



Programming Languages

CS 251
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Recap

Anonymous Functions

A lambda expression is an anonymous function.

`(define (fn))` is really short for `(define fn (lambda))`

`(define (hello-world) (display "hello world!"))`

`(define hello-world (lambda () (display "hello world!")))`



parameters

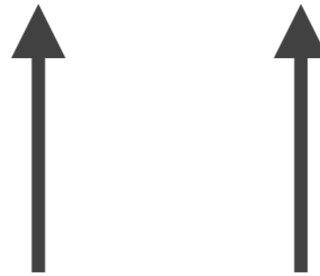


function body

Lambdas

Lambda: anonymous function

```
(lambda (x y) (+ x y))
```



list of arguments

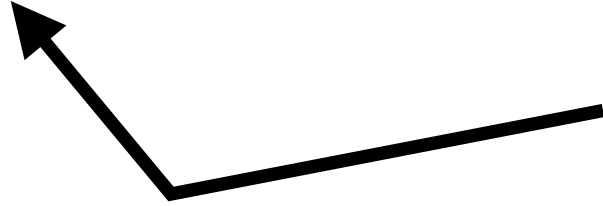
function body

Practice: write an anonymous function that returns the second item in a list.

Review: Local Binding

Normal local binding: bindings are parallel (right-hand side is ignorant of left-hand side)

```
(let ((cat-speak (printf "meow!"))
      (dog-speak (printf "woof!"))
      (unbound (cat-speak))))
```

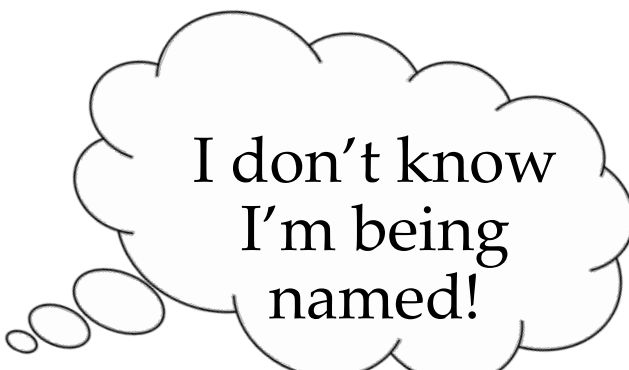


unbound,
going to
throw error

Review: Local Binding

Normal local binding: bindings are parallel
(right-hand side is ignorant of left-hand side)

```
(let ((bound  
      (lambda (x)  
        (if (= x 0)  
            (printf "zero!")  
            (bound (- x 1)))))))
```



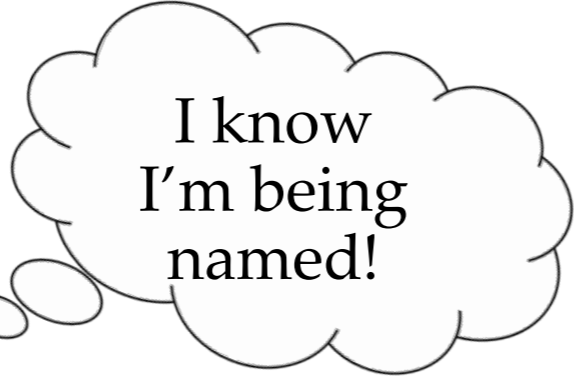
I don't know
I'm being
named!

uh oh! bound is
undefined here,
so we have no
way to call the
function in the
recursive step!


Review: Local Binding

Letrec: names given on the left are known on the right

```
(letrec ((bound
  (lambda (x)
    (if (= x 0)
        (printf "zero!")
        (bound (- x 1)))))))
```



bound by the
left-hand side,
can be called
recursively



String-reverse using letrec

```
(define (reverse str)
  (letrec ((helper
            (lambda (str x)
              (if (= x (string-length str))
                  ""
                  (string-append
                    (helper str (+ x 1))
                    (string (string-ref str x)))))))
    (helper str 0)))
```

define helper function

helper function arguments

base case

recursive call

call helper function

Practice:

Rewrite string-reverse to be tail-recursive.

