WELLESLEY

Alternative Evaluation Orders:

Delay and laziness

When are expressions evaluated?

Bonus: memoization

Eager evaluation: arguments first

call-by-value semantics

When do arguments/subexpressions evaluate (ML, Racket)?

- Function arguments: once, *before* calling function
- Conditional branches: only one branch, *after* checking condition

not eager...

```
fun iffy x y z =
  if x then y else z

fun facty n =
  iffy (n = 0)
       1
       (n * (facty (n - 1)))
```

What's wrong?

Delayed evaluation with *thunks*

explicit emulation of lexically-scoped call-by-name semantics

Thunk `fn () => e`

- *n.* a zero-argument function used to delay evaluation
- *v.* to create a thunk from an expression:
  "thunk the expression"

No new language features.

Type?

```
fun if_by_name x y z =
  if x () then y () else z ()

fun fact n =
  if_by_name (fn () => n = 0)
             (fn () => 1)
             (fn () => n * (fact (n - 1)))
```

Thunk: evaluate when value needed

explicit emulation of lexically-scoped call-by-name semantics

```
fun f1 th =
  if … then 7 else … th() …

fun f2 th =
  if … then 7 else th() + th()

fun f3 th =
  let val v = th ()
  in if … then 7 else v + v end

fun f4 th =
  if … then 7 else
  let val v = th () in v + v end
```

- # evaluations?
- Faster? Slower?
- Side effects?

Lazy evaluation: first time *value* is needed

call-by-need semantics

Argument/subexpression **evaluated zero or one times,** no earlier than first time result is actually needed.

Result reused (not recomputed) if needed again ***anywhere***.

Benefits of delayed evaluation, with minimized costs.

Explicit laziness with ***promises:***

- `Promise.delay (fn () => x * f x)`
- `Promise.force p`

Promises: explicit laziness

(a.k.a. suspensions)

```
signature PROMISE =
sig

  (* Type of promises for 'a. *)
  type 'a t

  (* Take a thunk for an 'a and
     make a promise to produce an 'a. *)
  val delay : (unit -> 'a) -> 'a t

  (* If promise not yet forced, call thunk and save.
     Return saved thunk result. *)
  val force : 'a t -> 'a

end
```

See code examples

Promises: delay and force

(a.k.a. suspensions)

```
structure Promise :> PROMISE =
struct
  datatype 'a promise = Thunk of unit -> 'a
                      | Value of 'a

  type 'a t = 'a promise ref

  fun delay thunk = ref (Thunk thunk)

  fun force p =
      case !p of
          Value v => v
        | Thunk th =>
            let val v = th ()
                val _ = p := Value v
            in v end
end
```

Limited mutation hidden in ADT.

Stream: infinite sequence of values

- Cannot make all the elements *now*.
- Make one when asked, delay making the rest.

- Interface/idiom for **division of labor**:
  - **Stream producer**
  - **Stream consumer**
  - Interleave production / consumption in *time*, but *not in code*.

- Examples:
  - UI events
  - UNIX pipes: `git diff delay.sml | grep "thunk"`
  - Sequential logic circuit updates (CS 240)

Streams in ML: false ~~start~~

Let a ***stream*** be a thunk that, *when called,* returns a pair of

- the next element; and
- the rest of the stream.

```
fn () => (next_element, next_thunk)
```

Given stream `s`, get elements:

- First: `let val (v1,s1) = s ()`
- Second: `val (v2,s2) = s1 ()`
- Third: `val (v3,s3) = s2 () ...`

Type of s? s1? s2? s3? ...?

Streams in ML: recursive types

Single-constructor datatype allows recursive type:

```
datatype 'a scons = Scons of 'a * (unit -> 'a scons)

type 'a stream = unit -> 'a scons
```

Type of s? s1? s2? s3? ...?

Given a stream `s`:

- First: `let val Scons(v1,s1) = s  ()`
- Second: `val Scons(v2,s2) = s1 ()`
- Third: `val Scons(v3,s3) = s2 ()`

...

Stream consumers

Find index of first element in `stream` for which `f` returns `true`.

```
fun firstindex f stream =
    let fun consume stream acc =
            let val Scons (v,s) = stream ()
            in
                if f v
                then acc
                else consume s (acc + 1)
            end
    in consume stream 0 end

: ('a -> bool) -> 'a stream -> int
```

Stream producers

```
fun ones () = Scons (1,ones)
val rec ones = fn () => Scons (1,ones)
```

Create next thunk via **delayed recursion!**

- Return a thunk that , when called, calls the outer function recursively.

```
val nats =
    let fun f x = Scons (x, fn () => f (x + 1))
    in fn () => f 0 end

val powers2 =
    let fun f x = Scons (x, fn () => f (x * 2))
    in fn () => f 1 end
```

Getting it wrong

Tries to use a variable before it is defined.

```
val ones_bad = Scons (1, ones_bad)
```

Would call `ones_worse` recursively *immediately* (infinitely).
Does not type-check.

```
fun ones_worse () = Scons (1, ones_worse ())
```

Correct: thunk that returns Scons of value and stream (thunk).

```
fun ones () = Scons (1, ones)
val rec ones = fn () => Scons (1, ones)
```

Bonus: Lazy by default?

ML:

- Eager evaluation. Explicitly emulate laziness when needed (promises).
- Immutable data, bindings. Explicit mutable cells when needed (refs).
- Side effects anywhere.

Pros: avoid unnecessary work, build elegant infinite data structures.

Cons: difficult to control/predict evaluation order:

- Space usage: when will environments become unreachable?
- Side-effect ordering: when will effects execute?

Haskell: canonical real-world example

- Non-strict evaluation, except pattern-matching. Explicit strictness when needed.
- Usually implemented as lazy evaluation.
- Immutable everything. Emulate mutation/state when needed.
- Side effects banned/restricted/emulated.

Bonus: Memoization

see memo.sml

Not delayed evaluation, but…

- Promises (call-by-need) are memoized thunks (call-by-name), though memoizaiton is more general (multiple arguments).
- Can use an indirect recursive style similar to streams (without delay)
  - Actually fixpoint…

Basic idea:

- Save results of expensive pure computations in mutable cache.
- Reuse earlier computed results instead of recomputing.
- Even for recursive calls.

Benefits:

- Save time when recomputing.
- Can reduce exponential recursion costs to linear (and amortized by repeated calls with same arguments).

See also: dynamic programming (CS 231)