Local Bindings and Scope

SOLUTIONS

These slides borrow heavily from Ben Wood's Fall '15 slides, some of which are in turn based on Dan Grossman's material from the University of Washington.

CS251 Programming Languages

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Motivation for local bindings

We want local bindings = a way to name things locally in functions and other expressions.

Why?

- For style and convenience
- Avoiding duplicate computations
- A big but natural idea: nested function bindings
- Improving algorithmic efficiency (*not* "just a little faster")

`let` expressions: Example

```
> (let {[a (+ 1 2)] [b (* 3 4)]} (list a b))
'(3 12)
```

Pretty printed form

```
> (let {[a (+ 1 2)]
        [b (* 3 4)]}
    (list a b))
'(3 12)
```


`let` in the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

```
(define (quadratic-roots a b c)
  (let {[-b (- b)]
        [sqrt-discriminant
          (sqrt (- (* b b) (* 4 a c)))]
        [2a (* 2 a)]}
    (list (/ (+ -b sqrt-discriminant) 2a)
          (/ (- -b sqrt-discriminant) 2a))))
```

```
> (quadratic-roots 1 -5 6)
'(3 2)
> (quadratic-roots 2 7 -15)
'(1½ -5)
```

Formalizing `let` expressions

2 questions:

a new keyword!

- Syntax: `(let {[Id1 E1] ... [Idn En]} Ebody)`
  - Each ***Idi*** is any *identiier*, and ***Ebody*** and each ***Ei*** are any *expressions*

- Evaluation:
  - Evaluate each expression ***Ei*** to value ***Vi*** in the current dynamic environment.
  - Evaluate ***Ebody[V1,…Vn/Id1,…,Idn]*** in the current dynamic environment.

  Result of whole **let** expression is result of evaluating ***Ebody***.

Parens vs. Braces vs. Brackets

As matched pairs, they are interchangeable.
Differences can be used to enhance readability.

```
> (let {[a (+ 1 2)] [b (* 3 4)]} (list a b))
'(3 12)

> (let ((a (+ 1 2)) (b (* 3 4))) (list a b))
'(3 12)

> (let [[a (+ 1 2)] [b (* 3 4)]] (list a b))
'(3 12)

> (let [{a (+ 1 2)} (b (* 3 4))] (list a b))
'(3 12)
```


`let` is an expression

A let-expression is ***just an expression***, so we can use it ***anywhere*** an expression can go.

Silly example:

```
(+ (let {[x 1]} x)
   (let {[y 2]
         [z 4]}
     (- z y)))
```


`let` is just syntactic sugar!

`(let {[Id1 E1] … [Idn En]} Ebody)`

desugars to

`((lambda (Id1 … Idn) Ebody) E1 … En)`

Example:

```
(let {[a (+ 1 2)] [b (* 3 4)]} (list a b))
```

desugars to

```
((lambda (a b) (list a b)) (+ 1 2) (* 3 4))
```

Avoid repeated recursion

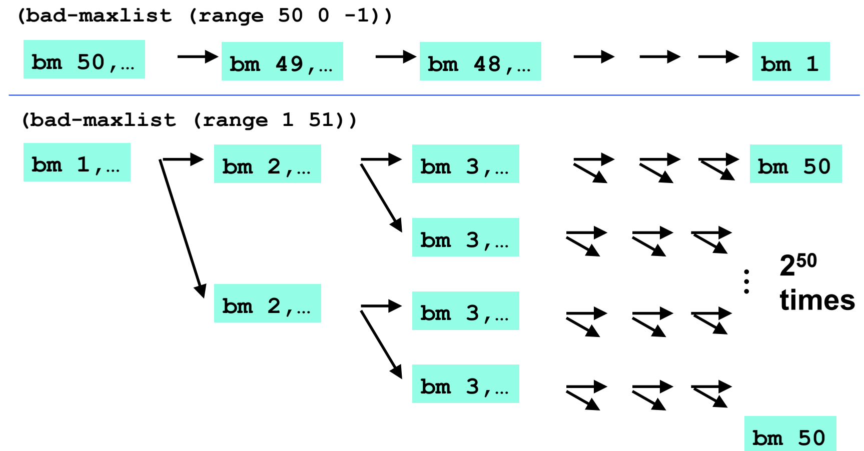
Consider this code and the recursive calls it makes

