First-Class Functions in Racket

First-Class Values

A value is **first-class** if it satisfies all of these properties:

- It can be named by a variable
- It can be passed as an argument to a function;
- It can be returned as the result of a function;
- It can be stored as an element in a data structure (e.g., a list);
- It can be created in any context.

Examples from Racket: numbers, boolean, strings, characters, lists, … and **functions**!

Functions can be Named

\[
\text{(define } \text{dbl} (\lambda (x) (* 2 x)))
\]
\[
\text{(define } \text{avg} (\lambda (a b) (/ (+ a b) 2))))
\]
\[
\text{(define } \text{pow}
\begin{align*}
\lambda (\text{base expt}) \\
&\quad (\text{if } (= \text{expt} 0) \\
&\quad \quad 1 \\
&\quad (* \text{base (pow base (- expt 1))}))
\end{align*}
\]

Recall syntactic sugar:

\[
\text{(define } \text{dbl} x) (* 2 x))
\]
\[
\text{(define } \text{avg} a b) (/ (+ a b) 2))
\]
\[
\text{(define } \text{pow base expt}) \ldots)
\]

Functions can be Passed as Arguments

\[
\text{(define } \text{app-3-5} (\lambda (f) (f 3 5)))
\]
\[
\text{(define } \text{sub2} (\lambda (x y) (- x y)))
\]
\[
\{\text{app-3-5} \text{ sub2}\}
\]
\[
\Rightarrow (((\lambda (f) (f 3 5)) \{\text{sub2}\}) \text{ [varref]}
\]
\[
\Rightarrow\{(\lambda (x y) (- x y)) 3 5\} \text{ [function call]}
\]
\[
\Rightarrow\{(- 3 5) \text{ [function call]}
\]
\[
\Rightarrow -2 \text{ [subtraction]}
\]
More Functions-as-Arguments

What are the values of the following?

\[(\text{app-3-5 \ avg})\]
\[(\text{app-3-5 \ pow})\]
\[(\text{app-3-5 \ (\lambda \ (a \ b) \ a)})\]
\[(\text{app-3-5 \ +})\]

Functions can be Returned as Results from Other Functions

\[(\text{define make-linear-function} \ (\lambda \ (a \ b) \ ; \ a \ \text{and} \ b \ \text{are numbers} \ (\lambda \ (x) \ (+ \ (* \ a \ x) \ b))))\]
\[(\text{define 4x+7} \ (\text{make-linear-function} 4 \ 7))\]
\[(4x+7 \ 0)\]
\[(4x+7 \ 1)\]
\[(4x+7 \ 2)\]
\[(\text{make-linear-function} 6 \ 1)\]
\[(((\text{make-linear-function} 6 \ 1) \ 2))\]
\[(((\text{app-3-5 make-linear-function}) \ 2))\]

More Functions-as-Returned-Values

\[(\text{define flip2} \ (\lambda \ (\text{binop}) \ (\lambda \ (x \ y) \ (\text{binop} \ y \ x))))\]
\[(((\text{flip2 sub2}) \ 4 \ 7)\]
\[(\text{app-3-5 (flip2 sub2)})\]
\[(((\text{flip2 pow}) \ 2 \ 3))\]
\[(\text{app-3-5 (flip2 pow)})\]
\[(\text{define g} \ ((\text{flip2 make-linear-function}) \ 4 \ 7))\]
\[(\text{list} \ (g \ 0) \ (g \ 1) \ (g \ 2))\]
\[(((\text{app-3-5 (flip2 make-linear-function)}) \ 2))\]

Functions can be Stored in Lists

\[(\text{define funs} \ (\text{list} \ \text{sub2} \ \text{avg} \ \text{pow} \ \text{app-3-5} \ \text{make-linear-function} \ \text{flip2}))\]
\[(((\text{first funs}) \ 4 \ 7))\]
\[(((\text{fourth funs}) \ (\text{third funs}))\]
\[(((\text{fourth funs}) \ ((\text{sixth funs}) \ (\text{third funs})))\]
\[(((\text{fourth funs}) \ ((\text{fifth funs})) \ 2)\]
\[(((\text{fourth funs}) \ ((\text{sixth funs}) \ (\text{fifth funs}))) \ 2)\]
Functions can be Created in Any Context

- In some languages (e.g., C) functions can be defined only at top-level. One function cannot be declared inside of another.

- Racket functions like `make-linear-function` and `flip2` depend crucially on the ability to create one function inside of another function.

Python Functions are First-Class!

```python
def sub2(x, y):
    return x - y

def app_3_5(f):
    return f(3, 5)

def make_linear_function(a, b):
    return lambda x: a * x + b

def flip2(binop):
    return lambda x, y: binop(y, x)
```

In [2]: app_3_5(sub2)
Out[2]: -2

In [3]: app_3_5(flip2(sub2))
Out[3]: 2

In [4]: app_3_5(make_linear_function)(2)
Out[4]: 11

In [5]: app_3_5(flip2(make_linear_function))(2)
Out[5]: 13

JavaScript Functions are First-Class!

```javascript
function sub2(x, y){
    return x-y;
}

function app_3_5(f){
    return f(3,5);
}

function make_linear_function(a,b){
    return function(x){
        return a*x + b;
    };
}

function flip2(binop){
    return function(x,y){
        return binop(y,x);
    };
}
```

> app_3_5(sub2)
< -2

> app_3_5(flip2(sub2))
< 2

> app_3_5(make_linear_function)(2)
< 11

> app_3_5(flip2(make_linear_function))(2)
< 13

Summary (and Preview!)

Data and procedures and the values they amass,
Higher-order functions to combine and mix and match,
Objects with their local state, the messages they pass,
A property, a package, a control point for a catch —
In the Lambda Order they are all first-class.
One Thing to name them all, One Thing to define them,
One Thing to place them in environments and bind them,
In the Lambda Order they are all first-class.

Abstract for the Revised4 Report on the Algorithmic Language Scheme (R4RS), MIT Artificial Intelligence Lab Memo 848b, November 1991

Emblem for the Grand Recursive Order of the Knights of the Lambda Calculus