CS 251 Part 1: How to Program

CS 251 Fall 2019
Principles of Programming Languages
Ben Wood
WELLESLEY

Defining Racket: Expressions and Bindings

via the meta-language of PL definitions

Topics / Goals

1. Basic language forms and evaluation model.
2. Foundations of defining syntax and semantics.
   – Informal descriptions (English)
   – Formal descriptions (meta-language):
     - Grammars for syntax.
     - Judgments, inference rules, and derivations for big-step operational semantics.
3. Learn Racket. *(an opinionated subset)*
   – Not always idiomatic or the full story. Setup for transition to Standard ML.

From AI to language-oriented programming

LISP: List Processing language, 1950s-60s, MIT AI Lab.
Advice Taker: represent logic as data, not just as a program.
Metaprogramming and programs as data:
- Symbolic computation (not just number crunching)
- Programs that manipulate logic (and run it too)

Scheme: child of Lisp, 1970s, MIT AI Lab.
Still motivated by AI applications, became more "functional" than Lisp.
Important design changes/additions/cleanup:
- simpler naming and function treatment
- lexical scope
- first-class continuations
- tail-call optimization, …

Racket: child of Scheme, 1990s-2010s, PLT group.
Revisions to Scheme for:
- Rapid implementation of new languages.
- Education.

Became *Racket* in 2010.

Defining Racket

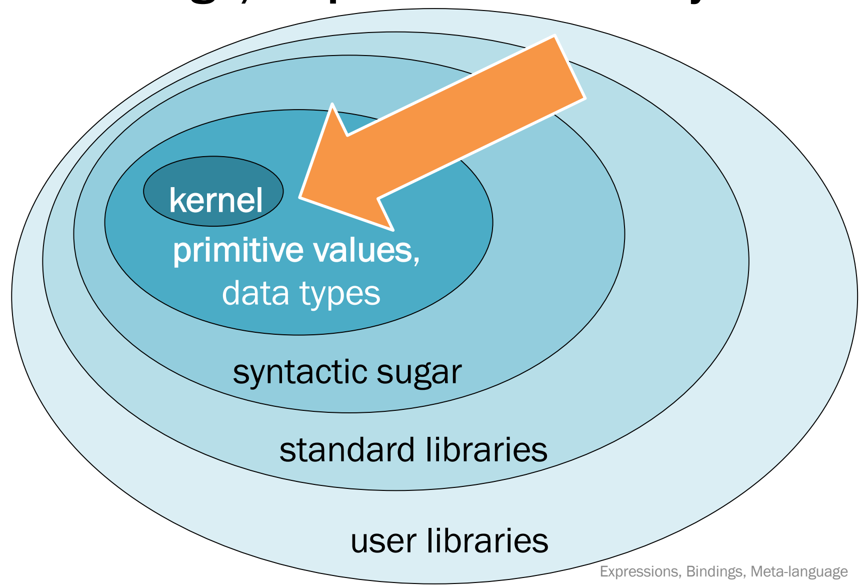
To define each new language feature:

- Define its **syntax.**
  How is it written?
- Define its **dynamic semantics** as **evaluation rules.**
  How is it evaluated?

Features

1. **Expressions**
   - A few today, more to come.
2. Bindings
3. That's all!
   - A couple more advanced features later.

PL design/implementation: layers



Values

Expressions that cannot be evaluated further.

Syntax:

Numbers:	`251`	`240`	`301`
Booleans:	`#t`	`#f`	
...			

Evaluation:

Values evaluate to themselves.

Addition expression

Syntax: `(+ e1 e2)`

- Parentheses required: no extras, no omissions.
- ***e1*** and ***e2*** stand in for *any expressions.*
- Note *prefix* notation.

Note recursive structure!

Examples:

`(+ 251 240)` `(+ (+ 251 240) 301)`

`(+ #t 251)`

Addition expression

Not quite!

Syntax: `(+ e1 e2)`

Note recursive structure!

Evaluation:

1. Evaluate ***e1*** to a value `v1`.
2. Evaluate ***e2*** to a value `v2`.
3. Return the arithmetic sum of `v1 + v2`.

Addition expression

Syntax: `(+ e1 e2)`

Evaluation:

Dynamic type checking

1. Evaluate ***e1*** to a value `v1`.
2. Evaluate ***e2*** to a value `v2`.
3. If `v1` and `v2` are numbers then
   return the arithmetic sum of `v1 + v2`.
   Otherwise there is a type error.

