Control

CS251 Handout #40
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What is Control?

In program execution, control is characterized by two components:

1. the expression/statement currently being evaluated:
   - CS111: the red control dot.
   - CS240: the program counter.
   - CS251: the argument to `eval` in the substitution model
2. The **continuation** = all pending operations to be performed when the value of current expression is returned:
   - CS111: the pending frames in the Java Execution Model.
   - CS240: the stack of procedure call activation frames.
   - CS251: the context surrounding the current expression in the substitution model

We will call the pair of (1) and (2) a **control point**. All computation is an iteration through control points.

Control Point Example 1

Expression	*Continuation*
`(/ (+ (* 6 5) (- 7 3)) 2)`	k_{top}
$\Rightarrow$ `(+ (* 6 5) (- 7 3))`	k_1=`(`λ `(`v_1`) (`k_{top} `(/` v_1 `2)))`
$\Rightarrow$ `(* 6 5)`	k_2=`(`λ `(`v_2`) (`k_1 `(+` v_2 `(- 7 3))))`
$\Rightarrow$ `(- 7 3)`	k_3=`(`λ `(`v_3`) (`k_1 `(+ 30` v_3`)))`
$\Rightarrow$ `(+ 30 4)`	k_1
$\Rightarrow$ `(/ 34 2)`	k_{top}
$\Rightarrow$ `17`	

Notes:

- Continuations are modeled as single-argument functions.
- k_{top} designates the top-level continuation (eg, prints result).
- The above assumes left-to-right evaluation of arguments. (MIT Scheme evaluates them right-to-left.)

Control Point Example 2: Recursive Factorial

```
(def (fact-rec n)
  (if (= n 0)
      1
      (* n (fact-rec (- n 1)))))
```

Expression	*Continuation*
$\Rightarrow$ `(fact-rec 3)`	k_{top}
$\Rightarrow$ `(fact-rec 2)`	k_1 = `(`λ `(`v_1`) (`k_{top} `(* 3` v_1`)))`
$\Rightarrow$ `(fact-rec 1)`	k_2 = `(`λ `(`v_2`) (`k_1 `(* 2` v_2`)))`
$\Rightarrow$ `(fact-rec 0)`	k_3 = `(`λ `(`v_3`) (`k_2 `(* 1` v_3`)))`
$\Rightarrow$ `(* 1 1)`	k_2
$\Rightarrow$ `(* 2 1)`	k_1
$\Rightarrow$ `(* 3 2)`	k_{top}
$\Rightarrow$ `6`	

Control Point Example 3: Iterative Factorial

```
(def (fact-iter n) (fact-tail n 1))

(def (fact-tail num ans)
  (if (= num 0)
      ans
      (fact-tail (- num 1) (* num ans))))
```

	Expression	*Continuation*
$\Rightarrow$	`(fact-iter 3)`	k_{top}
$\Rightarrow$	`(fact-tail 3 1)`	k_{top}
$\Rightarrow$	`(fact-tail 2 3)`	k_{top}
$\Rightarrow$	`(fact-tail 1 6)`	k_{top}
$\Rightarrow$	`(fact-tail 0 6)`	k_{top}
$\Rightarrow$	`6`	

Note: A function call is **tail recursive** if it does not alter continuation

Control Aspects of Familiar Constructs

- Evaluating nested subexpressions requires choosing an order and remembering what to do next.
  - Argument evaluation order is left-to-right in most language.
  - Evaluation order unspecified in Scheme (right-to-left in MIT-Scheme).
- Sequencing of statements in imperative language.
- Conditionals allow branches in control flow.
- Loops/tail recursion specify iterations.
- Function/procedure call and return:
  - In many execution models (e.g., C, Pascal, Java), calling a procedure pushes an activation frame on the call stack and returning from a procedure pops the activation from from the call stack.
  - In **properly tail-recursive languages** (e.g. Scheme, most ML implementations) stack is pushed by subexpression evaluation and procedure calls act like gotos that pass arguments (see Guy Steele's *The Expensive Procedure Call Myth or Lambda: The Ultimate Goto*).

