Higher-Order List Functions in Racket

CS251 Programming Languages
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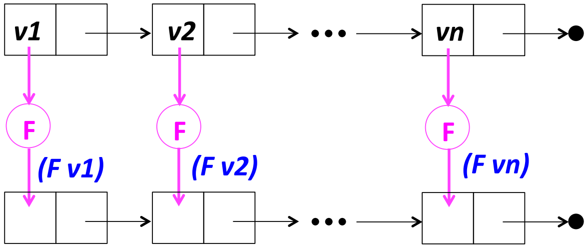
Higher-order List Functions

A function is **higher-order** if it takes another function as an input and/or returns another function as a result. E.g. `app-3-5`, `make-linear-function`, `flip2`.

We will now study **higher-order list functions** that capture the recursive list processing patterns we have seen.

Recall the List Mapping Pattern

`(mapF (list v1 v2 … vn))`



```
(define (mapF xs)
  (if (null? xs)
      null
      (cons (F (first xs))
            (mapF (rest xs)))))
```

Express Mapping via Higher-order `my-map`

```
(define (my-map f xs)
  (if (null? xs)
      null
      (cons (f (first xs))
            (my-map f (rest xs)))))
```

`my-map` Examples

```
> (my-map (λ (x) (* 2 x)) (list 7 2 4))


> (my-map first (list (list 2 3) (list 4) (list 5 6 7)))


> (my-map (make-linear-function 4 7) (list 0 1 2 3))


> (my-map app-3-5 (list sub2 + avg pow (flip pow)
                        make-linear-function))
```

Your turn

`(map-scale n nums)` returns a list that results from scaling each number in `nums` by `n`.

```
> (map-scale 3 (list 7 2 4))
'(21 6 12)

> (map-scale 6 (range 0 5))
'(0 6 12 18 24)
```

Currying

Haskell Curry

A curried binary function takes one argument at a time.

```
(define (curry2 binop)
   (λ (x) (λ (y) (binop x y)))

(define curried-mul (curry2 *))

> ((curried-mul 5) 4)

> (my-map (curried-mul 3) (list 1 2 3))

> (my-map ((curry2 pow) 4) (list 1 2 3))

> (my-map ((curry2 (flip2 pow)) 4) (list 1 2 3))

> (define lol (list (list 2 3) (list 4) (list 5 6 7)))

> (map ((curry2 cons) 8) lol)

> (map (??? 8) lol)
  `((2 3 8) (4 8) (5 6 7 8))
```

Mapping with binary functions

```
(define (my-map2 binop xs ys)
  (if (not (= (length xs) (length ys)))
      (error "my-map2 requires same-length lists")
      (if (or (null? xs) (null? ys))
          null
          (cons (binop (first xs) (first ys))
                (my-map2 binop (rest xs) (rest ys))))))
```

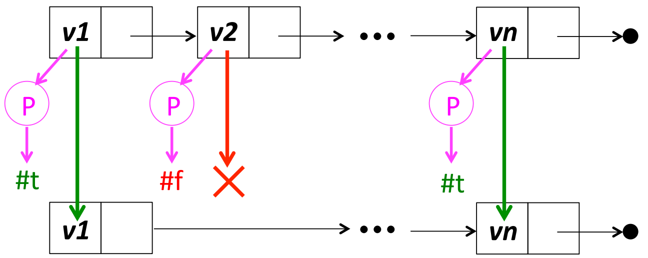
```
> (my-map2 pow (list 2 3 5) (list 6 4 2))
'(64 81 25)

> (my-map2 cons (list 2 3 5) (list 6 4 2))
'((2 . 6) (3 . 4) (5 . 2))

> (my-map2 cons (list 2 3 4 5) (list 6 4 2))
ERROR: my-map2 requires same-length lists
```

