

OPTIONAL PROBLEM SET 9
“Due” 5pm Monday, May 13, 2002

This is an *optional* problem set. If you turn in this problem set (or any part of it) it can only help your grade. Even though the problem set is optional, you are required to understand all of the material covered by this problem set for the final exam. Solutions to this problem set will be posted on the evening of Monday, May 13 so that you can study the solutions for the final exam if you desire.

Reading:

- 40 (Laziness) and 41-42 (Control)
- *SICP* 2.4-2.5 (Data-directed Programming) and 3.5 (streams)
- *MLWP*: 5.12—5.20 (Lazy data)
- Hughes’s *Why Functional Programming Matters*.

Submission:

- Your hardcopy submission for Problem 1 should include your paragraph for 1a, as well as your files `sqrt.scm` and `hamming.scm`.
- Your hardcopy submission for Problem 2 should be your file `trees.scm`.
- Your hardcopy submission for Problem 3 should be your file `HaskellFuns.hs`, which should contain (among other things) a function named `sqrt` and a variable named `hamming`.
- Your softcopy submission for this assignment should be your entire `ps9` directory.

Problem 1 [30]: Lazy Data

a [5] In his paper, “Why Functional Programming Matters”, John Hughes argues that lazy evaluation is an essential feature of the functional programming paradigm. Briefly summarize his argument in one paragraph.

b [10] In the file `~/cs251/ps9/sqrt.scm`, translate the Newton-Rhapson square-root example from pp. 27--29 of Hughes’s paper into Scheme using streams. Use your procedure to compute the square root of 2 with tolerances of 1, 0.1, and 0.01.

c [15] In the file `~/cs251/ps9/hamming.scm`, write the following procedures using Scheme streams:

- The `scale` procedure takes a scaling factor and a stream of integers and returns a new stream each of whose elements is a scaled version of the corresponding element of the original list.
- The `merge` procedure takes two streams of integers, each in sorted order, and returns a new stream, also in sorted order, that has all the elements of both input streams. The resulting list should not contain duplicates.
- The *Hamming numbers* are the set of positive integers whose prime factors only include the numbers 2, 3, and 5. For example, the first 15 Hamming numbers are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, and 24. Define an infinite stream named `hamming` that contains all of the Hamming numbers, in order. (*Hint*: use `scale` and `merge` from above.) Using the `take` procedure discussed in class, return a list of the first 52 Hamming numbers.

Problem 2 [50]: Non-Local Exits and Exceptions

Here we consider non-local exits and exceptions in the context of some Scheme procedures for manipulating binary trees. Below is a simple Scheme implementation of a binary tree ADT, along with some sample trees.

```
(define leaf (lambda () '()))
(define leaf? (lambda (thing) (null? thing)))
(define node (lambda (value left right) (list value left right)))
(define value (lambda (node) (first node)))
(define left (lambda (node) (second node)))
(define right (lambda (node) (third node)))

(define tree1 (node 3
                  (node 4
                       (node 2 (leaf) (leaf))
                       (leaf))
                  (node 5 (leaf) (leaf))))

(define tree2 (node 3
                  (node 4
                       (node 0 (leaf) (leaf))
                       (leaf))
                  (node 5 (leaf) (leaf))))

(define tree3 (node 3
                  (node 4
                       (node 'x (leaf) (leaf))
                       (leaf))
                  (node 5 (leaf) (leaf))))

(define tree4 (node 3
                  (node 0 (leaf) (leaf))
                  (node 'x (leaf) (leaf))))

(define tree5 (node 3
                  (node 'x (leaf) (leaf))
                  (node 0 (leaf) (leaf))))
```

The trees `tree1` and `tree2` contain only numeric values, but `tree3`, `tree4`, and `tree5` all contain a non-numeric value (the symbol `x`).

