

List Recursion



CS251 Programming Languages
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Recursive List Functions in Racket

Because Racket lists are defined recursively, it's natural to process them recursively.

Typically (but not always) a recursive function `recf` on a list argument `L` has two cases:

- **base case:** what does `recf` return when `L` is empty? (Use `null?` to test for an empty list.)
- **recursive case:** if `L` is the nonempty list `(cons Vfirst Vrest)` how are `Vfirst` and `(recf Vrest)` combined to give the result for `(recf L)`?

Note that we **always** apply `recf` directly to `Vrest` (and nothing else)!

Recursive List Functions: Divide/Conquer/Glue (DCG) strategy for the general case [in words]

Step 1 (concrete example): pick a concrete input list, typically 3 or 4 elements long. What should the function return on this input?

E.g. A `sum` function that returns the sum of all the numbers in a list:
`(sum '(5 7 2 4)) =>* 18`

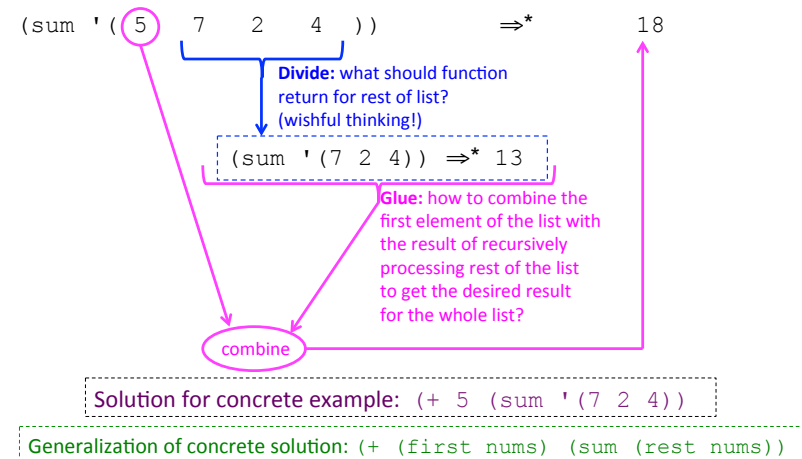
Step 2 (divide): without even thinking, **always** apply the function to the *rest* of the list. What does it return? `(sum '(7 2 4)) =>* 13`

Step 3 (glue): How to combine the first element of the list (in this case, 5) with the result from processing the rest (in this case, 13) to give the result for processing the whole list (in this case, 18)? `(+ 5 (sum '(7 2 4))) =>* 18`

Step 4 (generalize): Express the general case in terms of an arbitrary input:

```
(define (sum nums)
  ... (+ (first nums) (sum (rest nums))) ...)
```

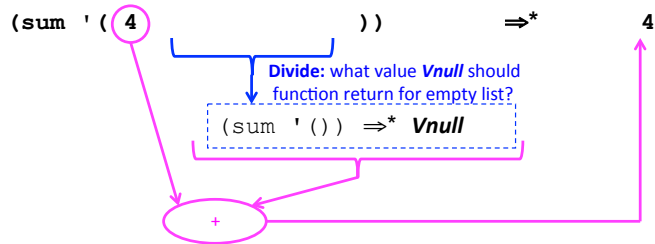
Recursive List Functions: Divide/Conquer/Glue (DCG) strategy for the general case [in diagram]



Recursive List Functions: base case via singleton case

Deciding what a recursive list function should return for the empty list is not always obvious and can be tricky. E.g. what should `(sum '())` return?

If the answer isn't obvious, consider the "penultimate case" in the recursion, which involves a list of one element:



In this case, **Vnull** should be 0, which is the identity element for addition.

But in general it depends on the details of the particular combiner determined from the general case. So solve the general case before the base case!