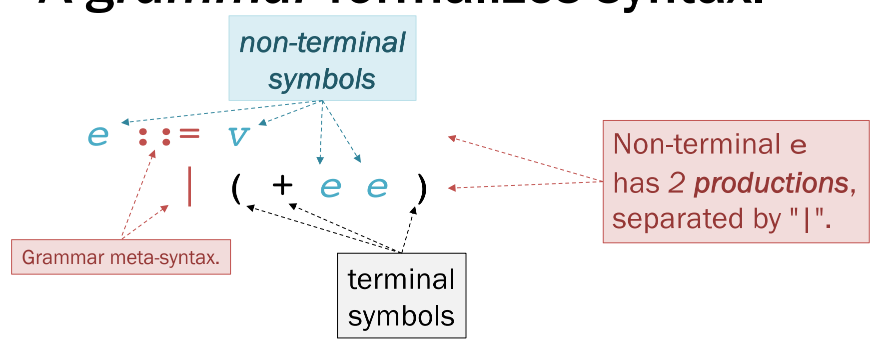
The language of languages

Because it pays to be precise.

Syntax:

– Formal grammar notation
– Conventions for writing syntax patterns

A *grammar* formalizes syntax.



"An expression *e* is one of:

– Any value *v*
– Any addition expression `(+ e e)` of any two expressions"

Racket syntax so far

Expressions

```
e ::= v
    | ( + e e )
```

Literal Values

```
v ::= #f | #t | n
```

Number values

```
n ::= 0 | 1 | 2 | ...
```


Notation conventions

Outside the grammar:

- Use of a non-terminal symbol, such as e, in syntax examples and evaluation rules means *any expression matching one of the productions of* e *in the grammar.*
- Two uses of e in the same context are aliases; they mean *the same expression.*
- Subscripts (or suffixes) distinguish separate instances of a single non-terminal, *e.g.*, e_1, e_2, ..., e_n or e1, e2, ..., en.

The language of languages 🙄

Because it pays to be precise.

Syntax:
- Formal grammar notation
- Conventions for writing syntax patterns

Semantics:
- Judgments:
  - formal assertions, like functions
- Inference rules:
  - implications between judgments, like cases of functions
- Derivations:
  - deductions based on rules, like applying functions

Judgments and rules formalize semantics.

Judgment e ↓ v
means "expression e evaluates to value v."

It is implemented by **inference rules** for different cases:

value rule:

$$\frac{}{v \downarrow v} \text{[value]}$$

addition rule:

if ***e1*** ↓ ***n1***
and ***e2*** ↓ ***n2***
and n is the arithmetic sum of **n1** and **n2**
then *(+ **e1 e2**)* ↓ *n*

$$\frac{e1 \downarrow n1 \quad e2 \downarrow n2 \quad n = n1 + n2}{(+\ e1\ e2) \downarrow n} \text{[add]}$$

...

Inference rules

If all premises hold
then the conclusion holds.

Inference rule notation and meaning

Axiom (no premises)
Bar is optional for axioms.

Conclusion

```
_______ [value]
 v ↓ v
```

Rule name

Number values, not just any values.
Models dynamic type checking.

"*v* is the arithmetic sum of the numbers *n1* and *n2*." (not Racket syntax)

Premises

Conclusion

```
 e1 ↓ n1
 e2 ↓ n2
 n = n1 + n2
_________________ [add]
(+ e1 e2) ↓ n
```

Rule name

Evaluation derivations

An evaluation **derivation** is a "proof" that an expression evaluates to a value using the evaluation rules.

`(+ 3 (+ 5 4)) ↓ 12` by the addition rule because:

- `3 ↓ 3` by the value rule, where `3` is a number
- and `(+ 5 4) ↓ 9` by the addition rule , where `9` is a number, because:
  - `5 ↓ 5` by the value rule, where `5` is a number
  - and `4 ↓ 4` by the value rule, where `4` is a number
  - and `9` is the sum of `5` and `4`
- and `12` is the sum of `3` and `9` .