– Don't worry about calls to **`first`**, **`rest`**, and **`null?`** because they do a small constant amount of work

```
(define (bad-maxlist xs)
  (if (null? xs)
      -inf.0
      (if (> (first xs) (bad-maxlist (rest xs)))
          (first xs)
          (bad-maxlist (rest xs)))))
```


Fast vs. unusable

```
(if (> (first xs)
       (bad-maxlist (rest xs)))
    (first xs)
    (bad-maxlist (rest xs)))
```



Some calculations

Suppose one **`bad-maxlist`** call's **`if`** logic and calls to **`null?`**, **`first?`**, **`rest`** take 10^{-7} seconds total

– Then (**`bad-maxlist (list 50 49 … 1))`** takes 50×10^{-7} sec

– And (**`bad-maxlist (list 1 2 … 50))`**
takes $(1 + 2 + 2^2 + 2^3 + \ldots + 2^{49}) \times 10^{-7}$
$= (2^{50} - 1) \times 10^{-7} = 1.12 \times 10^8$ sec = **over 3.5 years**

– And (**`bad-maxlist (list 1 2 … 55))`**
takes over 114 years

– And (**`bad-maxlist (list 1 2 … 100))`**
takes over 4×10^{15} years.
(Our sun is predicted to die in about 5×10^9 years)

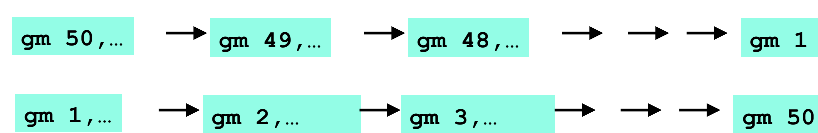
– Buying a faster computer won't help much ☺

The key is not to do repeated work!

– Saving recursive results in local bindings is essential…

Efficient maxlist

```
(define (good-maxlist xs)
  (if (null? xs)
      -inf.0
      (let {[rest-max (good-maxlist (rest xs))]}
        (if (> (first xs) rest-max)
            (first xs)
            rest-max))))
```



Transforming good-maxlist

```
(define (good-maxlist xs)
  (if (null? xs)
      -inf.0
      (let {[rest-max (good-maxlist (rest xs))]}
        (if (> (first xs) rest-max)
            (first xs)
            rest-max))))
```

```
(define (good-maxlist xs)
  (if (null? xs)
      -inf.0
      ((λ (fst rest-max) ; name fst too!
         (if (> fst rest-max) fst rest-max))
       (first xs)
       (good-maxlist (rest xs)))))
```

```
(define (good-maxlist xs)
  (if (null? xs)
      -inf.0
      (max (first xs) (good-maxlist (rest xs)))))
```

```
(define (max a b)
  (if (> a b) a b))
```


Your turn: sumProdList **Solution**

Given a list of numbers, `sumProdList` returns a **pair** of
(1) the sum of the numbers in the list and
(2) The product of the numbers in the list

```
(sumProdList '(5 2 4 3)) -> (14 . 120)
(sumProdList '()) -> (0 . 1)
```

Define `sumProdList`. Why is it a good idea to use `let` in your definition?

```
(define (sumProdList ns)
  (if (null? ns)
      '(0 . 1) ; (cons 0 1)
      (let {[sumProdRest (sumProdList (rest ns))]}
        (cons (+ (first ns) (car sumProdRest))
              (* (first ns) (cdr sumProdRest))))))
```


`and` and `or` sugar

`(and)` desugars to **`#t`**
`(and E1)` desugars to ***`E1`***
`(and E1 …)` desugars to **`(if E1 (and …) #f)`**

`(or)` desugars to **`#f`**
`(or E1)` desugars to ***`E1`***
`(or E1 …)` desugars to

```
(let ((Id1 E1))
  (if Id1 Id1 (or …))
```

where ***`Id1`*** must be **fresh** – i.e., not used elsewhere in the program.

- Why is `let` needed in `or` desugaring but not `and`?
- Why must ***`Id1`*** be fresh?

Scope and Lexical Contours

scope = area of program where declared name can be used.

Show scope in Racket via ***lexical contours*** in ***scope diagrams***.

```
(define add-n (λ ( x ) (+ n x )) )
(define add-2n (λ ( y ) (add-n (add-n y ))))
(define n 17)
(define f (λ ( z )
            (let {[ c (add-2n z ) ]
                  [ d (- z 3)     ]}
              (+ z (* c d )))    )    )
```

Declarations vs. References

A **declaration** introduces an identifier (variable) into a scope.

A **reference** is a use of an identifier (variable) within a scope.