For testing various tree-manipulating functions, we introduce the following function, which applies a given function to each of the five trees defined above:

```
(define test-trees
  (lambda (f)
    (map f (list tree1 tree2 tree3 tree4 tree5))))
```

The following `product` procedure calculates the product of a tree of numbers, but it handles several cases specially:

- If a node with a non-numeric value is encountered, the symbol `non-number` is returned as the product of that node without examining its left and right subtrees. This symbol is propagated as the result of `product` on the entire tree.

- If a node with a zero is encountered, a zero is returned as the product of that node without examining its left and right subtrees.
- If a node has a left subtree whose product is a zero, a zero is returned as the product of that node without examining its right subtree.

```
(define product
  (lambda (tree)
    (if (leaf? tree)
        1
        (let ((v (value tree)))
          (if (not (number? v))
              'non-number
              (if (= v 0)
                  0
                  (let ((left-result (product (left tree))))
                    (if (eq? left-result 'non-number)
                        'non-number
                        (if (= left-result 0)
                            0
                            (let ((right-result (product (right tree))))
                              (if (eq? right-result 'non-number)
                                  'non-number
                                  (* v (* left-result right-result))))))))))))))
```

For example, evaluating `(test-trees product)` yields `(120 0 non-number 0 non-number)`.

You should begin this problem by evaluating `(load "~/cs251/ps9/trees.scm")`. In addition to loading the tree code discussed above, it also loads the file `~/cs251/util/control.scm`, which extends Scheme with the `label`, `jump`, `handle`, `trap`, and `raise` constructs presented in class. Once this file is loaded, you can program in regular Scheme (not a toy language!) using these constructs.

Part a [10]: Non-local Exits

It is clumsy for `product` to perform checks that propagate `non-number` and zero. In a language that supports the `label` and `jump` constructs, such behavior can be expressed more elegantly by using `label` and `jump` to immediately return 0 when a 0 is encountered, or the symbol `non-number` when a non-number is encountered.

In this problem, you should flesh out in `trees.scm` the skeleton of the procedure `product-nonlocal-jump` in that behaves like `product` except that it performs non-local exits for the 0 and non-number cases via `label` and `jump`. As with the list `product` example discussed in class, `product-nonlocal-jump` should be defined in terms of a local recursive procedure `inner`. Your resulting procedure should be able to pass the following test: evaluating `(test-trees product-nonlocal-jump)` should yield `(120 0 non-number 0 non-number)`.

Warning: MIT-Scheme evaluates the arguments to a function from right to left rather than from left to right. Take this into account when writing `product-nonlocal-jump`, as this fact can affect the results you observe.

Part b [10]: Call-with-current-continuation

An alternative to using `label` and `jump` to perform a non-local exit is to use Scheme's built-in `call-with-current-continuation` procedure. Flesh out the `product-nonlocal-cwcc` procedure so that it behaves like `product-nonlocal-jump` but is implemented in terms of `call-with-current-continuation` rather than in terms of `label` and `jump`. Your resulting procedure should be able to pass the following test: evaluating `(test-trees product-nonlocal-cwcc)` should yield `(120 0 non-number 0 non-number)`.

Part c [15]: Continuation-Passing Style

An alternative to using Scheme's implicit continuations to perform a non-local exit is to use explicit continuations via continuation-passing style. Suppose that `product-nonlocal-cps` is defined as follows:

```
(define product-nonlocal-cps
  (lambda (tree)
    (product-cps tree (lambda (v) v))))
```

Here, `product-cps` is a function that finds the product of the numbers in its first argument (a tree) and "returns" the product by invoking its second argument (an explicit continuation) on the product. Your goal in this problem is to flesh out the definition of `product-cps`. You should not use `label`, `jump`, `call-with-current-continuation`, `raise`, `handle`, or `trap` in your definition. Nevertheless, your procedure should return immediately upon encountering a 0 or a non-number.

After you have defined `product-cps`, the enclosing `product-nonlocal-cps` procedure should be able to pass the following test: evaluating `(test-trees product-nonlocal-cps)` should yield `(120 0 non-number 0 non-number)`.