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Putting it all together: base & general cases

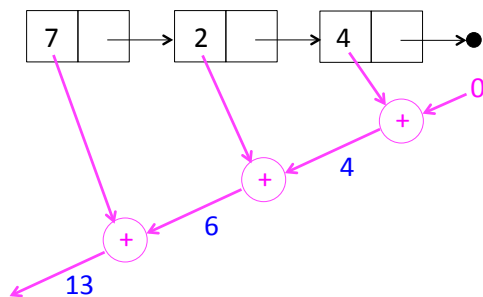
`(sum nums)` returns the sum of the numbers in the list `nums`

```
(define (sum ns)
  (if (null? ns)
      0
      (+ (first ns)
         (sum (rest ns)))))
```

List Recursion 6

Understanding sum: Approach #1

`(sum '(7 2 4))`



We'll call this the **recursive accumulation** pattern

List Recursion 5-7

Understanding sum: Approach #2

In `(sum (list 7 2 4))`, the list argument to `sum` is

```
(cons 7 (cons 2 (cons 4 null)))
```

Replace `cons` by `+` and `null` by `0` and simplify:

```
(+ 7 (+ 2 (+ 4 0)))
```

```
 $\Rightarrow$  (+ 7 (+ 2 4))
```

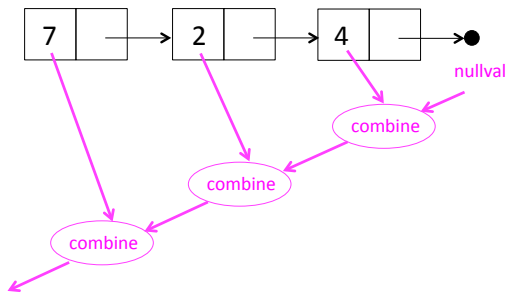
```
 $\Rightarrow$  (+ 7 6)
```

```
 $\Rightarrow$  13
```

Pairs and Lists 8

Generalizing sum: Approach #1

```
(recf (list 7 2 4))
```



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Generalizing sum: Approach #2

In (recf (list 7 2 4)), the list argument to recf is

```
(cons 7 (cons 2 (cons 4 null)))
```

Replace cons by **combine** and null by **nullval** and simplify:

```
(combine 7 (combine 2 (combine 4 nullval)))
```

List Recursion 10

Generalizing the sum definition

```
(define (recf ns)
  (if (null? ns)
      nullval
      (combine (first ns)
               (recf (rest ns)))))
```

List Recursion 11

Your turn



Define the following recursive list functions and test them in Racket:

(product ns) returns the product of the numbers in ns

(min-list ns) returns the minimum of the numbers in ns

Hint: use min and +inf.0 (positive infinity)

(max-list ns) returns the maximum of the numbers in ns

Hint: use max and -inf.0 (negative infinity)

(all-true? bs) returns #t if all the elements in bs are truthy; otherwise returns #f. *Hint: use and*

(some-true? bs) returns a truthy value if at least one element in bs is truthy; otherwise returns #f. *Hint: use or*

(my-length xs) returns the length of the list xs

List Recursion 12



Recursive Accumulation Pattern Summary

	combine	nullval
sum		
product		
min-list		
max-list		
all-true?		
some-true?		
my-length		

List Recursion 13

Define these using Divide/Conquer/Glue

```
> (snoc 11 '(7 2 4))
'(7 2 4 11)

> (my-append '(7 2 4) '(5 8))
'(7 2 4 5 8)

> (append-all '((7 2 4) (9) () (5 8)))
'(7 2 4 9 5 8)

> (my-reverse '(5 7 2 4))
'(4 2 7 5)
```

List Recursion 14

Mapping Example: map-double



(map-double ns) returns a new list the same length as ns in which each element is the double of the corresponding element in ns.

```
> (map-double (list 7 2 4))
'(14 4 8)
```

```
(define (map-double ns)
  (if (null? ns)
      ; Flesh out base case

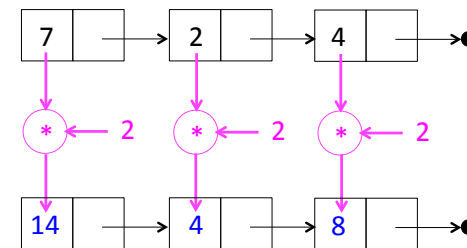
      ; Flesh out general case

  ))
```