We can box declarations, circle references, and draw a line from each reference to its declaration. Dr. Racket does this for us (except it puts ovals around both declarations and references).

An identifier (variable) reference is **unbound** if there is no declaration to which it refers.

Scope and Define Sugar

```
(define (add-n  x ) (+  n  x ) )
(define (add-2n y ) (add-n (add-n  y )) )
(define n 17)
(define (f z )
        (let {[ c (add-2n z ) ]
              [ d (- z 3)     ]}
          (+ z (* c d )))      )    )
```


Shadowing

An inner declaration of a name ***shadows*** uses of outer declarations of the same name.

```
(let {[x 2]}
  (- (let {[x (* x x)]}
       (+ x 3))   ; Can't refer to outer x here.
     x ))
```

Can't refer to outer x here.

Alpha-renaming

Can consistently rename identifiers as long as it doesn't change the "wiring diagram" between uses and declarations.

```
(define (f w z)
  (* w
     (let {[c (add-2n z)]
           [d (- z 3)]}
       (+ z (* c d))))))
```

OK

```
(define (f c d)
  (* c
     (let {[b (add-2n d)]
           [c (- d 3)]}
       (+ d (* b c))))))
```

Not OK (because y in (+ y ...) refers to let-bound y, not function parameter y.)

```
(define (f x y)
  (* x
     (let {[x (add-2n y)]
           [y (- y 3)]}
       (+ y (* x y))))))
```

Scope, Free Variables, and Higher-order Functions

In a lexical contour, an identifier is a ***free variable*** if it is not defined by a declaration within that contour.

Scope diagrams are especially helpful for understanding the meaning of free variables in higher order functions.

```
(define (make-sub n )
  (λ ( x ) (- x n)) )

(define (map-scale factor ns )
  (map (λ ( num ) (* factor num )) ns) )
```


Compare the Values of the Following **Solutions**

it's your turn

```
(let {[a (+ 2 3)] [b (* 3 4)]}
  (list a
        (let {[a (- b a)]
              [b (* a a)]} ; outer a
          (list a b))
        b)) ; outer a
⇒* '(5 (7 25) 12)
```

```
(let {[a (+ 2 3)] [b (* 3 4)]}
  (list a
        (let {[a (- b a)]}
          (let {[b (* a a)]} ; inner a
            (list a b)))
        b)) ; outer a
⇒* '(5 (7 49) 12)
```


More sugar: `let*`

`(let* {} Ebody)` desugars to ***`Ebody`***

`(let* {[Id1 E1] …} Ebody)`

desugars to `(let {[Id1 E1]}`
`(let* {…} Ebody))`

Example (same as 2nd example on previous slide)

```
(let {[a (+ 2 3)] [b (* 3 4)]}
  (list a
        (let* {[a (- b a)]
               [b (* a a)]}
          (list a b))
        b))
```


Local function bindings with `let`

- Silly example:

```
(define (quad x)
  (let ([square (lambda (x) (* x x))])
    (square (square x))))
```

- Private helper functions bound locally = good style.
- But can't use let for local recursion. Why not?

```
(define (up-to-broken x)
  (let {[between (lambda (from to)
                   (if (> from to)
                       null
                       (cons from
                             (between (+ from 1) to))))]}
    (between 1 x)))
```

?

`letrec` to the rescue!

```
(define (up-to x)
  (letrec {[between (lambda (from to)
                      (if (> from to)
                          null
                          (cons from
                                (between (+ from 1) to))))]}
    (between 1 x)))
```

In ***(letrec {[Id1 E1] ... [Idn E1]} Ebody)***, ***Id1*** … ***Idn*** are in the scope of ***E1*** … ***En*** .

Even Better

```
(define (up-to-better x)
  (letrec {[up-to-x (lambda (from)
                      (if (> from x)
                          null
                          (cons from
                                (up-to-x (+ from 1)))))]}
    (up-to-x 1)))
```

- Functions can use bindings in the environment where they are defined:
  - Bindings from "outer" environments
    - Such as parameters to the outer function
  - Earlier bindings in the let-expression
- Unnecessary parameters are usually bad style
  - Like **to** in previous example

Mutual Recursion with letrec

```
(define (test-even-odd num)
  (letrec {[even? (λ (x)
                    (if (= x 0)
                        #t
                        (odd? (- x 1))))]
           [odd? (λ (y)
                    (if (= y 0)
                        #f
                        (even? (- y 1))))]}
    (list (even? num) (odd? num))))
```

```
> (test-even-odd 42)
'(#t #f)

> (test-even-odd 17)
'(#f #t)
```


