Higher-Order List Functions in Racket

CS251 Programming Languages
Fall 2018, Lyn Turbak

Department of Computer Science
Wellesley College

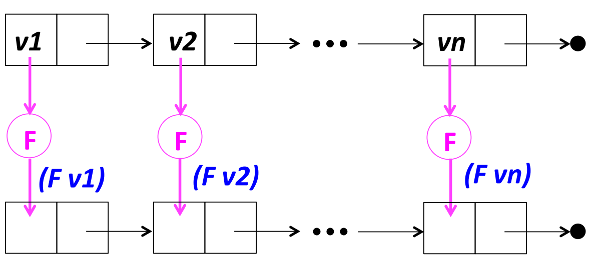
Higher-order List Functions

A function is **higher-order** if it takes another function as an input and/or returns another function as a result. E.g. `app-3-5`, `make-linear-function`, `flip2` from the previous lecture

We will now study **higher-order list functions** that capture the recursive list processing patterns we have seen.

Recall the List Mapping Pattern

`(mapF (list v1 v2 … vn))`




```
(define (mapF xs)
  (if (null? xs)
      null
      (cons (F (first xs))
            (mapF (rest xs)))))
```


Express Mapping via Higher-order `my-map`

```
(define (my-map f xs)
  (if (null? xs)
      null
      (cons (f (first xs))
            (my-map f (rest xs)))))
```

my-map Examples

```
> (my-map (λ (x) (* 2 x)) '(7 2 4))


> (my-map first '((2 3) (4) (5 6 7)))


> (my-map (make-linear-function 4 7) '(0 1 2 3))


> (my-map app-3-5 (list sub2 + avg pow (flip2 pow)
                        make-linear-function))
```


map-scale

Define `(map-scale n nums)`,which returns a list that results from scaling each number in `nums` by `n`.

```
> (map-scale 3 '(7 2 4))
'(21 6 12)

> (map-scale 6 (range 0 5))
'(0 6 12 18 24)
```


Currying

A curried binary function takes one argument at a time.

```
(define (curry2 binop)
   (λ (x) (λ (y) (binop x y))))

(define curried-mul (curry2 *))

> ((curried-mul 5) 4)

> (my-map (curried-mul 3) '(1 2 3))

> (my-map ((curry2 pow) 4) '(1 2 3))

> (my-map ((curry2 (flip2 pow)) 4) '(1 2 3))

> (define LOL '((2 3) (4) (5 6 7)))

> (my-map ((curry2 cons) 8) LOL)

> (my-map (                         8) LOL) ; fill in the blank
  '((2 3 8) (4 8) (5 6 7 8))
```

Haskell Curry

Mapping with binary functions

```
(define (my-map2 binop xs ys)
  (if (or (null? xs) (null? ys)) ; design decision:
                                 ; result has length of
                                 ; shorter list
      null
      (cons (binop (first xs) (first ys))
            (my-map2 binop (rest xs) (rest ys)))))
```

```
> (my-map2 pow '(2 3 5) '(6 4 2))
'(64 81 25)

> (my-map2 cons '(2 3 5) '(6 4 2))
'((2 . 6) (3 . 4) (5 . 2))

> (my-map2 + '(2 3 4 5) '(6 4 2))
'(8 7 6)
```

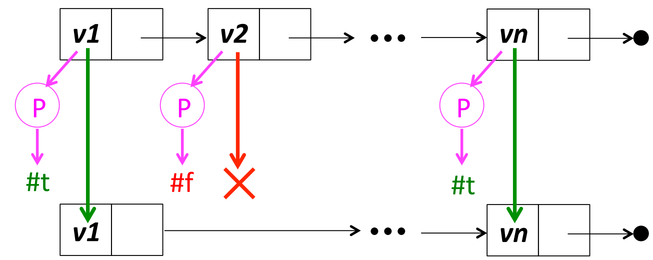
Built-in Racket `map` Function Maps over Any Number of Lists

```
> (map (λ (x) (* x 2)) (range 1 5))
'(2 4 6 8)

> (map pow '(2 3 5) '(6 4 2))
'(64 81 25)

> (map (λ (a b x) (+ (* a x) b))
       '(2 3 5) '(6 4 2) '(0 1 2))
'(6 7 12)

> (map pow '(2 3 4 5) '(6 4 2))
ERROR: map: all lists must have same size;
arguments were: #<procedure:pow> '(2 3 4 5) '(6 4 2)
```

