	$\delta_{10}(3)$

Dynamic Programming Table

$$\delta_t(j) = \begin{cases} b_1(O_1) & \text{if } t=1,\ j=1 \\ 0.0 & \text{if } t=1,\ j\neq 1 \end{cases}$$

base case, start in state 1
base case, do not start in
state other than state 1

$b_1(O_1)$									
0.0									
0.0									

Dynamic Programming Table

$$\delta_t(j) = \begin{cases} \max\limits_{1\leq i\leq N}(\delta_{t-1}(i)*a_{ij})*b_j(O_t) & \text{if } 2\leq t\leq T \end{cases}$$

recursive case

$\boldsymbol{\delta_7(2)}$ = max { $\boldsymbol{\delta_6(1)}$ * a_{12}, $\boldsymbol{\delta_6(2)}$ * a_{22}, $\boldsymbol{\delta_6(3)}$ * a_{32} } * $b_2(O_7)$

					$\delta_6(1)$				
					$\delta_6(2)$	$\delta_7(2)$			
					$\delta_6(3)$				

Dynamic Programming Table

$$\delta_t(j) = \begin{cases} \max\limits_{1\leq i\leq N}(\delta_{t-1}(i)*a_{ij})*b_j(O_t) & \text{if } 2\leq t\leq T \end{cases}$$

recursive case

$\boldsymbol{\delta_4(3)}$ = max { $\boldsymbol{\delta_3(1)}$ * a_{13}, $\boldsymbol{\delta_3(2)}$ * a_{23}, $\boldsymbol{\delta_3(3)}$ * a_{33} } * $b_3(O_4)$

		$\delta_3(1)$							
		$\delta_3(2)$							
		$\delta_3(3)$	$\delta_4(3)$						

Dynamic Programming Table

$$\delta_t(j) = \begin{cases} \\ \max\limits_{1 \le i \le N} (\delta_{t-1}(i) * a_{ij}) * b_j(O_t) & \text{if } 2 \le t \le T \end{cases}$$

recursive case

$\boldsymbol{\delta_9(1)} = \max \{ \boldsymbol{\delta_8(1)} * a_{11}, \boldsymbol{\delta_8(2)} * a_{21}, \boldsymbol{\delta_8(3)} * a_{31} \} * b_1(O_9)$

							$\delta_8(1)$	$\delta_9(1)$	
							$\delta_8(2)$		
							$\delta_8(3)$		

Probability of Optimal State Sequence

$$\delta_t(j) = \begin{cases} b_1(O_1) & \text{if } t=1,\ j=1 \\ 0.0 & \text{if } t=1,\ j \ne 1 \\ \max\limits_{1 \le i \le N} (\delta_{t-1}(i) * a_{ij}) * b_j(O_t) & \text{if } 2 \le t \le T \end{cases}$$

base case, start in state 1
base case, do not start in
state other than state 1
recursive case

$\delta_1(1)$	$\delta_2(1)$	$\delta_3(1)$	$\delta_4(1)$	$\delta_5(1)$	$\delta_6(1)$	$\delta_7(1)$	$\delta_8(1)$	$\delta_9(1)$	$\delta_{10}(1)$
$\delta_1(2)$	$\delta_2(2)$	$\delta_3(2)$	$\delta_4(2)$	$\delta_5(2)$	$\delta_6(2)$	$\delta_7(2)$	$\delta_8(2)$	$\delta_9(2)$	$\delta_{10}(2)$
$\delta_1(3)$	$\delta_2(3)$	$\delta_3(3)$	$\delta_4(3)$	$\delta_5(3)$	$\delta_6(3)$	$\delta_7(3)$	$\delta_8(3)$	$\delta_9(3)$	$\delta_{10}(3)$

Backtracking Table

$$\delta_t(j) = \begin{cases} b_1(O_1) & \text{if } t=1,\ j=1 \\ 0.0 & \text{if } t=1,\ j \ne 1 \\ \max\limits_{1 \le i \le N} (\delta_{t-1}(i) * a_{ij}) * b_j(O_t) & \text{if } 2 \le t \le T \end{cases}$$

base case, start in state 1
base case, do not start in
state other than state 1
recursive case