Built-in Racket `map` Function Maps over Any Number of Lists

```
> (map (λ (x) (* x 2)) (range 1 5))
'(2 4 6 8)

> (map pow (list 2 3 5) (list 6 4 2))
'(64 81 25)

> (map (λ (a b x) (+ (* a x) b))
       (list 2 3 5) (list 6 4 2) (list 0 1 2))
'(6 7 12)

> (map pow (list 2 3 4 5) (list 6 4 2))
ERROR: map: all lists must have same size;
arguments were: #<procedure:pow> '(2 3 4 5) '(6 4 2)
```

Recall the List Filtering Pattern

`(filterP (list v1 v2 … vn))`



```
(define (filterP xs)
  (if (null? xs)
      null
      (if (P (first xs))
          (cons (first xs) (filterP (rest xs)))
          (filterP (rest xs)))))
```

Express Filtering via Higher-order `my-filter`

```
(define (my-filter pred xs)
  (if (null? xs)
      null
      (if (pred (first xs))
          (cons (first xs)
                (my-filter pred (rest xs)))
          (my-filter pred (rest xs)))))
```

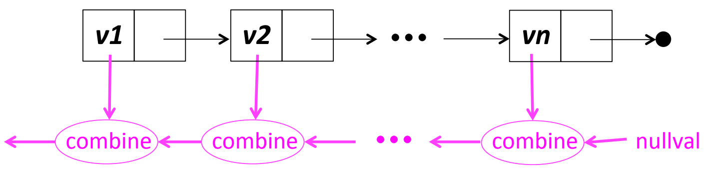
Built-in Racket `filter` function acts just like `my-filter`

`filter` Examples

```
> (filter (λ (x) (> x 0)) (list 7 -2 -4 8 5))

> (filter (λ (n) (= 0 (remainder n 2)))
          (list 7 -2 -4 8 5))

> (filter (λ (xs) (>= (len xs) 2))
          (list (list 2 3) (list 4) (list 5 6 7))

> (filter number?
          (list 17 #t 3.141 "a" (list 1 2) 3/4 5+6i))

> (filter (lambda (binop) (>= (app-3-5 binop)
                              (app-3-5 (flip2 binop))))
          (list sub2 + * avg pow (flip2 pow)))
```

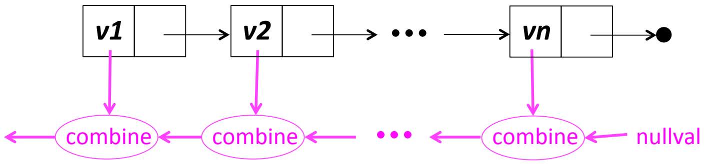
Recall the Recursive List Accumulation Pattern

`(recf (list v1 v2 … vn))`



```
(define (rec-accum xs)
  (if (null? xs)
      nullval
      (combine (first xs)
               (rec-accum (rest xs)))))
```

Express Recursive List Accumulation via Higher-order `my-foldr`



```
(define (my-foldr combine nullval vals)
  (if (null? vals)
      nullval
      (combine (first vals)
               (my-foldr combine
                         nullval
                         (rest vals)))))
```

`my-foldr` Examples

```
> (my-foldr + 0 (list 7 2 4))

> (my-foldr * 1 (list 7 2 4))

> (my-foldr - 0 (list 7 2 4))

> (my-foldr min +inf.0 (list 7 2 4))

> (my-foldr max -inf.0 (list 7 2 4))

> (my-foldr cons (list 8) (list 7 2 4))

> (my-foldr append null
            (list (list 2 3) (list 4)(list 5 6 7)))
```

More `my-foldr` Examples

```
> (my-foldr (λ (a b) (and a b)) #t (list #t #t #t))

> (my-foldr (λ (a b) (and a b)) #t (list #t #f #t))

> (my-foldr (λ (a b) (or a b)) #f (list #t #f #t))

> (my-foldr (λ (a b) (or a b)) #f (list #f #f #f))

;; This doesn't work. Why not?
> (my-foldr and #t (list #t #t #t))
```

