



## Alternative Evaluation Orders: Delay and laziness

*When are expressions evaluated?*

*Bonus: memoization*

<https://cs.wellesley.edu/~cs251/f19/>

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## Eager evaluation: arguments first

*call-by-value semantics*

When do arguments/subexpressions evaluate (ML, Racket)?

- Function arguments: once, *before* calling function
- Conditional branches: only one branch, *after* checking condition

not eager...

```
fun iff y x y z =
  if x then y else z
```

```
fun facty n =
  iff (n = 0)
    1
    (n * (facty (n - 1)))
```

What's wrong?

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## Delayed evaluation with thunks

*explicit emulation of lexically-scoped call-by-name semantics*

**Thunk** `fn () => e`

- **n.** a zero-argument function used to delay evaluation
- **v.** to create a thunk from an expression:  
"think the expression"

No new language features.

```
fun if_by_name x y z =
  if x () then y () else z ()
```

Type?

```
fun fact n =
  if_by_name (fn () => n = 0)
    (fn () => 1)
    (fn () => n * (fact (n - 1)))
```

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See code examples

## Thunk: evaluate when value needed

*explicit emulation of lexically-scoped call-by-name semantics*

```
fun f1 th =
  if ... then 7 else ... th() ...
```

```
fun f2 th =
  if ... then 7 else th() + th()
```

```
fun f3 th =
  let val v = th ()
  in if ... then 7 else v + v end
```

```
fun f4 th =
  if ... then 7 else
  let val v = th () in v + v end
```

- # evaluations?
- Faster?  
Slower?
- Side effects?

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## Lazy evaluation: first time *value* is needed

*call-by-need semantics*

Argument/subexpression **evaluated zero or one times**,  
no earlier than first time result is actually needed.

**Result reused** (not recomputed) if needed again *anywhere*.

Benefits of delayed evaluation, with minimized costs.

Explicit laziness with *promises*:

- Promise.delay (fn () => x \* f x)
- Promise.force p

## Promises: explicit laziness

(a.k.a. *suspensions*)

```
signature PROMISE =
sig

  (* Type of promises for 'a. *)
  type 'a t

  (* Take a thunk for an 'a and
     make a promise to produce an 'a. *)
  val delay : (unit -> 'a) -> 'a t

  (* If promise not yet forced, call thunk and save.
     Return saved thunk result. *)
  val force : 'a t -> 'a

end
```

See code examples

## Promises: delay and force

(a.k.a. *suspensions*)

```
structure Promise :> PROMISE =
struct
  datatype 'a promise = Thunk of unit -> 'a
                        | Value of 'a

  type 'a t = 'a promise ref

  fun delay thunk = ref (Thunk thunk)

  fun force p =
    case !p of
      Value v => v
    | Thunk th =>
        let val v = th ()
            val _ = p := Value v
        in v end

end
```

Limited mutation  
hidden in ADT.

## Stream: infinite sequence of values

- Cannot make all the elements *now*.
- Make one when asked, delay making the rest.
- Interface/idiom for **division of labor**:
  - **Stream producer**
  - **Stream consumer**
  - Interleave production / consumption in *time*, but *not in code*.
- Examples:
  - UI events
  - UNIX pipes: `git diff delay.sml | grep "thunk"`
  - Sequential logic circuit updates (CS 240)

## Streams in ML: false~~start~~

Let a **stream** be a thunk that, *when called*, returns a pair of

- the next element; and
- the rest of the stream.

```
fn () => (next_element, next_thunk)
```

Given stream *s*, get elements:

- First: `let val (v1,s1) = s ()`
- Second: `val (v2,s2) = s1 ()`
- Third: `val (v3,s3) = s2 () ...`

Type of *s*? *s1*?  
*s2*? *s3*? ...?

## Streams in ML: recursive types

Single-constructor datatype allows recursive type:

```
datatype 'a scons = Scons of 'a * (unit -> 'a scons)
```

```
type 'a stream = unit -> 'a scons
```

Type of *s*? *s1*?  
*s2*? *s3*? ...?

Given a stream *s*:

- First: `let val Scons(v1,s1) = s ()`
- Second: `val Scons(v2,s2) = s1 ()`
- Third: `val Scons(v3,s3) = s2 ()`
- ...

## Stream consumers

Find index of first element in stream for which *f* returns true.

```
fun firstindex f stream =  
  let fun consume stream acc =  
        let val Scons (v,s) = stream ()  
        in  
          if f v  
          then acc  
          else consume s (acc + 1)  
        end  
  in consume stream 0 end  
  
: ('a -> bool) -> 'a stream -> int
```

## Stream producers

```
fun ones () = Scons (1,ones)  
val rec ones = fn () => Scons (1,ones)
```

Create next thunk via **delayed recursion!**

- Return a thunk that , when called, calls the outer function recursively.

```
val nats =  
  let fun f x = Scons (x, fn () => f (x + 1))  
  in fn () => f 0 end  
  
val powers2 =  
  let fun f x = Scons (x, fn () => f (x * 2))  
  in fn () => f 1 end
```

## Getting it wrong

Tries to use a variable before it is defined.

```
val ones_bad = Scons (1, ones_bad)
```

Would call `ones_worse` recursively *immediately* (infinitely).  
Does not type-check.

```
fun ones_worse () = Scons (1, ones_worse ())
```

**Correct:** thunk that returns Scons of value and stream (thunk).

```
fun ones () = Scons (1, ones)  
val rec ones = fn () => Scons (1, ones)
```

## Bonus: Lazy by default?

**ML:**

- Eager evaluation. Explicitly emulate laziness when needed (promises).
- Immutable data, bindings. Explicit mutable cells when needed (refs).
- Side effects anywhere.

**Pros:** avoid unnecessary work, build elegant infinite data structures.

**Cons:** difficult to control/predict evaluation order:

- Space usage: when will environments become unreachable?
- Side-effect ordering: when will effects execute?

**Haskell:** canonical real-world example

- Non-strict evaluation, except pattern-matching. Explicit strictness when needed.
- Usually implemented as lazy evaluation.
- Immutable everything. Emulate mutation/state when needed.
- Side effects banned/restricted/emulated.

## Bonus: Memoization

see `memo.sml`

Not delayed evaluation, but...

- Promises (call-by-need) are memoized thunks (call-by-name), though memoization is more general (multiple arguments).
- Can use an indirect recursive style similar to streams (without delay)
  - Actually fixpoint...

Basic idea:

- Save results of expensive pure computations in mutable cache.
- Reuse earlier computed results instead of recomputing.
- Even for recursive calls.

Benefits:

- Save time when recomputing.
- Can reduce exponential recursion costs to linear (and amortized by repeated calls with same arguments).

See also: dynamic programming (CS 231)