Altering the Normal Flow of Control

Sometimes want to alter the normal flow of control:

- to immediately stop execution of the program, due to a user request (typing Control-C) or encountering an error. E.g. halt opcode in assembly language; `error` in HOFL, Scheme;
- to return an answer immediately without processing all pending computations. E.g. encountering a zero when finding the product of elements in a list, array, or tree.
- to handle an unusual situation that may need to be handled differently in different contexts (an exception). E.g., division by zero, out-of-bounds array access, unbound variables in environment lookup.

Altering normal flow of control can be very convenient and efficient, but can lead to "spaghetti code". Dijkstra's *Goto Considered Harmful* and the structured programming movement of the 1970s advocated control constructs with one control input and one control output.

Non-local Exits: `return`

In C, C++, and Java, `return` can force early exit of a function/method.

Example (Java): calculating array product. Want to return early if encounter a zero. Also suppose that encountering any negative number should cause the result to be -1.

```
public static int arrayProd (int[] a)
{
    int prod = 1;
    for (int i = 0; i < a.length; i++) {
        if (a[i] == 0)
            return 0;   // Non-local exit from loop
        else if (a[i] < 0) then
            return -1;  // Non-local exit from loop
        else
            prod = a[i] * prod;
    }
    return prod;
}
```

Non-local Exits: `break`

Java has labeled `break` statements for breaking out of a loop and `continue` statement to jump to end of loop. C's unlabeled `break` and `continue` work on innermost enclosing loop.

```
public static int sumArrayProds (int[][] a)
{
    int sum = 0;
    outer:for (int i = 0; i < a.length; i++) {
        int prod = 1;
        inner:for (int j = 0; i < a[i].length; j++) {
            if (a[i][j] < 0)  // return current sum
                break outer;  // on negative num
            else if (a[i][j] == 0)
                prod = 0; break inner;
            // Alternatively: continue outer;
            else
                prod = a[i][j] * prod;
        }
        sum = sum + prod;
    }
    return sum;
}
```


Non-Local Exits: `goto`

In Pascal, can only express non-local exits via goto:

```
function product (outer_lst: intlist): integer;
  label 17; {labels are denoted by numbers 0 to 9999}
  function inner (lst: intlist): integer;
   begin
     if lst = nil then
      inner := 1
     else if lstˆ.head = 0 then
      begin
       product := 0; {sets return value of function}
       goto 17; {control jumps to label 17}
      end;
     else
      inner := lstˆ.head * inner(lstˆ.tail)
   end;
begin
    product := inner (outer_lst);
    17: {end of program}
end;
```

Non-Local Exits: `label` and `jump`

We will study non-local exits in Scheme by extending it with the following label and jump constructs:

- `(label` I_{cp} E_{body}`)` evaluates E_{body} in a lexical environment in which the name I_{cp} is bound to a first-class control point that represents the continuation of the entire label expression. `label` returns the value of E_{body} unless `jump` is called on I_{cp}, in which case the value supplied to `jump` is returned.
- `(jump` E_{cp} E_{val}`)` returns the value of E_{val} to the control point that is the value of E_{cp}. `jump` signals an error if E_{cp} is not a control point.

`label` and `jump`: Simple Examples

```
(+ 1 (label exit (* 2 (- 3 (/ 4 1)))))
```

```
(+ 1 (label exit (* 2 (- 3 (/ 4 (jump exit 5))))))
```

```
(+ 1 (label exit
       (* 2 (- 3 (/ 4 (jump exit (+ 5 (jump exit 6))))))))
```

```
(+ 1 (label exit1
       (* 2 (label exit2
              (- 3 (/ 4 (+ (jump exit2 5)
                           (jump exit1 6)))))))))
```

`label` and `jump`: List Product

```
(define product
  (lambda (outer-list)
    (label return
      (letrec ((inner (lambda (lst)
                        (if (null? lst)
                            1
                            (if (= (car lst) 0)
                                (jump return 0)
                                (* (car lst)
                                   (inner (cdr lst))))))))
        (inner outer-list)))))
```


`label` and `jump`: List Product Alternative

```
(define product
  (lambda (outer-list)
    (label return
      (foldr (lambda (x ans)
               (if (= x 0)
                   (jump return 0)
                   (* x ans)))
             1
             outer-list))))
```

Unlike the previous version, a `jump` is performed here on the way out of the recursion rather than on the way in.