Racket makes different design decision than my-map2: generate error when lists have different length

Recall the List Filtering Pattern

(filter**P** (list ***v1 v2*** … ***vn***))




```
(define (filterP xs)
  (if (null? xs)
      null
      (if (P (first xs))
          (cons (first xs) (filterP (rest xs)))
          (filterP (rest xs)))))
```


Express Filtering via Higher-order `my-filter`

```
(define (my-filter pred xs)
  (if (null? xs)
      null
      (if (pred (first xs))
          (cons (first xs)
                (my-filter pred (rest xs)))
          (my-filter pred (rest xs)))))
```

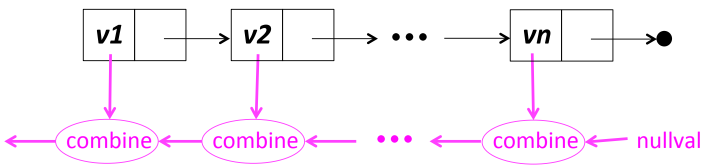
Built-in Racket `filter` function acts just like `my-filter`

`filter` Examples

```
> (filter (λ (x) (> x 0)) '(7 -2 -4 8 5))

> (filter (λ (n) (= 0 (remainder n 2)))
          '(7 -2 -4 8 5))

> (filter (λ (xs) (>= (len xs) 2))
          '((2 3) (4) (5 6 7))

> (filter number?
          '(17 #t 3.141 "a" (1 2) 3/4 5+6i))

> (filter (lambda (binop) (>= (app-3-5 binop)
                              (app-3-5 (flip2 binop))))
          (list sub2 + * avg pow (flip2 pow)))
```

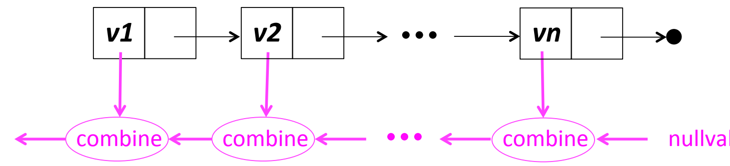
Recall the Recursive List Accumulation Pattern

(recf (list *v1 v2* … *vn*))




```
(define (rec-accum xs)
  (if (null? xs)
      nullval
      (combine (first xs)
               (rec-accum (rest xs)))))
```


Express Recursive List Accumulation via Higher-order `my-foldr`




```
(define (my-foldr combine nullval vals)
  (if (null? vals)
      nullval
      (combine (first vals)
               (my-foldr combine
                         nullval
                         (rest vals)))))
```


`my-foldr` Examples

it's your turn

```
> (my-foldr + 0 '(7 2 4))

> (my-foldr * 1 '(7 2 4))

> (my-foldr - 0 '(7 2 4))

> (my-foldr min +inf.0 '(7 2 4))

> (my-foldr max -inf.0 '(7 2 4))

> (my-foldr cons '(8) '(7 2 4))

> (my-foldr append null '((2 3) (4) (5 6 7)))
```


More `my-foldr` Examples

it's your turn

```
> (my-foldr (λ (a b) (and a b)) #t (list #t #t #t))

> (my-foldr (λ (a b) (and a b)) #t (list #t #f #t))

> (my-foldr (λ (a b) (or a b)) #f (list #t #f #t))

> (my-foldr (λ (a b) (or a b)) #f (list #f #f #f))

;; This doesn't work. Why not?
> (my-foldr and #t (list #t #t #t))
```