-1	1	1	1	1	1	3	3	1	1
-1	1	2	2	2	2	2	2	2	2
-1	1	3	3	2	2	2	2	2	2

Determine Optimal State Sequence

* Assuming optimal state sequence terminates in state #2

$Q^1\ Q^2\ Q^2\ Q^2\ Q^2\ Q^2\ Q^2\ Q^2\ Q^2\ Q^2$

-1	1	1	1	1	1	3	3	1	1
-1	1	2	2	2	2	2	2	2	2
-1	1	3	3	2	2	2	2	2	2

Runtime of Viterbi Algorithm?

$$\delta_t(j) = \begin{cases} b_1(O_1) & \text{if } t=1,\ j=1 \\ 0.0 & \text{if } t=1,\ j\neq 1 \\ \max_{1\leq i\leq N} (\delta_{t-1}(i) * a_{ij}) * b_j(O_t) & \text{if } 2\leq t\leq T \end{cases}$$

base case, start in state 1
base case, do not start in
state other than state 1
recursive case

$\delta_1(1)$	$\delta_2(1)$	$\delta_3(1)$	$\delta_4(1)$	$\delta_5(1)$	$\delta_6(1)$	$\delta_7(1)$	$\delta_8(1)$	$\delta_9(1)$	$\delta_{10}(1)$
$\delta_1(2)$	$\delta_2(2)$	$\delta_3(2)$	$\delta_4(2)$	$\delta_5(2)$	$\delta_6(2)$	$\delta_7(2)$	$\delta_8(2)$	$\delta_9(2)$	$\delta_{10}(2)$
$\delta_1(3)$	$\delta_2(3)$	$\delta_3(3)$	$\delta_4(3)$	$\delta_5(3)$	$\delta_6(3)$	$\delta_7(3)$	$\delta_8(3)$	$\delta_9(3)$	$\delta_{10}(3)$

Extension: Initial Probabilities

We may not always want to start in the first state. Perhaps the first observation character was generated by a different state (other than the first).

We can have a probability of starting in each state:

$\pi_1 = 0.6$ $\pi_2 = 0.1$ $\pi_3 = 0.3$

HMM: Model, λ

$\lambda = (A, B, \pi)$

Transition Probabilities, *A*

$a_{11} = 0.90$
$a_{12} = 0.05$
$a_{13} = 0.05$

$a_{21} = 0.02$
$a_{22} = 0.70$
$a_{23} = 0.28$

$a_{31} = 0.30$
$a_{32} = 0.30$
$a_{33} = 0.40$

Emission Probabilities, *B*

$b_1(H) = 0.9$
$b_1(T) = 0.1$
$b_2(H) = 0.5$
$b_2(T) = 0.5$
$b_3(H) = 0.2$
$b_3(T) = 0.8$

Initial Probabilities, π

$\pi_1 = 0.6$
$\pi_2 = 0.1$
$\pi_3 = 0.3$

Dynamic Programming Table

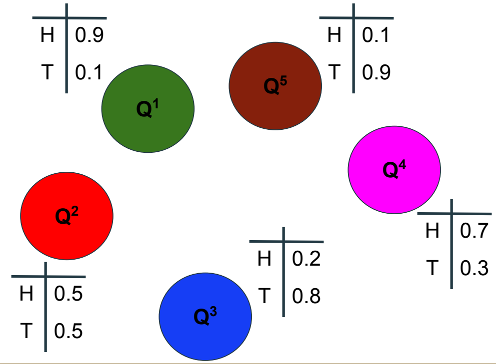
$$\delta_t(j) = \begin{cases} \pi_j b_j(O_1) & \text{if } t=1 \\ \max_{1\leq i\leq N} (\delta_{t-1}(i) * a_{ij}) * b_j(O_t) & \text{if } 2\leq t\leq T \end{cases}$$

base case
recursive case

$\delta_1(1)$	$\delta_2(1)$	$\delta_3(1)$	$\delta_4(1)$	$\delta_5(1)$	$\delta_6(1)$	$\delta_7(1)$	$\delta_8(1)$	$\delta_9(1)$	$\delta_{10}(1)$
$\delta_1(2)$	$\delta_2(2)$	$\delta_3(2)$	$\delta_4(2)$	$\delta_5(2)$	$\delta_6(2)$	$\delta_7(2)$	$\delta_8(2)$	$\delta_9(2)$	$\delta_{10}(2)$
$\delta_1(3)$	$\delta_2(3)$	$\delta_3(3)$	$\delta_4(3)$	$\delta_5(3)$	$\delta_6(3)$	$\delta_7(3)$	$\delta_8(3)$	$\delta_9(3)$	$\delta_{10}(3)$