Control Points Introduced by `label` are First-Class

```
(define fact
  (lambda (n)
    (let ((loop 'later) ; don't care about initial value
          (ans 1))
      (begin
        (label top (set! loop (lambda ()
                                (jump top 'ignore))))
        (if (= n 0)
            ans
            (begin
              (set! ans (* n ans))
              (set! n (- n 1))
              (loop)))))))
```


First-class Control Points are Strange and Powerful

```
(let ((g (lambda (x) x)))
  (letrec ((fact (lambda (n)
                   (if (= n 0)
                       (label base
                         (begin
                           (set! g (lambda (y)
                                     (begin
                                       (set! g (lambda (z) z))
                                       (jump base y))))
                           1))
                       (* n (fact (- n 1)))))))
    (+ (g 1)
       (+ (fact 3) ; Cont. = (λ (v) (+ 1 (+ v ...)))
          (+ (g 10)
             (+ (fact 4) ;Cont. = (λ (v) (+ 1 (+ 60 (+ 10 (+ v ...)))))
                (g 100)))))))
```

Scheme's `call-with-current-continuation`

Off-the-shelf Scheme does not support `label` and `jump`. But it does support `call-with-current-continuation` (sometimes abbreviated `cwcc`) which encapsulates both `label` and `jump` and can be used to implement many advanced control constructs.

`(call-with-current-continuation` E_{proc}`)` behaves like:

```
(let ((body-proc E_proc))
  (label return
    (body-proc (lambda (val)
                 (jump return val)))))
```


Example of `call-with-current-continuation`

```
(define product
  (lambda (outer-list)
    (call-with-current-continuation
      (lambda (return)
        (letrec
          ((inner (lambda (lst)
                    (cond ((null? lst) 1)
                          ((= 0 (car lst)) (return 0))
                          (else (* (car lst)
                                   (inner (cdr lst))))
                          ))))
          (inner outer-list))))))
```

Continuation Passing Style (CPS)

The constructs we have seen so far rely on implicit continuations. It is possible to model non-local control flow by passing explicit continuations in a style known as **continuation-passing style (CPS)**.

For example, here is a CPS version of recursive factorial:

```
(define fact-rec-cps
  (lambda (n k) ; k is the explicit continuation
    (if (= n 0)
        (k 1)
        (fact-rec-cps (- n 1)
                      (lambda (v) (k (* n v)))))))

(fact-rec-cps 3 (lambda (v) v))

(fact-rec-cps 4 (lambda (v) (+ 1 (* 2 v))))
```


Exception Handling

Want to be able to “signal” exceptional situations and handle them differently in different contexts.

Many languages provide exception systems:

- Java’s `throw` and `try`/`catch`
- OCaml’s `raise` and `try`/`with`
- Common Lisp’s `throw` and `catch`

`raise`, `handle`, and `trap`

We study exception handling in Scheme extended with:

- `(raise` I_{tag} E`)` Evaluate E to value V and raise exception with tag I_{tag} and value V.
- `(handle` I_{tag} $E_{handler}$ E_{body}`)` First evaluate $E_{handler}$ to a one-argument handler function $V_{handler}$. Then evaluate E_{body} to value V_{body}. If no exception is encountered, return V_{body}. If an exception is raised with tag I_{tag} and value V_{body}, the call to `handle` returns with the value of the application `(`$V_{handler}$ V_{body}`)` evaluated at the *point of the handle* (**termination semantics**).
- `(trap` I_{tag} $E_{handler}$ E_{body}`)` is evaluated like `(handle` I_{tag} $E_{handler}$ E_{body}`)` except that if an exception is raised with tag I_{tag} and value V_{body}, the call to `raise` returns with the value of the application `(`$V_{handler}$ V_{body}`)` evaluated at the *point of the raise* (**resumption semantics**).

