



**CS 251** Spring 2020  
Principles of Programming Languages  
Ben Wood



# CS 251 Part 3: When Things Happen



# Delay and Laziness

*When are expressions evaluated?*

*Bonus: memoization*

# Topics

- Eager evaluation order (review)
  - *call-by-value*
- Delayed evaluation with ***thunks***
  - emulating *call-by-name*
- Lazy evaluation with ***promises***
  - emulating *call-by-need*
- Infinite sequences with ***streams***
- Memoization (bonus)

# Eager evaluation: arguments first

*call-by-value semantics*

When do arguments/subexpressions evaluate (ML, Racket)?

- Function arguments: once, *before* calling function
- Conditional branches: only one branch, *after* checking condition

```
fun fact n =  
  if (n = 0) then 1 else (n * (fact (n - 1)))
```

not eager...

```
fun iffy x y z =  
  if x then y else z
```

```
fun facty n =  
  iffy (n = 0)  
    1  
    (n * (facty (n - 1)))
```

What's wrong?

# Delayed evaluation with thunks

explicit emulation of lexically-scoped call-by-name semantics

*Thunk*    `fn () => e`

- *n*. a zero-argument function used to delay evaluation
- *v*. to create a thunk from an expression:  
    *"think the expression"*

No new language features.

```
fun if_by_name x y z =  
  if x () then y () else z ()
```

Type?

```
fun fact n =  
  if_by_name (fn () => n = 0)  
             (fn () => 1)  
             (fn () => n * (fact (n - 1)))
```

# Think: evaluate when value needed

*explicit emulation of lexically-scoped call-by-name semantics*

```
fun f1 th =  
  if ... then 7 else ... th() ...
```

```
fun f2 th =  
  if ... then 7 else th() + th()
```

```
fun f3 th =  
  let val v = th ()  
  in if ... then 7 else v + v end
```

```
fun f4 th =  
  if ... then 7 else  
  let val v = th () in v + v end
```

- # evaluations?
- Faster?  
Slower?
- Side effects?

# ***Lazy evaluation: first time value is needed*** ***call-by-need semantics***

Argument/subexpression **evaluated zero or one times**,  
no earlier than first time result is actually needed.

**Result reused** (not recomputed) if needed again *anywhere*.

Benefits of delayed evaluation, with minimized costs.

Explicit laziness with ***promises***:

- `Promise.delay (fn () => x * f x)`
- `Promise.force p`

# Promises: explicit laziness

*(a.k.a. suspensions)*

```
signature PROMISE =
sig

  (* Type of promises for 'a. *)
  type 'a t

  (* Take a thunk for an 'a and
     make a promise to produce an 'a. *)
  val delay : (unit -> 'a) -> 'a t

  (* If promise not yet forced, call thunk and save.
     Return saved thunk result. *)
  val force : 'a t -> 'a

end
```



# Promises: delay and force

(a.k.a. suspensions)

```

structure Promise :> PROMISE =
struct
  datatype 'a promise = Thunk of unit -> 'a
                        | Value of 'a

  type 'a t = 'a promise ref

  fun delay thunk = ref (Thunk thunk)

  fun force p =
    case !p of
      Value v => v
    | Thunk th =>
      let val v = th ()
          val _ = p := Value v
      in v end
end

```

Limited mutation  
hidden in ADT.

# *Stream*: infinite sequence of values

Infinite sequence:

- Cannot make all the elements *now*.
- Make one when asked, delay making the rest.

Interface/idiom for **division of labor**:

- **Stream producer**
- **Stream consumer**
- Interleave production / consumption in *time*, but *not in code*.

Examples:

- UI events
- UNIX pipes: `git diff delay.sml | grep "thunk"`
- Sequential logic circuit updates (CS 240)

# Streams in ML: ~~false start~~

Let a *stream* be a thunk that, *when called*, returns a pair of

- the next element; and
- the rest of the stream.

```
fn () => (next_element, next_thunk)
```

Given stream *s*, get elements:

- First: `let val (v1, s1) = s ()`
- Second: `val (v2, s2) = s1 ()`
- Third: `val (v3, s3) = s2 () ...`

Type of *s*? *s1*?  
*s2*? *s3*? ...?

# Streams in ML: recursive types

Single-constructor datatype allows recursive type:

```
datatype 'a scon =  
  Scons of 'a * (unit -> 'a scon)
```

```
type 'a stream = unit -> 'a scon
```

Given a stream `s`:

- First: `let val Scons (v1, s1) = s ()`
- Second: `val Scons (v2, s2) = s1 ()`
- Third: `val Scons (v3, s3) = s2 ()`
- ...

Type of `s`? `s1`?  
`s2`? `s3`? ...?

# Stream consumers

Find index of first element in `stream` for which `f` returns `true`.

```
fun firstindex f stream =  
  let fun consume stream acc =  
        let val Scons (v,s) = stream ()  
        in  
          if f v  
          then acc  
          else consume s (acc + 1)  
        end  
  in consume stream 0 end  
  
: ('a -> bool) -> 'a stream -> int
```

# Stream producers

```
fun ones () = Scons (1, ones)
val rec ones = fn () => Scons (1, ones)
```

Create next thunk via **delayed recursion!**

- Return a thunk that, when called, calls the outer function recursively.

```
val nats =
  let fun f x = Scons (x, fn () => f (x + 1))
      in fn () => f 0 end
```

```
val powers2 =
  let fun f x = Scons (x, fn () => f (x * 2))
      in fn () => f 1 end
```

# Getting it wrong

Tries to use a variable before it is defined.

```
val ones_bad = Scons (1, ones_bad)
```

Would call `ones_worse` recursively *immediately* (infinitely).

Does not type-check.

```
fun ones_worse () = Scons (1, ones_worse ())
```

**Correct:** `thunk` that returns `Scons` of value and stream (`thunk`).

```
fun ones () = Scons (1, ones)
val rec ones = fn () => Scons (1, ones)
```

# Bonus: Lazy by default?

## ML:

- Eager evaluation. Explicitly emulate laziness when needed (promises).
- Immutable data, bindings. Explicit mutable cells when needed (refs).
- Side effects anywhere.

**Pros:** avoid unnecessary work, build elegant infinite data structures.

**Cons:** difficult to control/predict evaluation order:

- Space usage: when will environments become unreachable?
- Side-effect ordering: when will effects execute?

**Haskell:** canonical real-world example

- Non-strict evaluation, except pattern-matching. Explicit strictness when needed.
- Usually implemented as lazy evaluation.
- Immutable everything. Emulate mutation/state when needed.
- Side effects banned/restricted/emulated.



# Bonus: Memoization

see memo.sml

Not delayed evaluation, but...

- Promises (call-by-need) are memoized thunks (call-by-name), though memoization is more general (multiple arguments).
- Can use an indirect recursive style similar to streams (without delay)
  - Actually fixpoint...

Basic idea:

- Save results of expensive pure computations in mutable cache.
- Reuse earlier computed results instead of recomputing.
- Even for recursive calls.

Benefits:

- Save time when recomputing.
- Can reduce exponential recursion costs to linear (and amortized by repeated calls with same arguments).

See also: dynamic programming (CS 231)