Your turn: sumProdList

Define `sumProdList` (from scope lecture) in terms of `foldr`.
Is `let` necessary here like it was in scoping lecture?

```
(sumProdList '(5 2 4 3)) -> (14 . 120)
(sumProdList '()) -> (0 . 1)
```

```
(define (sumProdList nums)
  (foldr                          ; combiner



                                  ; nullval
         nums))
```


Mapping & Filtering in terms of `my-foldr`

```
(define (my-map f xs)
  (my-foldr                          ; combiner



                                     ; nullval
             xs))

(define (my-filter pred xs)
  (my-foldr                          ; combiner



                                     ; nullval
             xs))
```


Built-in Racket `foldr` Function Folds over Any Number of Lists

```
> (foldr + 0 '(7 2 4))
13
> (foldr (lambda (a b sum) (+ (* a b) sum))
         0
         '(2 3 4)
         '(5 6 7))
56
> (foldr (lambda (a b sum) (+ (* a b) sum))
         0
         '(1 2 3 4)
         '(5 6 7))
ERROR: foldr: given list does not have the same size
as the first list: '(5 6 7)
```

Same design decision as in map

Problematic for `foldr`

`(keepBiggerThanNext nums)` returns a new list that keeps all nums that are bigger than the following num. It never keeps the last num.

```
> (keepBiggerThanNext '(7 5 3 9 8))
'(7 5 9)

> (keepBiggerThanNext '(2 7 5 3 9 8))
'(7 5 9)

> (keepBiggerThanNext '(4 2 7 5 3 9 8))
'(4 7 5 9)
```

`keepBiggerThanNext` cannot be defined by fleshing out the following template. Why not?

```
(define (keepBiggerThanNext nums)
   (foldr <combiner> <nullvalue> nums))
```

`keepBiggerThanNext` with `foldr`

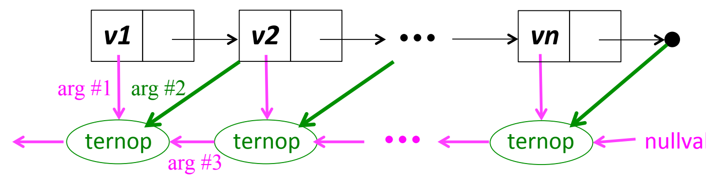
`keepBiggerThanNext` needs (1) next number and (2) list result from below.
With `foldr`, we can provide both #1 and #2, and then return #2 at end

```
(define (keepBiggerThanNext nums)
  (second
    (foldr (λ (thisNum nextNum&subResult)
             (let {[nextNum (first nextNum&subResult)]
                   [subResult (second nextNum&subResult)]}
               (list thisNum ; becomes nextNum for elt to left
                     (if (> thisNum nextNum)
                         (cons thisNum subResult) ; keep
                         subResult))))            ; don't keep
           (list +inf.0 '()) ; +inf.0 guarantees last num
                             ; in nums won't be kept
           nums)))
```


`foldr-ternop`: more info for combiner

In cases like `keepBiggerThanNext`, it helps for the combiner
to also take rest of list as an extra arg

`(foldr-ternop` ***`ternop`*** *`nullval`* `(list` *`v1 v2`* … *`vn`*`))`




```
(define (foldr-ternop ternop nullval vals)
  (if (null? vals)
      nullval
      (ternop (first vals) ; arg #1
              (rest vals) ; extra arg # 2 to ternop
              ; arg #3
              (foldr-ternop ternop nullval (rest vals)))))
```


`keepBiggerThanNext` with `foldr-ternop`

```
(define (keepBiggerThanNext nums)
  (foldr-ternop
                                        ; combiner





                                         ; nullval
     nums))

> (keepBiggerThanNext '(4 2 7 5 3 9 8))
'(4 7 5 9)
```