



CS 251 Part 3: When Things Happen



Delay and Laziness

When are expressions evaluated?

Bonus: memoization

Topics

- Eager evaluation order (review)
 - *call-by-value*
- Delayed evaluation with *thunks*
 - emulating *call-by-name*
- Lazy evaluation with *promises*
 - emulating *call-by-need*
- Infinite sequences with *streams*
- Memoization (bonus)

Eager evaluation: arguments first

call-by-value semantics

When do arguments/subexpressions evaluate (ML, Racket)?

- Function arguments: once, *before* calling function
- Conditional branches: only one branch, *after* checking condition

```
fun fact n =
  if (n = 0) then 1 else (n * (fact (n - 1)))
```

not eager...

```
fun iffy x y z =
  if x then y else z
```

```
fun facty n =
  iffy (n = 0)
    1
    (n * (facty (n - 1)))
```

What's wrong?

Delayed evaluation with *thunks*

explicit emulation of lexically-scoped call-by-name semantics

Thunk `fn () => e`

- `n`. a zero-argument function used to delay evaluation
- `v`. to create a thunk from an expression:
"think the expression"

No new language features.

```
fun if_by_name x y z =  
  if x () then y () else z ()
```

Type?

```
fun fact n =  
  if_by_name (fn () => n = 0)  
             (fn () => 1)  
             (fn () => n * (fact (n - 1)))
```

Delay and Laziness 5

See code examples

Thunk: evaluate when value needed

explicit emulation of lexically-scoped call-by-name semantics

```
fun f1 th =  
  if ... then 7 else ... th() ...  
  
fun f2 th =  
  if ... then 7 else th() + th()  
  
fun f3 th =  
  let val v = th ()  
  in if ... then 7 else v + v end  
  
fun f4 th =  
  if ... then 7 else  
  let val v = th () in v + v end
```

- # evaluations?
- Faster?
Slower?
- Side effects?

Delay and Laziness 6

Lazy evaluation: first time *value* is needed

call-by-need semantics

Argument/subexpression **evaluated zero or one times**,
no earlier than first time result is actually needed.

Result reused (not recomputed) if needed again *anywhere*.

Benefits of delayed evaluation, with minimized costs.

Explicit laziness with *promises*:

- `Promise.delay (fn () => x * f x)`
- `Promise.force p`

Delay and Laziness 7

Promises: explicit laziness

(a.k.a. *suspensions*)

```
signature PROMISE =  
sig  
  
  (* Type of promises for 'a. *)  
  type 'a t  
  
  (* Take a thunk for an 'a and  
  make a promise to produce an 'a. *)  
  val delay : (unit -> 'a) -> 'a t  
  
  (* If promise not yet forced, call thunk and save.  
  Return saved thunk result. *)  
  val force : 'a t -> 'a  
  
end
```

Delay and Laziness 8

Promises: delay and force

(a.k.a. suspensions)

```
structure Promise :> PROMISE =
struct
  datatype 'a promise = Think of unit -> 'a
                        | Value of 'a

  type 'a t = 'a promise ref

  fun delay thunk = ref (Think thunk)

  fun force p =
    case !p of
      Value v => v
    | Think th =>
      let val v = th ()
          val _ = p := Value v
      in v end
end
```

Limited mutation
hidden in ADT.

Stream: infinite sequence of values

Infinite sequence:

- Cannot make all the elements *now*.
- Make one when asked, delay making the rest.

Interface/idiom for **division of labor**:

- **Stream producer**
- **Stream consumer**
- Interleave production / consumption in *time*, but *not in code*.

Examples:

- UI events
- UNIX pipes: `git diff delay.sml | grep "think"`
- Sequential logic circuit updates (CS 240)

Streams in ML: false ~~start~~

Let a **stream** be a thunk that, *when called*, returns a pair of

- the next element; and
 - the rest of the stream.
- ```
fn () => (next_element, next_thunk)
```

Given stream *s*, get elements:

- First: `let val (v1,s1) = s ()`
- Second: `val (v2,s2) = s1 ()`
- Third: `val (v3,s3) = s2 () ...`

Type of *s*? *s1*?  
*s2*? *s3*? ...?

## Streams in ML: recursive types

Single-constructor datatype allows recursive type:

```
datatype 'a scon =
 Scons of 'a * (unit -> 'a scon)
```

```
type 'a stream = unit -> 'a scon
```

Given a stream *s*:

- First: `let val Scons(v1,s1) = s ()`
- Second: `val Scons(v2,s2) = s1 ()`
- Third: `val Scons(v3,s3) = s2 ()`
- ...

Type of *s*? *s1*?  
*s2*? *s3*? ...?

## Stream consumers

Find index of first element in stream for which `f` returns true.

```
fun firstindex f stream =
 let fun consume stream acc =
 let val Scons (v,s) = stream ()
 in
 if f v
 then acc
 else consume s (acc + 1)
 end
 in consume stream 0 end
: ('a -> bool) -> 'a stream -> int
```

Delay and Laziness 14

## Stream producers

```
fun ones () = Scons (1,ones)
val rec ones = fn () => Scons (1,ones)
```

Create next thunk via **delayed recursion!**

- Return a thunk that, when called, calls the outer function recursively.

```
val nats =
 let fun f x = Scons (x, fn () => f (x + 1))
 in fn () => f 0 end

val powers2 =
 let fun f x = Scons (x, fn () => f (x * 2))
 in fn () => f 1 end
```

Delay and Laziness 15

## Getting it wrong

Tries to use a variable before it is defined.

```
val ones_bad = Scons (1, ones_bad)
```

Would call `ones_worse` recursively *immediately* (infinitely).  
Does not type-check.

```
fun ones_worse () = Scons (1, ones_worse ())
```

**Correct:** thunk that returns Scons of value and stream (thunk).

```
fun ones () = Scons (1, ones)
val rec ones = fn () => Scons (1, ones)
```

Delay and Laziness 16

optional

## Bonus: Lazy by default?

**ML:**

- Eager evaluation. Explicitly emulate laziness when needed (promises).
- Immutable data, bindings. Explicit mutable cells when needed (refs).
- Side effects anywhere.

**Pros:** avoid unnecessary work, build elegant infinite data structures.

**Cons:** difficult to control/predict evaluation order:

- Space usage: when will environments become unreachable?
- Side-effect ordering: when will effects execute?

**Haskell:** canonical real-world example

- Non-strict evaluation, except pattern-matching. Explicit strictness when needed.
- Usually implemented as lazy evaluation.
- Immutable everything. Emulate mutation/state when needed.
- Side effects banned/restricted/emulated.

Delay and Laziness 17

# Bonus: Memoization

see memo.sml

Not delayed evaluation, but...

- Promises (call-by-need) are memoized thunks (call-by-name), though memoization is more general (multiple arguments).
- Can use an indirect recursive style similar to streams (without delay)
  - Actually fixpoint...

Basic idea:

- Save results of expensive pure computations in mutable cache.
- Reuse earlier computed results instead of recomputing.
- Even for recursive calls.

Benefits:

- Save time when recomputing.
- Can reduce exponential recursion costs to linear (and amortized by repeated calls with same arguments).

See also: dynamic programming (CS 231)