Extension: Number of States

HMM: Emission Probabilities, B



<u>Emission Probabilities, *B*</u>

$b_1(H) = 0.9$

$b_1(T) = 0.1$

$b_2(H) = 0.5$

$b_2(T) = 0.5$

$b_3(H) = 0.2$

$b_3(T) = 0.8$

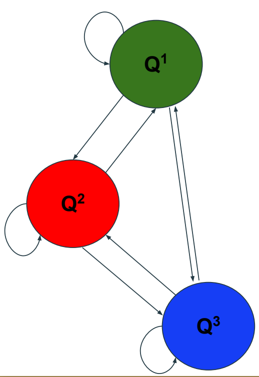
$b_4(H) = 0.7$

$b_4(T) = 0.3$

$b_5(H) = 0.1$

$b_5(T) = 0.9$

HMM: Transmission Probabilities, A



<u>Transition Probabilities, *A*</u>

$a_{11} = 0.90$ $a_{21} = 0.10$

$a_{12} = 0.02$ $a_{22} = 0.60$

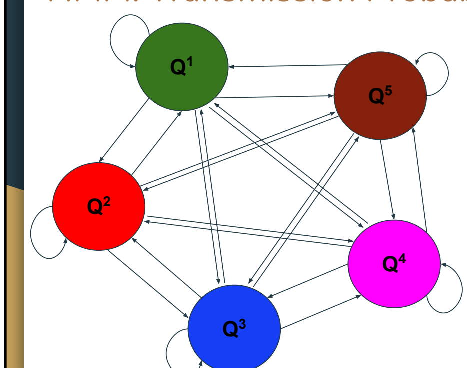
$a_{13} = 0.02$ $a_{23} = 0.15$

$a_{31} = 0.20$

$a_{32} = 0.15$

$a_{33} = 0.30$

HMM: Transmission Probabilities, A



<u>Transition Probabilities, *A*</u>

$a_{11} = 0.90$ $a_{21} = 0.10$

$a_{12} = 0.02$ $a_{22} = 0.60$

$a_{13} = 0.02$ $a_{23} = 0.15$

$a_{14} = 0.03$ $a_{24} = 0.09$

$a_{15} = 0.03$ $a_{25} = 0.06$

$a_{31} = 0.20$ $a_{41} = 0.04$ $a_{51} = 0.07$

$a_{32} = 0.15$ $a_{42} = 0.04$ $a_{52} = 0.22$

$a_{33} = 0.30$ $a_{43} = 0.04$ $a_{53} = 0.10$

$a_{34} = 0.25$ $a_{44} = 0.84$ $a_{54} = 0.11$

$a_{35} = 0.10$ $a_{45} = 0.04$ $a_{55} = 0.50$

HMM: Model, λ

$$\lambda = (A, B)$$

Transition Probabilities, A

$a_{11} = 0.90$
$a_{12} = 0.02$
$a_{13} = 0.02$

$a_{21} = 0.10$
$a_{22} = 0.60$
$a_{23} = 0.15$

$a_{31} = 0.20$
$a_{32} = 0.15$
$a_{33} = 0.30$

Emission Probabilities, B

$b_1(H) = 0.9$
$b_1(T) = 0.1$

$b_2(H) = 0.5$
$b_2(T) = 0.5$

$b_3(H) = 0.2$
$b_3(T) = 0.8$

HMM: Model, λ

$$\lambda = (A, B)$$

Transition Probabilities, A

$a_{11} = 0.90$
$a_{12} = 0.02$
$a_{13} = 0.02$
$a_{14} = 0.03$
$a_{15} = 0.03$

$a_{21} = 0.10$
$a_{22} = 0.60$
$a_{23} = 0.15$
$a_{24} = 0.09$
$a_{25} = 0.06$

$a_{31} = 0.20$
$a_{32} = 0.15$
$a_{33} = 0.30$
$a_{34} = 0.25$
$a_{35} = 0.10$

$a_{41} = 0.04$
$a_{42} = 0.04$
$a_{43} = 0.04$
$a_{44} = 0.84$
$a_{45} = 0.04$

$a_{51} = 0.07$
$a_{52} = 0.22$
$a_{53} = 0.10$
