



Type Checking and Type Inference

Type checking

Static:

Can reject a program before it runs to prevent possibility of some errors.

Dynamic:

Little/no static checking.

May try to treat a number as a function during evaluation. Report error then.

Part of language definition,
not an implementation detail.

static types \neq explicit types

```
fun f x = (* infer val f : int -> int *)
  if x > 3
  then 42
  else x * 2

fun g x = (* report type error *)
  if x > 3
  then true
  else x * 2
```

Type inference

Problem:

- Give every binding/expression a type such that type checking succeeds.
- Fail *if and only if* no solution exists

Implementation:

- Could be a pass before type checker
- Often implemented in type checker

Easy, difficult, or *impossible*:

- Easy: Accept all programs
- Easy: Reject all programs
- Subtle, elegant, and *not magic*: ML

Human type inference...

What is the type of `x`?

What is the type of `f`?

Describe your process.

```
val x = 42
fun f (y, z, w) =
  if y
  then z + x
  else 0
```

Next:

- More examples, but:
 - General algorithm is a slightly more advanced topic
 - Supporting nested functions also a bit more advanced
- Enough to “do type inference in your head”
 - And appreciate it is not magic

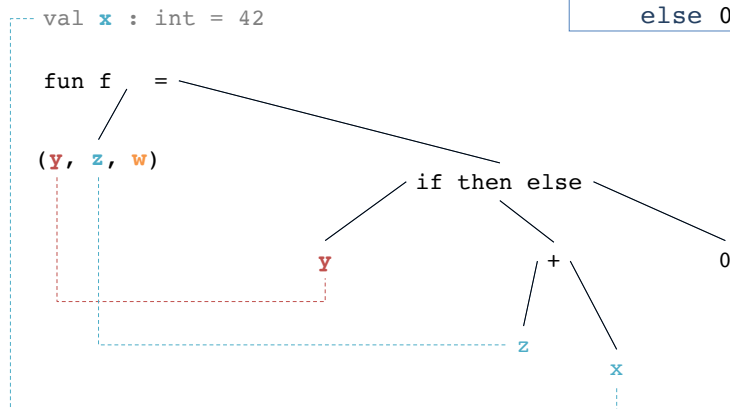
Key steps

1. Determine types of bindings in order
 - Cannot use later bindings.
2. For each `val` or `fun` binding:
 - Analyze definition for all necessary facts (**constraints**).
 - Example: `x > 0` \Rightarrow `x : int`
 - Type error if no way for all facts to hold (over-constrained)
3. Use type variables (`'a` ...) for any unconstrained types.

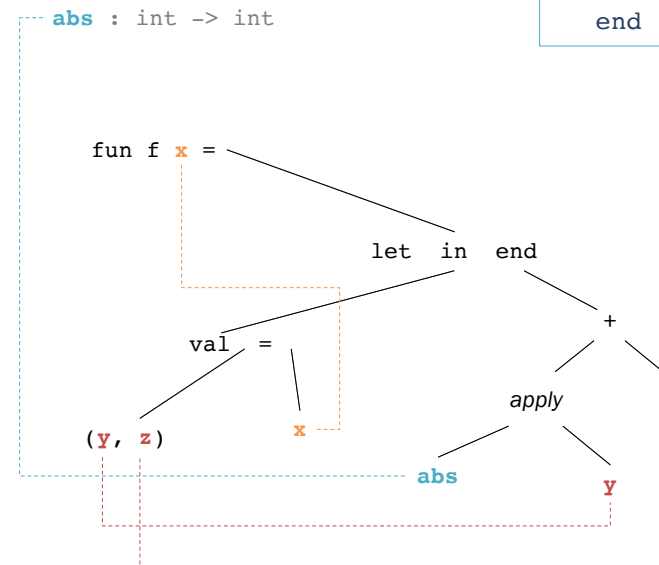
Inference and polymorphism are orthogonal; together = "sweet spot".
Results in *most general feasible type*.
4. Enforce the *value restriction*, discussed later.

See code examples in `inf.sml`.

```
val x = 42
fun f (y, z, w) =
  if y
  then z + x
  else 0
```



```
fun f x =
  let val (y, z) = x in
    (abs y) + z
  end
```



Problem: unsoundness!

Combine polymorphism and mutation:

```
val thing = ref NONE (* : 'a option ref *)
val _ = thing := SOME "hi"
val i = 1 + case !thing of NONE => 0 | SOME x => x
```

- Assignment type-checks:
 - (op:=) : 'a ref * 'a -> unit
 - instantiate **string** for 'a
 - use as **string** ref * **string** -> unit
- Dereference type-checks:
 - ! : 'a ref -> 'a
 - instantiate **int** for 'a
 - use as **int** ref -> **int**
- **val i : int = "hi"**

Solution

Reject at least one of these lines

```
val thing = ref NONE (* : 'a option ref *)
val _ = thing := SOME "hi"
val i = 1 + case !thing of NONE => 0 | SOME x => x
```

Cannot just special-case ref types. Abstract types!

```
signature HIDE = sig
  type 'a hidden
  val make : 'a -> 'a hidden
  val thing : 'a hidden
end
structure Hide :> HIDE = struct
  type 'a hidden = 'a ref
  val make = ref
  val thing = make NONE
end
```

The Value Restriction

```
val thing = ref NONE (* : ?.X1 option ref *)
val _ = thing := SOME "hi"
val i = 1 + case !thing of NONE => 0 | SOME x => x
```

A variable-binding can have a polymorphic type only if the expression is a variable or value.

- Function calls like `ref NONE` are neither

Otherwise

Warning: type vars not generalized because of value restriction are instantiated to dummy types (Basically unusable)

Not obvious: suffices to make type system sound.

Value Restriction downside

Causes problems when unnecessary (no mutation) because:

```
val pairWithOne = List.map (fn x => (x,1))
(* does not get type 'a list -> ('a*int) list *)
```

Type-checker does not know `List.map` is not making a mutable ref.

Workarounds for partial application:

wrap in a function binding to keep it polymorphic

```
fun pairWithOne xs = List.map (fn x => (x,1)) xs
(* 'a list -> ('a*int) list *)
```

give up on polymorphism; write explicit non-polymorphic type

```
val pairWithOne : int list -> (int * int) list =
  List.map (fn x => (x,1))
val pairWithOne = List.map (fn (x : int) => (x,1))
```

A local optimum

ML type inference is elegant and fairly easy to understand, despite the value restriction.

More difficult *without* polymorphism

- What type should length-of-list have?

More difficult *with* subtyping (later)

- Suppose pairs are supertypes of wider tuples
- Then `val (y, z) = x` constrains `x` to have at least two fields, not exactly two fields.
- Sometimes languages can support this, but types are often more difficult to infer and understand.