Evaluation derivations

Adding vertical bars helps clarify nesting.

```
| 3 ↓ 3 [value]
|  | 5 ↓ 5 [value]
|  | 4 ↓ 4 [value]
|  | 9 = 5 + 4
|  |____________ [add]
| (+ 5 4) ↓ 9
| 12 = 3 + 9
|________________ [add]
(+ 3 (+ 5 4)) ↓ 12
```

Rules defining the evaluation judgment

```
e ↓ v

_______ [value]
 v ↓ v

 e1 ↓ n1
 e2 ↓ n2
 n = n1 + n2
_______________ [add]
(+ e1 e2) ↓ n
```


Errors are modeled by "stuck" derivations.

How to evaluate
`(+ #t (+ 5 4))?`

```
| #t ↓ n  X
|  | 5 ↓ 5  [value]
|  | 4 ↓ 4  [value]
|  | 9 = 5 + 4
|  |____________ [add]
| (+ 5 4) ↓ 9
|
|-------------------- [add]
```

Stuck. Can't apply the [add] rule because there is no rule that allows `#t` to evaluate to a number.

How to evaluate
`(+ (+ 1 2) (+ 5 #f))?`

```
 | 1 ↓ 1 [value]
 | 2 ↓ 2 [value]
 | 3 = 1 + 2
 |____________ [add]
 (+ 1 2) ↓ 3
| 5 ↓ 5 [value]
| #f ↓ n  X
|-------------------- [add]
```

Stuck. Can't apply the [add] rule because there is no rule that allows `#t` to evaluate to a number.

Other number expressions

Similar syntax and evaluation for:

`+ - * / quotient < > <= >= =`

Some small differences.

Build syntax and evaluation rules for: `quotient` and `>`

Conditional `if` expressions

Syntax: `(if e1 e2 e3)`

Evaluation:

1. Evaluate `e1` to a value `v1`.
2. If `v1` is not the value `#f` then
   evaluate `e2` and return the result
   otherwise
   evaluate `e3` and return the result

Evaluation rules for `if` expressions.

```
  e1 ↓ v1
  e2 ↓ v2
  v1 is not #f
─────────────────────
(if e1 e2 e3) ↓ v2
```

[if nonfalse]

e3 is not evaluated!

```
  e1 ↓ #f
  e3 ↓ v3
─────────────────────
(if e1 e2 e3) ↓ v3
```

[if false]

e2 is not evaluated!

Notice: at most one of these rules can have its premises satisfied!

`if` expressions

if expressions are ***expressions***.

Racket has no "statements!"

```
(if (< 9 (- 251 240))
    (+ 4 (* 3 2))
    (+ 4 (* 3 3)))

(+ 4 (* 3 (if (< 9 (- 251 240)) 2 3)))

(if (if (< 1 2) (> 4 3) (> 5 6))
    (+ 7 8)
    (* 9 10))
```

`if` expression evaluation

Will either of these expressions result in an error (stuck derivation) when evaluated?

```
(if (> 251 240) 251 (/ 251 0))

(if #f (+ #t 251) 251)
```


Language design choice: `if` semantics

v1 not required to be a Boolean value

```
e1 ↓ v1
e2 ↓ v2
v1 is not #f
-------------------------- [if nonfalse]
(if e1  e2  e3) ↓ v2
```

Alternative design

```
e1 ↓ #t
e2 ↓ v2
-------------------------- [if true]
(if e1  e2  e3) ↓ v2
```


Variables and environments

How do we know the value of a variable?

```
(define x (+ 1 2))
(define y (* 4 x))
(define diff (- y x))
(define test (< x diff))
(if test (+ (* x y) diff) 17)
```

Keep a ***dynamic environment***:

- A sequence of ***bindings*** mapping ***identifier*** (variable name) to ***value***.
- "Context" for evaluation, used in evaluation rules.