$a_{54} = 0.11$
$a_{55} = 0.50$

Emission Probabilities, B

$b_1(H) = 0.9$
$b_1(T) = 0.1$

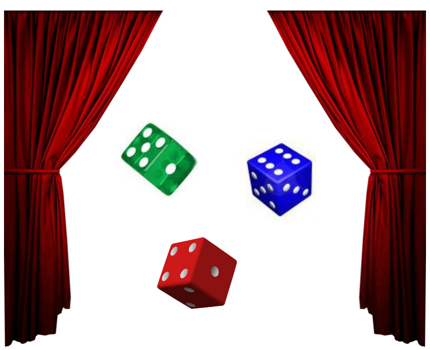
$b_2(H) = 0.5$
$b_2(T) = 0.5$

$b_3(H) = 0.2$
$b_3(T) = 0.8$

$b_4(H) = 0.7$
$b_4(T) = 0.3$

$b_5(H) = 0.1$
$b_5(T) = 0.9$

Extension: Emission Alphabet



HMM: Emission Probabilities, B

Q^1

1	0.11
2	0.25
3	0.04
4	0.30
5	0.08
6	0.22

Q^3

1	0.166
2	0.166
3	0.166
4	0.166
5	0.166
6	0.166

Q^2

1	0.42
2	0.09
3	0.10
4	0.23
5	0.10
6	0.06

Emission Probabilities, B

$b_1(1) = 0.11$
$b_1(2) = 0.25$
$b_1(3) = 0.04$
$b_1(4) = 0.30$
$b_1(5) = 0.08$
$b_1(6) = 0.22$

$b_1(1) = 0.42$
$b_1(2) = 0.09$
$b_1(3) = 0.10$
$b_1(4) = 0.23$
$b_1(5) = 0.10$
$b_1(6) = 0.06$

$b_1(1) = 0.166$
$b_1(2) = 0.166$
$b_1(3) = 0.166$
$b_1(4) = 0.166$
$b_1(5) = 0.166$
$b_1(6) = 0.166$

HMM: Model, λ

$$\lambda = (A, B)$$

Transition Probabilities, A

$a_{11} = 0.90$
$a_{12} = 0.05$
$a_{13} = 0.05$

$a_{21} = 0.02$
$a_{22} = 0.70$
$a_{23} = 0.28$

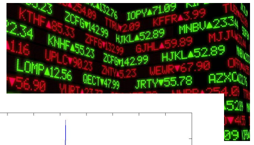
$a_{31} = 0.30$
$a_{32} = 0.30$
$a_{33} = 0.40$

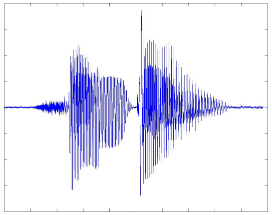
Emission Probabilities, B

$b_1(1) = 0.11$
$b_1(2) = 0.25$
$b_1(3) = 0.04$
$b_1(4) = 0.30$
$b_1(5) = 0.08$
$b_1(6) = 0.22$

$b_1(1) = 0.42$	$b_1(1) = 0.166$
$b_1(2) = 0.09$	$b_1(2) = 0.166$
$b_1(3) = 0.10$	$b_1(3) = 0.166$
$b_1(4) = 0.23$	$b_1(4) = 0.166$
$b_1(5) = 0.10$	$b_1(5) = 0.166$
$b_1(6) = 0.06$	$b_1(6) = 0.166$

Applications





suit case

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CACCATGGTGCACCTGACTCCTGAGGAGAAGTCTGCGGTTACTGCC
TGGTCTACCCTTGGACCCAGAGGTTCTTTGAGTCCTTTGGGGATCT
GTCCACTCCTGATGCTGTTATGGGCAACCCTAAGGTGAAGGCTCAT
GGCAAGAAAGTGCTCGGTGCCTTTAGTGATGGCCTGGCTCACCTGG
ACAACCTCAAGGGCACCTTTGCCACACTGAGTGAGCTGCACTGTGA
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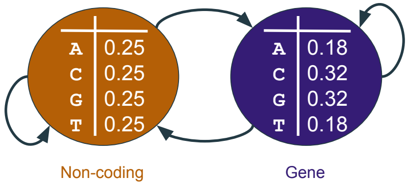

Overview