List Recursion 15

Understanding map-double

(map-double '(7 2 4))

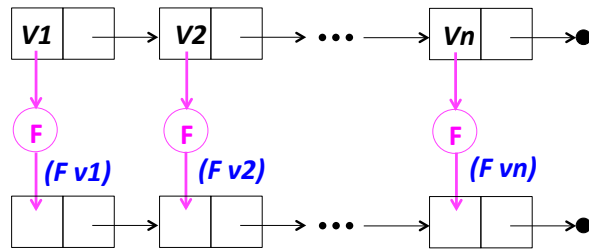


We'll call this the **mapping** pattern

List Recursion 16

Generalizing map-double

(map^F (list *v1 v2 ... vn*))



```
(define (mapF xs)
  (if (null? xs)
      null
      (cons (F (first xs))
            (mapF (rest xs))))))
```

List Recursion 17

Expressing map^F as an accumulation



```
(define (mapF xs)
  (if (null? xs)
      null
      ((λ (fst subres)
         ) ; Flesh this out
        (first xs)
        (mapF (rest xs)))))
```

List Recursion 18

Some Recursive Listfuns Need Extra Args

```
(define (map-scale factor ns)
  (if (null? ns)
      null
      (cons (* factor (first ns))
            (map-scale factor (rest ns)))))
```

List Recursion 19

Filtering Example: filter-positive



(filter-positive ns) returns a new list that contains only the positive elements in the list of numbers ns, in the same relative order as in ns.

```
> (filter-positive (list 7 -2 -4 8 5))
'(7 8 5)
```

```
(define (filter-positive ns)
  (if (null? ns)
      ; Flesh out base case

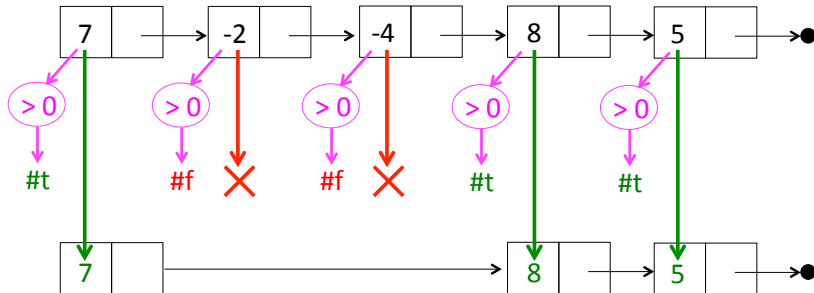
      ; Flesh out recursive case

  ))
```

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Understanding filter-positive

(filter-positive (list 7 -2 -4 8 5))

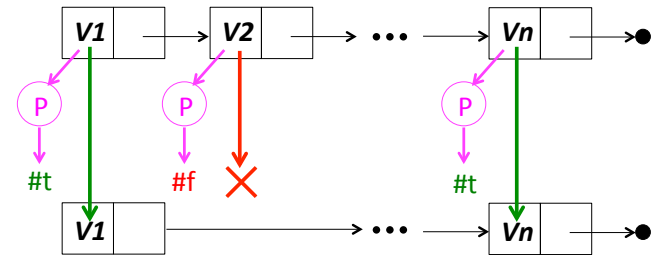


We'll call this the **filtering** pattern

List Recursion 21

Generalizing filter-positive

(filter P (list $V1$ $V2$... Vn))



```
(define (filter $P$  xs)
  (if (null? xs)
      null
      (if ( $P$  (first xs))
          (cons (first xs) (filter $P$  (rest xs)))
          (filter $P$  (rest xs)))))
```

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Expressing filter P as an accumulation



```
(define (filter $P$  xs)
  (if (null? xs)
      null
      ((lambda (fst subres)
          ; Flesh this out
          (cons (first xs)
                (filter $P$  (rest xs))))))
```

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