Mapping & Filtering in terms of `my-foldr`

```
(define (my-map f xs)
  (my-foldr ???

            ???
            xs))

(define (my-filter pred xs)
  (my-foldr ???

            ???
            xs))
```

Built-in Racket `foldr` Function Folds over Any Number of Lists

```
> (foldr + 0 (list 7 2 4))
13
> (foldr (lambda (a b sum) (+ (* a b) sum))
         0
         (list 2 3 4)
         (list 5 6 7))
56
> (foldr (lambda (a b sum) (+ (* a b) sum))
         0
         (list 1 2 3 4)
         (list 5 6 7))
ERROR: foldr: given list does not have the same size
as the first list: '(5 6 7)
```

More `foldr` Examples

```
> (foldr + 0 (list 7 2 4))
13
> (foldr (lambda (a b sum) (+ (* a b) sum))
         0
         (list 2 3 4)
         (list 5 6 7))
56
> (foldr (lambda (a b sum) (+ (* a b) sum))
         0
         (list 1 2 3 4)
         (list 5 6 7))
ERROR: foldr: given list does not have the same size
as the first list: '(5 6 7)
```

Problematic for `foldr`

`(locallyBig nums)` returns a new list that keeps all nums that are bigger than the following num. It always keeps the last num.

```
> (locallyBig '(7 5 3 9 8))
'(7 5 9 8)

> (locallyBig '(2 7 5 3 9 8))
'(7 5 9 8)

> (locallyBig '(4 2 7 5 3 9 8))
'(4 7 5 9 8)
```

`locallyBig` cannot be defined by fleshing out the following template. Why not?

```
(define (locallyBig nums)
   (foldr <combiner> <nullvalue> nums))
```

`locallyBig` with `foldr`

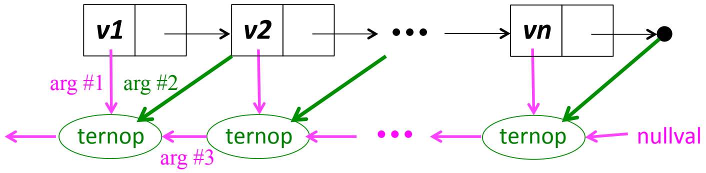
`locallyBig` needs (1) next number as well as (2) list from below.
With `foldr`, we can provide both #1 and #2, and then return #2 at end

```
(define (locallyBig nums)
  (second
    (foldr (λ (thisNum nextNum&locallyBigRest)
             (let ((nextNum (first nextNum&locallyBigRest))
                   (locallyBigRest
                     (second nextNum&locallyBigRest)))
               (list thisNum ; #1: nextNum for elt to left
                     ; #2: list from below
                     (if (> thisNum nextNum)
                         (cons thisNum locallyBigRest)
                         locallyBigRest))))
           (list -inf.0 ; #1 initial nextNum
                 '())   ; #1 initial list
           nums)))
```

`foldr-ternop`: more info for combiner

In cases like `locallyBig`, helps for combiner to also take rest of list.

```
(foldr-ternop ternop nullval (list v1 v2 … vn))
```



```
(define (foldr-ternop ternop nullval vals)
  (if (null? vals)
      nullval
      (ternop (first vals) ; arg #1
              (rest vals) ; extra arg # 2 to ternop
              ; arg #3
             (foldr-ternop ternop nullval (rest vals)))))
```

`locallyBig` with `foldr-ternop`

```
(define (locallyBigTernop nums)
  (foldr-ternop
      (λ (thisNum restNums locallyBigRest)
        (if (null? restNums)
            (list thisNum) ; Always include last num in nums
            (let ((nextNum (first restNums))) ; Key info from
                                              ; extra arg
              (if (> thisNum nextNum)
                  (cons thisNum locallyBigRest)
                  locallyBigRest))))
      '()
      nums))

> (locallyBigTernop '(4 2 7 5 3 9 8))
'(4 7 5 9 8)
```