Formal Semantics

Formal Semantics

- ◆ Gives a way of representing the semantics of the language
- ◆ Helps us reason mathematically about program behavior
- ◆ Basic idea: write rules for how expressions get evaluated
- ◆ Two main styles:
 - ◆ **Big step semantics**: models the evaluation of each expression
 - ◆ **Small step semantics**: models the **step-by-step** evaluation of each expression

Language components

- ◆ **Expressions:** bits of the language

(+ 1 2) “cat” (define (foo n) n)

- ◆ **Values:** expressions that cannot be reduced any further

“cat” (define (foo n) n)

- ◆ **Declarations:** bind variables to values

(let

Big step semantics: values

◆ Syntax:

- Numbers: 0, 1, ...
- Strings: “donut”, “cookie”, ...
- Functions: (define (foo x) x), ...
- Booleans: #t, #f

◆ Evaluation:

- Values evaluate to themselves.

Big step semantics: addition

- ◆ Syntax: $(+ e1 e2)$
 - $e1$ and $e2$ stand for arbitrary expressions
- ◆ Evaluation:
 1. Evaluate $e1$ to a value $v1$.
 2. Evaluate $e2$ to a value $v2$.
 3. Return the arithmetic sum of $v1$ and $v2$.

Big step semantics: addition

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Let's test it out!

Big step semantics: addition

- ◆ Syntax: $(+ e1 e2)$
 - $e1$ and $e2$ stand for arbitrary expressions
- ◆ Evaluation:
 1. Evaluate $e1$ to a value $v1$.
 2. Evaluate $e2$ to a value $v2$.
 3. **If $v1$ and $v2$ are numbers**, return the arithmetic sum of $v1$ and $v2$.
 4. Otherwise, a dynamic type-checking error occurs.

Practice:

Write the big step semantics for string concatenation.

Big step semantics: string concatenation

- ◆ Syntax: (string-append e1 e2)
- ◆ Evaluation:
 1. Evaluate e1 to a value v1.
 2. Evaluate e2 to a value v2.
 3. **If v1 and v2 are strings**, return the concatenation of v1 and v2.
 4. Otherwise, a dynamic type-checking error occurs.

Let's test it out!

Approach 1: change the big step semantics

- ◆ Syntax: (string-append $e_1 \dots e_n$)
- ◆ Evaluation:
 1. For all e from e_1 to e_n , evaluate e to a value v .
 2. **If $v_1 \dots v_n$ are strings**, return the concatenation of $v_1 \dots v_n$.
 3. Otherwise, a dynamic type-checking error occurs.

Approach 2: treat it as “syntactic sugar”

- ◆ Syntax: (string-append e1 e2 e3)

- ◆ Evaluation:

(string-append e1 e2 e3) evaluates to

(string-append (string-append e1 e2) e3).

Big step semantics: notation

- ◆ The \downarrow symbol is used to express evaluation assertions:

$e \downarrow v$ is pronounced “e evaluates to v”.

- ◆ Translating our rules:

Values: $v \downarrow v$

Addition: If:

- $e_1 \downarrow v_1$,
- $e_2 \downarrow v_2$, and
- v_1 and v_2 are numbers that sum to v ,

Then $(+ e_1 e_2) \downarrow v$

Big step semantics: notation

We can use this notation to write our semantics even more concisely as a “proof tree.”:

Addition:

$$\frac{\begin{array}{l} e1 \downarrow v1 \\ e2 \downarrow v2 \end{array}}{(+ e1 e2) \downarrow v}$$

where $v1$ and $v2$ are numbers and v is the sum of $v1$ and $v2$



This part is important!

Practice:

Model the derivation of $(+ 5 (+ 10 9))$ in the proof tree format.

First class functions

In Racket, functions are values. This is because Racket has **first class functions**: functions have all the rights and privileges of other values.

Function Bill of Rights:

We the Racketeers hereby declare that functions:

- ◆ Do not need to be named (lambdas)
- ◆ Can be returned by functions
- ◆ Can be arguments to functions

Functions as values

Do we need to do anything special for functions?

No! Like other values, functions can be evaluated any further... *until they are applied.*

Syntax: **lambda** (id_1, \dots, id_n) e

parameters

function body

Semantics: $v \downarrow v$

Functions as values

Since functions are values, they need to be handled by our big step semantics for values.

Do we need to do anything special for functions?

Syntax: **(lambda** (id_1, \dots, id_n) e)



parameters

function body

Semantics: ????