`handle`/`trap` effectively bind $V_{handler}$ in a *dynamically scoped* exception handler namespace, and `(raise` I_{tag} E`)` looks up I_{tag} in this namespace.

Exception Handling Examples 1

```
(define test
  (lambda ()
    (let ((raiser (lambda (x)
                    (if (< x 0)
                        (raise negative x)
                        (if (even? x)
                            (raise even x)
                            x)))))
      (+ (raiser 1) (+ (raiser -3) (raiser 4))))))
```

What is the value of the following, where *`handler_1`* and *`handler_2`* range over {`handle`,`trap`}? First assume left-to-right argument evaluation, then right-to-left.

```
(handler_1 negative (lambda (v) (- v))
  (handler_2 even (lambda (v) (* v v))
    (test)))
```

Exception Handling Examples 2

What are the value of the following expressions, where *handler* ranges over {handle,trap}?

```
; Expression 1
(handler a (lambda (x) (+ 4000 x))
  (handler b (lambda (x) (+ 300 (raise a (+ x 4)))
    (handler a (lambda (x) (+ 20 x))
      (+ 1 (raise b 2)))))

; Expression 2
(handler c (lambda (x) (* x 10))
  (+ 1 (raise c (+ 2 (raise c 4)))))
```


Exception Handling In OCaml

OCaml's `raise` and `try`/`with` uses termination semantics.

In `raise` E, E must evaluate to an exception packet created by an exception constructor (where exceptions are effectively an extensible datatype).

`try` E_{body} `with` *clauses* evaluates E_{body} and returns its value unless an exception is raised, in which case the matching clause in *clauses* is evaluated and its value is returned as the value of `try`.

OCaml Exception Example

```
exception Neg of int
exception Even of int

let raiser x =
  if x < 0 then
    raise (Neg x)
  else if (x mod 2) = 0 then
    raise (Even x)
  else
    x

let test () = (raiser 1) + (raiser -3) + (raiser 4)

let innerTest () = try test() with
                     Neg y -> raiser(7 + -y)
                   | Even z -> 3 * z

let outerTest () = try innerTest() with
                     Neg y -> -y
                   | Even z -> z * z
```

Can translate this example into Java using `throw` and `try`/`catch`.

Implementing `raise`

(raise I_{tag} E) $\rightsquigarrow$ (raise-tag 'I_{tag} E)

```
(define raise-tag
  (lambda (tag value)
    (let ((handler
            ;; Look up handler in current handler env.
            ;; Handlers are dynamically scoped!
            (env-lookup tag (get-handler-env))))
      (if (unbound? handler)
          (error (string-append "Unhandled exception "
                                (symbol->string tag)
                                ": "))
          (handler value)))))
```

Implementing `handle` and `trap` 1

```
(define with-handler
  (lambda (tag make-handler try-thunk)
    (begin
      (let ((old-env (get-handler-env)))
        (begin
          ;; Remember handler in dynamic environment
          (set-handler-env! (env-bind tag
                                      (make-handler old-env)
                                      (get-handler-env)))
          ;; Evaluate try-thunk
          (let ((try-value (try-thunk)))
            ;; In normal case, pop handler
            (begin
              (set-handler-env! old-env) ; reinstate old handler env.
              try-value)))))))) ;; Return value
```


Implementing `handle` and `trap` 2

```
(trap tag handler body) desugars to
  (let ((*handler* handler) ; only evaluate once
        (*thunk* (lambda () body))) ; avoid capturing *handler*
    (with-handler 'tag
      (lambda (old-env)
        (lambda (value) (*handler* value))) ; ignores old-env
      *thunk*))
```

```
(handle tag handler body) desugars to
  (let ((*handler* handler) ; only evaluate once
        (*thunk* (lambda () body))) ;avoid capturing *handler*
     (call-with-current-continuation
      (lambda (handle-cont)
        (with-handler 'tag
          (lambda (old-env)
            (lambda (value)
              ;; Invoking HANDLE-CONT returns directly to
              ;; appropriate handle, ignoring current continuation.
              (begin
                (set-handler-env! old-env) ; reinstall old-env
                (handle-cont (*handler* value)))))
          *thunk*))))
```