More Racket syntax

Bindings

b ::= **(define** *x* *e***)**

Expressions

e ::= *v* | *x* | (+ *e* *e*) | ... | (if *e* *e* *e*)

Literal Values (booleans, numbers)

v ::= #f | #t | *n*

Identifiers (variable names)

x (see valid identifier explanation)

Dynamic environments

Grammar for environment notation:

```
E ::= .              (empty environment)
    | x ↦ v, E       (one binding, rest of environment)
```

where:

- *x* is any legal variable identifier
- *v* is any value

Concrete example:

```
num ↦ 17, absZero ↦ -273, true ↦ #t, .
```

Abstract example:

```
x1 ↦ v1, x2 ↦ v2, …, xn ↦ vn, .
```


Variable reference expressions

Syntax: ***x***

x is any *identifier*

Evaluation rule:

Look up x in the current environment, E, and return the value, v, to which x is bound. If there is no binding for x, a name error occurs.

Search from most to least recent (left to right).

```
E ⊢ e ↓ v
```

Revised expression evaluation judgment uses environment.

```
 E(x) = v
-----------  [var]
 E ⊢ x ↓ v
```


Expression evaluation rules must pass the environment.

```
E ⊢ x ↓ v
```

```
E ⊢ v ↓ v [value]

 E(x) = v
-----------  [var]
 E ⊢ x ↓ v

 E ⊢ e1 ↓ n1
 E ⊢ e2 ↓ n2
 n = n1 + n2
----------------------  [add]
E ⊢(+ e1  e2) ↓ n

 E ⊢ e1 ↓ v1
 E ⊢ e2 ↓ v2
 v1 is not #f
---------------------------  [if nonfalse]
E ⊢ (if e1  e2  e3) ↓ v2

 E ⊢ e1 ↓ #f
 E ⊢ e3 ↓ v3
---------------------------  [if false]
E ⊢ (if e1  e2  e3) ↓ v3
```


Derivation with environments

Let `E = test ↦ #t, diff ↦ 9, y ↦ 12, x ↦ 3`

```
E ⊢ test ↓ #t    [var]
    E ⊢ x ↓ 3    [var]
    E ⊢ 5 ↓ 5    [value]
    ------------------ [mult]
  E ⊢(* x 5) ↓ 15
  E ⊢ diff ↓ 9   [var]
  ---------------------------- [add]
 E ⊢ (+ (* x 5) diff) ↓ 24
------------------------------------------ [if nonfalse]
E ⊢ (if test (+ (* x 5) diff) 17) ↓ 24
```

`define` bindings

Syntax: `(define x e)`

`define` is a keyword, ***x*** is any *identifier,* ***e*** is any expression

Evaluation rule:

1. **Under the current environment,** `E`, evaluate `e` to a value `v`.
2. **Produce a new environment,** `E'`, by extending the current environment, `E`, with the binding `x` ⟼ `v`.

$E \vdash b \Downarrow E'$

$$\frac{E \vdash e \downarrow v \qquad E' = x \mapsto v,\ E}{E \vdash (\texttt{define}\ x\ e) \Downarrow E'}\ [\text{define}]$$

Environment example

```
  ; E0 = .
(define x (+ 1 2))
  ; E1 = x ⟼ 3, .
```
(abbreviated `x ⟼ 3`; write as `x --> 3` in text)
```
(define y (* 4 x))
  ; E2 = y ⟼ 12, x ⟼ 3
```
(most recent binding first)
```
(define diff (- y x))
  ; E3 = diff ⟼ 9, y ⟼ 12, x ⟼ 3
(define test (< x diff))
  ; E4 = test ⟼ #t, diff ⟼ 9, y ⟼ 12, x ⟼ 3
(if test (+ (* x 5) diff) 17)
  ;
```
(environment here is still E4)

Racket identifiers

Most character sequences are allowed as identifiers, except:

- those containing
  - whitespace
  - special characters `( ) [ ] { } " , ' ` ; # | \`
- identifiers syntactically indistinguishable from numbers (e.g., `-45`)

Fair game: `! @ $ % ^ & * . - + _ : < = > ? /`

- `myLongName`, `my_long__name`, `my-long-name`
- `is_a+b<c*d-e?`
- `64bits`

Why are other languages less liberal with legal identifiers?

Big-step vs. small-step semantics

We defined **a big-step operational semantics:** evaluate "all at once"

A **small-step operational semantics** defines step by step evaluation:

```
(- (* (+ 2 3) 9) (/ 18 6))
→ (- (* 5 9) (/ 18 6))
→ (- 45 (/ 18 6))
→ (- 45 3)
→ 42
```

A small-step view helps define evaluation orders later in 251.