Exercise: `let` vs. `let*` vs. `letrec` Solutions

```
(let {[f (λ (x) (/ x 2))]
      [g (λ (y) (+ y 1))]
      [h (λ (a b) (+ a b))]}
  (let {[f (λ (y) (- y 1))]
        [g (λ (n)
             (if (<= n 0)
                 1
                 (h n (g (f n)))))]
        [h (λ (a b) (* a b))]}
    (list (f 10) (g 4) (h 2 3))))
```

- What is the value of the above expression? `'(9 7 6)`
- What is its value if the inner `let` is replaced by `let*`? `'(9 8 6)`
- What is its value if the inner `let` is replace by `letrec`? `'(9 24 6)`
  (in this case, g is the factorial function!)

Local definitions are sugar for `letrec`

The following internal **`define`**s desugar to the **`letrec`**s studied in previous slides

```
(define (up-to-alt x)
  (define (up-to-x from)
    (if (> from x)
        null
        (cons from
              (up-to-x (+ from 1)))))
  (up-to-x 1))

(define (test-even-odd num)
  (define (even? x)
    (if (= x 0) #t (not (odd? (- x 1)))))
  (define (odd? y)
    (if (= y 0) #f (not (even? (- y 1)))))
  (list (even? num) (odd? num)))
```


Nested functions: style

- Good style to define helper functions inside the functions they help if they are:
  - Unlikely to be useful elsewhere
  - Likely to be misused if available elsewhere
  - Likely to be changed or removed later
- A fundamental trade-off in code design: reusing code saves effort and avoids bugs, but makes the reused code harder to change later

Local Scope in other languages

Java

```java
public static int w = 2;
public static int x = 3;

public static int f (int y)
{
    int z;
    if (y > x) {
      z = y - x;
    } else {
      z = y * w;
    }
    w = y + z;
    return y * z;
}
```

JavaScript

```javascript
var w = 2;
var x = 3;

function f(y) {
  if (y > x) {
    var z = y - x;
  } else {
    var z = y * w;
  }
  w = y + z;
  return y * z;
}
```

Python

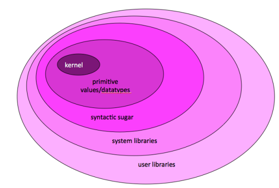
```python
w = 2
x = 3

def f(y):
    global w
    if y > x:
      z = y - x
    else:
      z = y * w
    w = y + z
    return y * z
```

In all 3 languages, f(8) returns 28
and a following f(10) returns 70

- Java requires z to be declared outside **`if`** if it's used in both branches, because each { … } defines a new scope. But in JavaScript and Python, any declaration has scope of entire funciton body regardless of where declaration is.
- Python uses = to both declare and re-assign, so needs **global** declaration when assigning to global variable.
- JavaScript and Python allow local function decls; Java has local class (not method) decls
- No `let`-like expression in Python/JavaScript, but can be simulated by calling local or anonymous function.

Racket Language Summary So Far



Racket kernel declarations:

- definitions: `(define` ***Id E*** `)`

Racket kernel expressions

- literal values (numbers, boolean, strings): e.g. `251`, `3.141`, `#t`, `"Lyn"`
- variable references: e.g., `x`, `fact`, `positive?`, `fib_n-1`
- conditionals: `(if` ***Etest Ethen Eelse*** `)`
- function values: `(lambda (`***Id1 … Idn***`)` ***Ebody*** `)`
- function calls: `(`***Erator Erand1 … Erandn***`)`
  Note: arithmetic and relational operations are *really* just function calls!
- **(new) local recursion**: `(letrec {[`***Id1 E1***`]` … `[`***Idn En***`]}` ***Ebody*** `)`

Racket Syntactic Sugar

- `(define (`***Idfun Id1 … Idn***`)` ***Ebody*** `)`
- `(and` ***E1 … E2*** `)`
- `(or` ***E1 … E2*** `)`
- `(let {[`***Id1 E1***`]` … `[`***Id1 E1***`]}` ***Ebody*** `)`
- `(let* {[`***Id1 E1***`]` … `[`***Id1 E1***`]}` ***Ebody*** `)`

Racket Built-in Functions

`+`, `-`, `*`, `/`, `min`, `max`, …
`<`, `<=`, `=`, `>=`, `>`,
`cons`, `car`, `cdr`,
`list`, `first`, `second`, …, `rest`