Probabilistic Programming
A computation graph is a directed graph where nodes represent operations and variables and edges define the order of computation.
Computation Graphs

\[ c = b + z \]

\[ z = x + y \]

\[ b = a \times a \]

\[ x \]

\[ y \]

\[ a \]
Probabilistic Programming Languages

Probabilistic programming languages are useful for problems that require reasoning under uncertainty.

Key concept: the programs are probability models.
A probability model is a formal representation of a problem that involves non-determinism (randomness).

Three key parts: sample space, events, and probabilities for the events.
Cereal Sampling

Imagine that we draw a single piece of Lucky Charms cereal.

Source: www.village-bakery.com

Source: https://www.walmart.com
Cereal sampling

Imagine that we draw two pieces of Lucky Charms cereal out of a bowl.

Events:
Picking a horseshoe + a rainbow
Picking a rainbow + a letter
Picking two letters
…

Source: www.village-bakery.com
Cereal Sampling

Sample space (set of all outcomes):
horseshoe + rainbow
rainbow + a letter
two letters
two rainbows
two horseshoes
horseshoe + letter
....
Probabilities:
\[
p(\text{horseshoe}) = 0.1 \\
p(\text{rainbow}) = 0.2 \\
p(\text{letter}) = 0.7 \\
p(\text{shooting star}) = 0.0 \\
p(\text{balloon}) = 0.0 \\
\]
...

The probability of the sample space always sums to 1.
Key concept: the programs are probability models.

PPLs have stochastic elements whose values are sampled on every run of the program. The meaning of the program is the probability of every possible execution of the program.
Two main modes: prediction and inference. Prediction uses observed causes to guess unseen results; inference uses observed results to try to understand unseen causes.
From a computation graph point-of-view, prediction is the forward propagation of data through the graph, while inference is the backwards propagation of data.
Figaro

Figaro is a probabilistic programming language that uses Scala syntax.

This is another example of a domain-specific language embedded in a general-purpose language.
val sunnyToday = Flip(0.2)

Flip(0.2) is an instance of Element[Boolean].
val sunnyToday = Flip(0.2)
Every morning, I wake up, lean out my window, and shout a greeting.

When the weather is good, I usually say, “Hello world!” or “Howdy, universe!”.

When the weather is bad, I’m grumpier. Sometimes I say, “Hello world!”, but sometimes I say, “Oh no, not again.”
<table>
<thead>
<tr>
<th><strong>Today’s weather</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunny</td>
<td>0.2</td>
</tr>
<tr>
<td>Not sunny</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Today’s greeting</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>If today's weather is sunny</td>
<td>“Hello, world!”</td>
</tr>
<tr>
<td></td>
<td>“Howdy, universe!”</td>
</tr>
<tr>
<td>If today's weather isn’t sunny</td>
<td>“Hello, world!”</td>
</tr>
<tr>
<td></td>
<td>“Oh no, not again”</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th><strong>Tomorrow’s weather</strong></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>If today's weather is sunny</td>
<td>Sunny</td>
</tr>
<tr>
<td></td>
<td>Not sunny</td>
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Morning greeting application

Let’s see how we would model two days of my morning routine.

There are three tasks that we want our model to be able to do:

1. Predict the greeting today
2. Given an observation of the greeting, infer the weather
3. Learn from an observation of today’s greeting in order to predict tomorrow’s greeting.
Elements

An element is a language construct that represents a process that probabilistically produces a value.

The value of an element isn’t known until the computation graph is run.

Figaro elements let you specify the probabilistic process used to sample a value more explicitly.
Figaro is one of the newest, most powerful probabilistic programming languages, partly because of its strong interface with Scala. Older statistical modeling languages: STAN, BayesianLab
Others include:
Church, WebPPL
Probabilistic programming languages are relatively new (~2000), and we’re still figuring out useful applications for them.

**Example applications:**

- evaluating online game players (Microsoft)
- identifying nuclear test treaty violations (Stuart Russell)
- identifying malware (Charles River Analytics)
- modeling language learning and conversation dynamics (lots of folks, including me!)