Part d [15]: Exception Handling

The `product` and `product-nonlocal` procedures defined above contain hardwired assumptions about how to handle zeroes and non-numeric values. Suppose we want a version of `product` that always returns `non-number` if the tree argument contains a non-numeric values, regardless of whether it contains any zeroes. Then we must delve into the code for `product` and change the way it handles zeroes.

In a language that supports exception handling, a more flexible approach is to use exceptions to handle special cases like zero and non-numeric values. Consider the following `product-exception` procedure, which is defined in `~/cs251/ps9/trees.scm`.

```
(define product-exception
  (lambda (tree)
    (if (leaf? tree)
        1
        (let ((v (value tree)))
          (if (not (number? v))
              (raise non-number tree)
              (if (= v 0)
                  (raise zero tree)
                  ;; Use LET to explicitly process left subtree before right
```

```

(let ((left-prod (product-exception (left tree))))
  (let ((right-prod (product-exception (right tree))))
    (* v (* left-prod right-prod))))))

```

This procedure raises exceptions for the special cases where the node value is zero or not a number. In both cases, the current node is passed as the argument to the exception handler. This allows the caller of `product-exception` to determine how the special cases should be handled. In particular, different callers can deal with the special cases in different ways.

In the following parts, you will write procedures `product1`, `product2`, and `product3` that handle these cases in three different ways. Each of your `producti` procedures should have the following form:

```

(define producti
  (lambda (tree)
    (<handler1> non-number
      (lambda (tree) <exp1>)
      <handler2> zero
      (lambda (tree) <exp2>)
      (product-exception tree))))

```

where `<handler1>` and `<handler2>` are either `handle` or `trap`, whichever is appropriate.

i. [5]. `product1` has the same behavior as the `product` and `product-nonlocal` procedures above:

```

(test-trees product1)
; Value: (120 0 non-number 0 non-number)

```

ii. [5]. `product2` is like `product` except that for trees containing a non-numeric values, it returns the symbol `non-number` regardless of whether there are any zeroes in the tree.

```

(test-trees product2)
; Value: (120 0 non-number non-number non-number)

```

iii. [5]. `product3` is like `product` except that it treats every non-numeric value as 1 for the purposes of calculating the product.

```

(test-trees product3)
; Value: (120 0 60 0 0)

```

Problem 3 [20]: Haskell

In this problem, you will write some simple functions in Haskell and execute them by using the Hugs interpreter.

1. Write your Haskell code in the file `~/cs251/ps9/HaskellFuns.hs`
2. To use the Hugs interpreter:
 - First execute `cd ~/cs251/ps9` to go to the correct directory.
 - To launch Hugs, execute: `hugs`
 - When in the hugs interpreter, you may do the following:
 - Load a Haskell file via `:l filename`. E.g `:l HaskellFuns.hs`
 - Reload a Haskell file via `:r` (without any arguments). Once you have used `:l` once, you can use `:r` every time thereafter.
 - Evaluate a Haskell expression by typing it in and hitting Return/Enter.
 - To exit Hugs, execute: `:q`

For more information about the Haskell language, consult Simon Thompson's *Haskell: The Craft of Functional Programming* and/or the Haskell materials linked from the CS251 home page.

a [10] Translate the Newton-Rhapson `sqrt` function from Problem 1b to Haskell.

b [10] Translate `hamming` from Problem 1c to Haskell. `hamming` should denote the infinite list of Hamming numbers.

*Problem Set Header Page:
Please make this the first page of your submission.*

CS251 *Optional* Problem Set 9
“Due” 5pm, May 13, 2002

Name:

Date & Time Submitted (*only if late*):

Collaborators (*anyone you collaborated with in the process of doing the problem set*):

*In the **Time** column, please estimate the time you spent on the parts of this problem set. Please try to be as accurate as possible; this information will help me to design future problem sets. I will fill out the **Score** column when grading your problem set.*

| Part | Time | Score |
|--------------------|-------------|--------------|
| General Reading | | |
| Problem 1 [30] | | |
| Problem 2 [50] | | |
| Problem 3 [20] | | |
| Total [100] | | |